A boundary integral equation method for the two-dimensional diffusion equation subject to a nonlocal condition

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Abstract

A boundary integral equation method is proposed for the numerical solution of the two-dimensional diffusion equation subject to a nonlocal condition. The nonlocal condition is in the form of a double integral giving the specification of mass in a region which is a subset of the solution domain. A specific test problem is solved using the method.

KEY WORDS: boundary element method; diffusion equation; Laplace transform; nonlocal condition

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1 INTRODUCTION

In non-dimensionalised form, the partial differential equation that governs two-dimensional linear and isotropic diffusion processes is given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t}. \tag{1}$$

A class of problems of practical interest is to solve (1) for the unknown function $u(x, y, t)$ for time $t \geq 0$ in a two-dimensional region R (on the $0xy$ plane) subject to the initial and boundary conditions

$$u(x, y, 0) = f(x, y) \text{ for } (x, y) \in R, \tag{2}$$

$$u(x, y, t) = g(x, y, t) \text{ for } (x, y) \in C_1 \text{ and } t \geq 0, \tag{3}$$

$$u(x, y, t) = h(x, y)z(t) \text{ for } (x, y) \in C_2 \text{ and } t \geq 0, \tag{4}$$

$$\frac{\partial}{\partial n}\left[u(x, y, t)\right] = k(x, y, t) \text{ for } (x, y) \in C_3 \text{ and } t \geq 0, \tag{5}$$

and the non-local (integral) condition

$$\iint_S u(x, y, t)dxdy = m(t) \text{ for } t \geq 0. \tag{6}$$

where f, g, h, k and m are known and suitably prescribed functions, z is an unknown function to be determined, the region R is bounded by a simple closed curve C, the curves C_1, C_2 and C_3 are non-intersecting and such that $C_1 \cup C_2 \cup C_3 = C$, S is a given subregion of R that is independent of time t and is bounded by a simple closed curve D given by $D = C_2 \cup C_4$, the open curve C_4 lies completely in the interior of R, and $\partial u/\partial n = \mathbf{n} \cdot \nabla u$, $\mathbf{n}$ is the unit normal vector on C pointing away from R. From a physical standpoint, (6) specifies the total amount of mass of the diffusing quantity u (or the total amount of heat energy, in the case of heat diffusion) which the the region S can possess at any time t. Condition (4) with $z(t)$ being unknown implies that the concentration of the diffusing quantity (or the temperature) on some part of the boundary must be controlled in a certain way in order that the

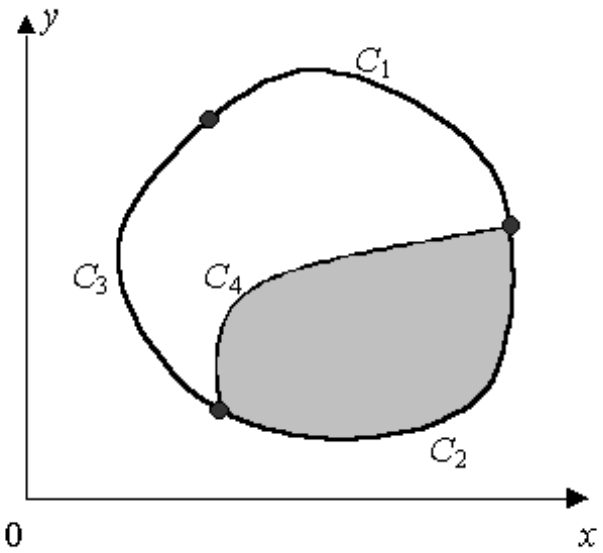

Figure 1: A sketch of the geometry of the problem. The solution domain is bounded by a simple closed curve C which comprises three distinct parts C_1, C_2 and C_3 over which different boundary conditions are given. The specification of mass (nonlocal condition) is given in the shaded region bounded by $C_2 \cup C_4$. The curve C_4 lies in the interior of the solution domain.

region S carries the required amount of mass (or heat energy). For a sketch of the geometry of the problem, refer to Figure 1.

