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Preface

This book explains how plane elastostatic crack problems may be formulated and solved in terms of hypersingular integral equations. The unknown functions in the hypersingular integral equations are the crack opening displacements. Once the hypersingular integral equations are solved, the crack tip stress intensity factors, which play an important role in fracture analysis, may be easily computed.

Chapter 1 gives a brief account on the linear theory of fracture mechanics and the importance of the crack tip stress intensity factors in predicting crack extension, lays down the mathematical equations in linear elasticity needed in subsequent chapters, and provides basic definitions of the Hadamard finite-part integrals which appear in hypersingular integral equations for crack problems.

Chapter 2 shows how Fourier integral representations for plane elastostatic displacements and stresses may be used to derive hypersingular integral equations for coplanar cracks in idealised elastic spaces. A detailed step by step derivation of the hypersingular integral equations is given for coplanar cracks in an anisotropic elastic full space under antiplane deformations. The analysis is extended to include generalised plane deformations, periodic coplanar cracks, and coplanar cracks in an infinitely long elastic slab and between two dissimilar elastic half spaces. Formulae for the stresses near the crack tips are also given in this chapter.

Chapter 3 presents two different numerical methods for solving a general system of hypersingular integral equations in linear crack problems. The first method approximates globally the unknown crack opening displacements over a crack by using Chebyshev polynomials of the second kind. In the second method, each of the cracks is discretised into small elements and the crack opening displacements are approximated locally over each of the elements using spatial functions of a relatively simple form.

Chapter 4 shows how the boundary integral equations in linear elasticity may be employed to obtain hypersingular boundary integral equations for the numerical solution of a plane elastostatic problem involving arbitrarily located planar cracks in a two-dimensional body of finite extent. The boundary integral equations are also used together with special Green's functions to derive hypersingular integral equations for arbitrarily located planar cracks in elastic spaces with certain idealised geometries.

Chapter 5 describes the numerical construction of an elastostatic Green's function for arbitrarily located traction free planar cracks in an elastic full space. The Green's function, which is constructed by solving numerically a suitable system of hypersingular integral equations for the cracks, is used to formulate a plane elastostatic problem involving planar cracks in a elastic body of finite extent in terms of boundary integral equations that do not contain any integral over the cracks.

Chapter 6 deals with the extension of the analyses in the earlier chapters to include edge and curved cracks and plane electro-elastostatic crack problems.

FORTRAN 77 programmes for generating the numerical results in Figure 5.1 and Tables 3.1, 3.2, 4.2, 6.1 and 6.2 are listed in full for the benefit of readers who may be interested in programming the numerical algorithms in this book.

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Singapore W. T. ANG
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