The class of problems defined by (1)-(6) arises in many practical applications in heat transfer, control theory, thermoelasticity and medical sciences. A specific application which involves the use of the absorption of light to measure the concentration of a diffusing chemical is described in Noye and Dehghan [6]. Special cases of the problem, such as R being a rectangular region, have been solved directly by many researchers, e.g. Gumel, Ang and Twizell [5]; Noye and Dehghan [6, 7]; Noye, Dehghan and van der Hoek [8]; Cannon, Lin and Matheson [3]; Wang and Lin [11] using the finite-difference methods. With the exception of [6], the case $S = R$, i.e. $C_4 = C_1 \cup C_3$, was studied in all the references just cited.

The present paper makes use of a boundary integral equation method (BIEM) for the numerical solution of (1)-(6) in the Laplace transform (LT) space. The physical solution is recovered by using the Stehfest's algorithm [10] for the numerical inversion of the Laplace transformation. This approach should provide a useful and interesting alternative to the existing finite-difference methods. It can be easily implemented on the computer for solution domains of arbitrary shape. The BIEM for solving (1) with either u or $\partial u/\partial n$ completely specified at each and every point on the boundary of the solution domain is well established but continues to be a research subject of considerable interest, see e.g. Rizzo and Shippy [9] and Chen *et al* [4]. The application of the BIEM to diffusion problems with non-local conditions, such as the one defined by (1)-(6), would lead to a formulation involving unknowns at nodal points throughout the physical solution domain, if the non-local conditions are not properly treated. In the present paper, the LT of the non-local condition (6) is recast into a form which involves the LT of u at points (x, y) on only the boundary C. Consequently, the linear algebraic equations in the BIEM do not involve unknowns at points in the interior of the physical domain R.

2 FORMULATION IN LT SPACE

We shall reformulate the problem described in Section 1 by taking the LT of all functions and equations involved with respect to the time parameter t. To this end, let us define the LT operator $\mathcal{L}$ on a function $r(x, y, t)$ $(t \geq 0)$ by

$$\mathcal{L}\{r(x, y, t); t \to p\} \stackrel{\text{def}}{=} \int_0^\infty r(x, y, t) \exp(-pt) dt \tag{7}$$

where p is the LT parameter. In the present paper, we shall assume that p is real and positive.

Applying $\mathcal{L}$ on (1)-(6), we find that the problem in the LT space is then to solve

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - pU = -f(x, y) \tag{8}$$

for $U(x, y; p) = \mathcal{L}\{u(x, y, t); t \to p\}$ (for $(x, y) \in R$ and a suitably selected value of p) subject to

$$U(x, y; p) = G(x, y; p) \text{ for } (x, y) \in C_1, \tag{9}$$

$$U(x, y; p) = h(x, y)\Phi(p) \text{ for } (x, y) \in C_2, \tag{10}$$

$$\frac{\partial}{\partial n}\left[U(x, y; p)\right] = K(x, y; p) \text{ for } (x, y) \in C_3, \tag{11}$$

and

$$\iint_S U(x, y; p) dx dy = M(p), \tag{12}$$

where $G(x, y; p) = \mathcal{L}\{g(x, y, t); t \to p\}$, $\Phi(p) = \mathcal{L}\{z(t); t \to p\}$, $K(x, y; p) = \mathcal{L}\{k(x, y, t); t \to p\}$ and $M(p) = \mathcal{L}\{m(t); t \to p\}$. Notice that $\Phi(p)$ is an unknown function to be determined.

Using (8) and the divergence theorem, we find that (12) can be rewritten as

$$\int_{C_2 \cup C_4} \frac{\partial}{\partial n}\left[U(x, y; p)\right] ds = pM(p) - \iint_S f(x, y) dx dy. \tag{13}$$

Notice that the right hand side of (13) is known and the left hand side containing an unknown function is expressed in terms of a boundary integral.

To facilitate the task of solving (8)-(11) together with (13), we make the substitution

$$U(x,y;p) = U_{\text{part}}(x,y;p) + V(x,y;p), \tag{14}$$

where $U_{\text{part}}(x,y;p)$ is any particular solution of (8).

According to Atkinson [2], a particular solution of (8) is given by

$$U_{\text{part}}(x,y;p) = -\iint_T f(\xi,\eta)\Gamma(x,y;\xi,\eta;p)d\xi d\eta, \tag{15}$$

where $\Gamma(x,y;\xi,\eta;p)$ is given by (23) below (in Section 3) and T is a two-dimensional region which may be chosen to assume any specific geometry convenient for computing numerically the double integral as long as $R \subseteq T$. In the specific example considered in Section 5, we take $T = R$ because R is a simple square region.

With the substitution in (14), (8)-(11) and (13) become:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} - pV = 0 \tag{16}$$

$$V(x,y;p) = G(x,y;p) - U_{\text{part}}(x,y;p) \text{ for } (x,y) \in C_1, \tag{17}$$

$$V(x,y;p) = h(x,y)\Phi(p) - U_{\text{part}}(x,y;p) \text{ for } (x,y) \in C_2, \tag{18}$$

$$\frac{\partial}{\partial n}\left[V(x,y;p)\right] = K(x,y;p) - \frac{\partial}{\partial n}\left[U_{\text{part}}(x,y;p)\right] \text{ for } (x,y) \in C_3, \tag{19}$$

$$\int_{C_2 \cup C_4} \frac{\partial}{\partial n}\left[V(x,y;p)\right] ds = pM(p) - \iint_S f(x,y)dxdy - \int_{C_2 \cup C_4} \frac{\partial}{\partial n}\left[U_{\text{part}}(x,y;p)\right] ds, \tag{20}$$

Since U_{part} satisfies (8), the integral over $C_2 \cup C_4$ on the right hand side of (20) can be transformed back to a domain integral over S in order to rewrite (20) in the form

$$\int_{C_2 \cup C_4} \frac{\partial}{\partial n}\left[V(x,y;p)\right] ds = pM(p) - p\iint_S U_{\text{part}}(x,y;p)dxdy, \tag{21}$$

if one wishes to avoid evaluating $\partial[U_{\text{part}}(x,y;p)]/\partial n$ when dealing with the non-local condition.

3 BIEM

For $(\xi,\eta) \in R \cup C$, the standard boundary integral solution of the homogeneous modified Helmholtz equation (16) is given by (see, e.g. Rizzo and Shippy [9])

$$\lambda(\xi,\eta)V(\xi,\eta;p) = \int_C \{V(x,y;p)\frac{\partial}{\partial n}\left[\Gamma(x,y;\xi,\eta;p)\right] - \Gamma(x,y;\xi,\eta;p)\frac{\partial}{\partial n}\left[V(x,y;p)\right]\}ds, \quad (22)$$

where $\lambda(\xi,\eta) = 1$ if $(\xi,\eta) \in R$ and $0 < \lambda(\xi,\eta) < 1$ if $(\xi,\eta) \in C$ and

$$\Gamma(x,y;\xi,\eta;p) = -\frac{1}{2\pi}K_0(r\sqrt{p}) \quad (23)$$

where $r = \sqrt{(x-\xi)^2 + (y-\eta)^2}$ and K_0 is the modified Bessel function of the second kind and of order zero.

The boundary C is discretized by putting N closely-packed points on it. Two consecutive points are joined by straight line segments or elements $C^{(1)}$, $C^{(2)}$, ..., $C^{(N-1)}$ and $C^{(N)}$. We make the following approximations:

$$C \approx C^{(1)} \cup C^{(2)} \cup ... \cup C^{(N-1)} \cup C^{(N)}, \quad (24)$$

$$V(x,y;p) \approx V^{(k)} \text{ for } (x,y) \in C^{(k)} \ (k = 1,2,...,N), \quad (25)$$

$$\frac{\partial}{\partial n}\left[V(x,y;p)\right] \approx W^{(k)} \text{ for } (x,y) \in C^{(k)} \ (k = 1,2,...,N), \quad (26)$$

where $V^{(k)}$ and $W^{(k)}$ $(k = 1,2,...,N)$ are constants to be determined.

With (24)-(26), by letting (ξ,η) be the midpoint of $C^{(i)}$ given by $(\xi^{(i)},\eta^{(i)})$, (22) can be approximately rewritten as

$$\frac{1}{2}V^{(i)} = \sum_{k=1}^{N}\left\{V^{(k)}\int_{C^{(k)}}\frac{\partial}{\partial n}\left[\Gamma(x,y;\xi^{(i)},\eta^{(i)};p)\right]ds - W^{(k)}\int_{C^{(k)}}\Gamma(x,y;\xi^{(i)},\eta^{(i)};p)ds\right\} \text{ for } i = 1,2,...,N. \quad (27)$$

In (27), in taking $\lambda\left(\xi^{(i)},\eta^{(i)}\right)=1/2$, we assume that $\left(\xi^{(i)},\eta^{(i)}\right)$ lies on a smooth part of C.

Now if $\Phi(p)$ in (18) is given then from (17)-(19) either $V^{(k)}$ or $W^{(k)}$ (not both) is known over $C^{(k)}$. It follows that (27) constitutes a system of N linear algebraic equations in N unknowns. Since $\Phi(p)$ is not known, additional equations are needed to complete the system of linear algebraic equations. The additional equations are obtained as outlined below.

Discretize the curve C_4 into J straight line segments denoted by $C^{(N+1)}$, $C^{(N+2)}$, $\cdots$, $C^{(N+J-1)}$ and $C^{(N+J)}$ and make the approximation

$$\partial\left[V(x,y;p)\right]/\partial n \approx W^{(N+k)} \text{ for } (x,y)\in C^{(N+k)}(k=1,2,...,J), \tag{28}$$

where $W^{(N+k)}$ are constants to be determined and the normal derivative of V on $C^{(N+k)}$ (or C_4) is in the direction pointing away from S. If the boundary elements obtained by discretizing C_2 are labelled $C^{(1)}$, $C^{(2)}$, $\cdots$, $C^{(Q-1)}$ and $C^{(Q)}$ $(Q<N)$ then (21) may be used to give the approximation

$$\sum_{k=1}^{Q} W^{(k)}L^{(k)}+\sum_{k=1}^{J} W^{(N+k)}L^{(N+k)}=pM(p)-p\iint_{S} U_{\text{part}}(x,y;p)dxdy, \tag{29}$$

where $L^{(k)}$ is the length of the straight line $C^{(k)}$.

Although (29) caters for the unknown parameter $\Phi(p)$, it contains an additional J unknown constants $W^{(N+k)}$ $(k=1,2,\cdots,J)$. More equations are required. Applying the standard boundary integral equation in (22) for the regions S and $R\backslash S$ respectively, and letting (ξ,η) be the midpoint of

$C^{(N+i)}$ $(i = 1, 2, \cdots, J)$, we obtain

$$\begin{aligned}\frac{1}{2}V^{(N+i)} &= \sum_{k=1}^{Q}\left\{V^{(k)}\int_{C^{(k)}}\frac{\partial}{\partial n}\left[\Gamma(x,y;\xi^{(N+i)},\eta^{(N+i)};p)\right]ds\right. \\ &\quad \left.-W^{(k)}\int_{C^{(k)}}\Gamma(x,y;\xi^{(N+i)},\eta^{(N+i)};p)ds\right\} \\ &\quad +\sum_{k=1}^{J}\left\{V^{(N+k)}\int_{C^{(N+k)}}\frac{\partial}{\partial n}\left[\Gamma(x,y;\xi^{(N+i)},\eta^{(N+i)};p)\right]ds\right. \\ &\quad \left.-W^{(N+k)}\int_{C^{(N+k)}}\Gamma(x,y;\xi^{(N+i)},\eta^{(N+i)};p)ds\right\} \quad \text{for } i=1,2,...,J,\end{aligned} \tag{30}$$

and

$$\begin{aligned}\frac{1}{2}V^{(N+i)} &= \sum_{k=Q+1}^{N}\left\{V^{(k)}\int_{C^{(k)}}\frac{\partial}{\partial n}\left[\Gamma(x,y;\xi^{(N+i)},\eta^{(N+i)};p)\right]ds\right. \\ &\quad \left.-W^{(k)}\int_{C^{(k)}}\Gamma(x,y;\xi^{(N+i)},\eta^{(N+i)};p)ds\right\} \\ &\quad -\sum_{k=1}^{J}\left\{V^{(N+k)}\int_{C^{(N+k)}}\frac{\partial}{\partial n}\left[\Gamma(x,y;\xi^{(N+i)},\eta^{(N+i)};p)\right]ds\right. \\ &\quad \left.-W^{(N+k)}\int_{C^{(N+k)}}\Gamma(x,y;\xi^{(N+i)},\eta^{(N+i)};p)ds\right\} \quad \text{for } i=1,2,...,J,\end{aligned} \tag{31}$$

where $V^{(N+i)}$ is the approximate constant value of $V(x,y)$ over $C^{(N+i)}$ and $(\xi^{(N+i)},\eta^{(N+i)})$ is the midpoint of $C^{(N+i)}$.

Thus, after employing the boundary conditions (17)-(19), the system of linear algebraic equations (27)-(31) can be solved for the unknowns which include $\Phi(p)$, $V^{(i)}$ and $W^{(i)}$ for $i = 1, 2, \cdots, Q$ and also for $i = N+1, N+2, \cdots, N+J$, and either $V^{(i)}$ or $W^{(i)}$ for $i = Q+1, Q+2, \cdots, N$. Notice that if the boundary condition (17) or (19) holds over $C^{(i)}$ then $V^{(i)}$ or $W^{(i)}$ is known respectively. This is why either $V^{(i)}$ or $W^{(i)}$ (not both) for

$i = Q+1, Q+2, \cdots, N$ are unknowns. (Boundary elements where (17) or (19) holds are labelled $C^{(Q+1)}$, $C^{(Q+2)}, \cdots, C^{(N-1)}$ and $C^{(N)}$.)

The derivation above is for $S \subset R$. What about the special case where $S = R$? For this special case, (29)-(31) can be replaced by just a single equation, i.e.

$$\sum_{k=1}^{N} W^{(k)} L^{(k)} = pM(p) - p \iint_R U_{\text{part}}(x, y; p) dx dy, \tag{32}$$

and the unknowns are $\Phi(p)$, $V^{(i)}$ and $W^{(i)}$ for $i = 1, 2, \cdots, Q$ and either $V^{(i)}$ or $W^{(i)}$ for $i = Q+1, Q+2, \cdots, N$.

Once $V^{(k)}$ and $W^{(k)}$ are determined, we can compute $V(x, y; p)$ at any interior point (ξ, η) approximately via

$$\begin{aligned} V(\xi, \eta; p) \approx \sum_{k=1}^{N} \Bigg\{ & V^{(k)} \int_{C^{(k)}} \frac{\partial}{\partial n} \left[\Gamma(x, y; \xi, \eta; p) \right] ds \\ & - W^{(k)} \int_{C^{(k)}} \Gamma(x, y; \xi, \eta; p) ds \Bigg\}. \end{aligned} \tag{33}$$

4 LT INVERSION

The physical solution $u(x, y, t)$ and $z(t)$ can be recovered approximately from $U(x, y; p)$ and $\Phi(p)$ by using a LT inversion technique. According to the Stehfest's algorithm [10] which is nowadays increasingly used in applied mechanics for the numerical inversion of LT (e.g. Ang [1]), we obtain

$$\begin{aligned} u(x, y, t) &\approx \frac{\ln(2)}{t} \sum_{n=1}^{2P} c_n U\left(x, y; \frac{n \ln(2)}{t}\right), \\ z(t) &\approx \frac{\ln(2)}{t} \sum_{n=1}^{2P} c_n \Phi\left(\frac{n \ln(2)}{t}\right), \end{aligned} \tag{34}$$

where P is a positive integer and

$$c_n = (-1)^{n+P} \sum_{m=[(n+1)/2]}^{\min(n,P)} \frac{m^P (2m)!}{(P-m)! m! (m-1)! (n-m)! (2m-n)!}, \tag{35}$$

where $[r]$ denotes the integer part of the real number r.

As numerical techniques for inverting Laplace transform are highly susceptible to round-off errors, P cannot be selected to be as large as we like. On the other hand, choosing P to be too small may yield numerical results of lower accuracy. The optimum choice of P depends on the arithmetical precision of the computer (Stehfest [10]). Perhaps the best way to choose the optimum P is through testing the computer code of (34) on inverting known Laplace transforms of some elementary test functions. For the test problem in Section 5 below, programming in double precision using the Fortran 77 language on a Pentium III 450 MHz (64 Mb SDRAM) desktop computer, we find that to achieve satisfactory results the range of P which may be used is from 4 to 6, i.e. between 8 and 12 terms may be used in the approximation (34). With better machines, it may be possible to use a higher value of P, e.g. $P = 10$, as reported in (Stehfest [10]).

5 A TEST PROBLEM

For testing the BIEM, we construct a problem involving a square region R given by $0 < x < 1$, $0 < y < 1$. The region S where the specification of mass is given is

$$S = \{(x, y) : y < x(1 - x),\ x \geq 0,\ y \geq 0\}. \tag{36}$$

The initial-boundary conditions and the non-local condition are:

$$u(x,y,0) = \sin(\frac{\pi}{2}\,[x+y]) + \exp(\frac{\pi x}{2})\sin(\frac{\pi y}{2}) \text{ for } (x,y) \in R, \tag{37}$$

$$u(0,y,t) = [1+\exp(-\frac{\pi^2 t}{2})]\sin(\frac{\pi y}{2}) \text{ for } 0<y<1, \tag{38}$$

$$u(1,y,t) = \exp(-\frac{\pi^2 t}{2})\sin(\frac{\pi}{2}\,[1+y]) + \exp(\frac{\pi}{2})\sin(\frac{\pi y}{2}) \text{ for } 0<y<1, \tag{39}$$

$$u(x,1,t) = \exp(-\frac{\pi^2 t}{2})\sin(\frac{\pi}{2}\,[x+1]) + \exp(\frac{\pi x}{2}) \text{ for } 0<x<1, \tag{40}$$

$$u(x,0,t) = \sin(\frac{\pi x}{2})z(t) \text{ for } 0<x<1, \tag{41}$$

$$\iint\limits_S u(x,y,t)dxdy = \alpha\exp(-\frac{\pi^2 t}{2}) + \beta, \tag{42}$$

where

$$\begin{aligned} \alpha &= \int_0^1\int_0^{x(1-x)} \sin(\frac{\pi}{2}\,(x+y))dydx \approx 0.126\,280\,295\,9, \\ \beta &= \int_0^1\int_0^{x(1-x)} \exp(\frac{\pi x}{2})\sin(\frac{\pi y}{2})dydx \approx 5.\,941796\,306\times 10^{-2}. \end{aligned} \tag{43}$$

The exact solution for the specific problem above is given by

$$u(x,y,t) = \exp(-\frac{\pi^2 t}{2})\sin(\frac{\pi}{2}\,[x+y]) + \exp(\frac{\pi x}{2})\sin(\frac{\pi y}{2}) \tag{44}$$

with

$$z(t) = \exp(-\frac{\pi^2 t}{2}). \tag{45}$$

For the purpose of executing the BIEM, following (15), we take a particular solution of (8) to be given by

$$\begin{aligned} U_{\text{part}}(x,y;p) = -\int_0^1\int_0^1 &[\sin(\frac{\pi}{2}\,[\xi+\eta]) + \exp(\frac{\pi\xi}{2})\sin(\frac{\pi\eta}{2})] \\ &\times \Gamma(x,y;\xi,\eta;p)d\xi d\eta \end{aligned} \tag{46}$$

which can be evaluated using a numerical quadrature. The integrand in (46) behaves like $\ln(r)$ for small r, where r is the distance separating (x,y)

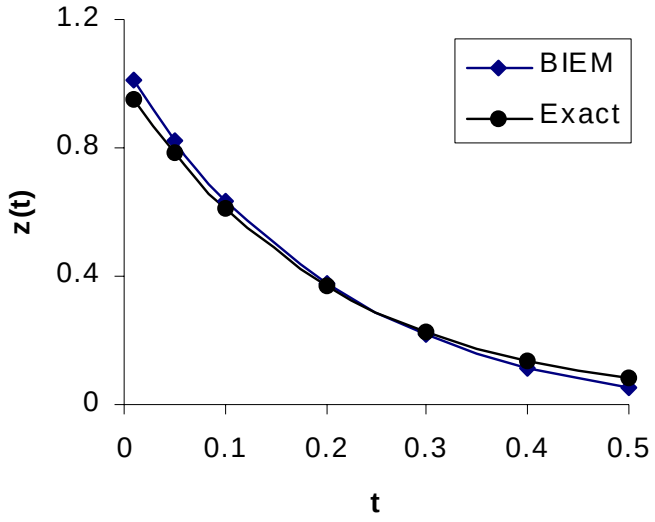

Figure 2: A graphical comparison of the numerical and exact $z(t)$.

and (ξ, η). From numerical experiments, we found that integrals with such integrands may be evaluated with reasonably good accuracy by merely using a simple midpoint rule, provided that the domain of integration is finely discretized into many square cells. Thus, the midpoint rule is applied to compute (46).

Discretizing the boundary of the square region into 80 equal length elements (each of length 0.05 units) and the curve $y = x(1-x)$ for $x \in [0,1]$ into 20 elements with endpoints given by $(k/20, k[20-k]/400)$ for $k = 0, 1, 2, \cdots, 20$, we apply the BIEM to solve the specific problem described above in the LT space. We use $P = 4$ in (34) to recover the solution in the physical space. The numerical results for the function $z(t)$ thus obtained for $t \in [0, 1/2]$ are graphed and compared in Figure 2 with the exact solution given by (45). As can be seen in Figure 2, there is a reasonably good agreement between the numerical and the exact values of z. For $t = 1/10$, we also compute u numerically at selected points (x, y) in the interior of the square

(solution) domain. The numerical values are compared and found to agree well with the exact ones given by (44). Refer to Table 1. When the number of elements used in the computation is doubled or trebled, an improvement in the accuracy of the numerical results may be observed.

Table 1: Numerical results for $u(x, y, 1/10)$ at selected points (x, y) in the interior of the solution domain.

Point (x, y)	BIEM	Exact
$(0.50, 0.50)$	2.170	2.161
$(0.30, 0.80)$	2.129	2.127
$(0.20, 0.30)$	1.059	1.053
$(0.70, 0.10)$	1.067	1.050
$(0.10, 0.10)$	0.3745	0.3717
$(0.90, 0.80)$	4.194	4.187

6 CONCLUSION

The problem of solving the two-dimensional diffusion equation subject to a nonlocal condition in the form of an integral taken over part of the solution domain is formulated in the LT space. A simple BIEM is then proposed for the numerical solution of the problem in the LT space. The physical solution is recovered through the use of a numerical technique for inverting LT. Such a method of solution is used to solve a specific problem which has a known exact solution. The numerical results obtained agree well with the exact solution.

In the proposed method, the nonlocal condition in the LT space is recast in such a way that the LT of the unknown function u (in the diffusion equation) appears only in the integrand of a boundary integral. Although domain integrals do appear in the integral formulation, their integrands do not contain unknown functions. Thus, the manipulation of unknown data is carried out on only the boundary of the solution domain, as in a typical BIEM. This makes the proposed method easier to implement on the computer, compared with other techniques such as the finite difference and the finite element

methods. It is particularly suitable for problems having solution domains that are irregular in shape.

The execution speed of the BIEM can be greatly improved if the computation for the different values of the LT parameter p is carried out in parallel using a computer with multiple processors.

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