# Magnetoelectroelastodynamic interaction of multiple arbitrarily oriented planar cracks*

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**Abstract**

The problem of multiple arbitrarily oriented planar cracks in an infinite magnetoelectroelastic space under dynamic loadings is considered. An explicit solution to the problem is given in the Laplace transform domain in terms of suitable exponential Fourier integral representations. The unknown functions in the Fourier integrals are directly related to the Laplace transform of the jumps in the displacements, electric potential and magnetic potential across opposite crack faces and are to be determined by solving a system of hypersingular integral equations. Once the hypersingular integral equations are solved, the displacements, electric potential, magnetic potential and other quantities of interest such as the crack tip intensity factors may be easily computed in the Laplace transform domain and recovered in the physical space with the help of a suitable algorithm for inverting Laplace transforms.

*Keywords*: multiple cracks, transient loads, magnetoelectroelastic materials, hypersingular integral equations

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# 1 Introduction

In recent years, there has been considerable interest in the analysis of cracks in magnetoelectroelastic solids. Most of the analytic or semi-analytic solutions for magnetoelectroelastic cracks are for static loadings and specific crack configurations. For example, Wang and Mai [12] had considered a single planar crack in magnetoelectroelastic solids and Gao *et al* [5], Li and Lee [9] and Zhong [15] had derived solutions for collinear cracks.

It appears that there are relatively few solutions of magnetoelectroelastodynamic crack problems especially for inplane deformations. Li [8] reduced the solution of a dynamic problem of a single permeable planar crack in an infinite transversely isotropic magnetoelectroelastic material under pure electric load and antiplane deformations to solving a Fredholm integral solution of the second kind in the Laplace transform domain. Using Laplace and Fourier transforms, Zhong *et al* [16]-[17] had obtained solutions for a single planar crack and a pair of coplanar cracks in an infinite magnetoelectroelastic space under impact loadings. Rojas-Diaz *et al* [10] and Wünsche *et al* [14] had presented hypersingular traction boundary element solutions for dynamic planar cracks in a magnetoelectroelastic solid under inplane deformations. The boundary element approach requires the derivation of fundamental solutions for the governing partial equations. For static problems, the fundamental solution is given in terms of simple elementary functions (Hong and Chen [6] and Chen and Hong [4]). Nevertheless, the fundamental solution for dynamic problems is rather complicated to derive, as it is expressed in terms of a line integral over a unit circle with integrand in the form of exponential integrals (Wang and Zhang [13]).

In the current paper, a semi-analytic solution is given for an arbitrary number of arbitrarily oriented planar cracks in a magnetoelectroelastic full-space under dynamic loadings. The cracks are assumed to open up by internal stresses and are either electrically impermeable or permeable and either magnetically impermeable or permeable. The displacements, electric potential and magnetic potential in the Laplace transform domain are expressed in terms of suitably constructed exponential Fourier transform representations. The Fourier integrals contain unknown functions that are directly related to the jumps in the Laplace transforms of the displacements, electrical potential and magnetic potential across opposite crack faces. The unknown functions are to be determined by solving a system of hypersingular integral equations. Once they are determined, the displacements, electric potential, magnetic potential and other physical quantities of interest, such as the crack tip stress and electric displacement intensity factors, may be easily computed in the Laplace transform domain and recovered in the physical space by using a suitable algorithm for inverting Laplace transforms. The crack tip stress, electric displacement and magnetic induction intensity factors are computed for some specific cases of the problem. For cases involving a single planar crack and two coplanar cracks, the values of the stress, electric displacement and magnetic induction intensity factors computed are compared with those in the literature.

# 2 A magnetoelectroelastic crack problem

With reference to an $Ox_1x_2x_3$ Cartesian coordinate system, consider an infinite magnetoelectroelastic space with $N_0$ arbitrarily oriented non-intersecting planar cracks with geometries that do not change along the $x_3$ axis. The cracks are denoted by $\Gamma^{(1)}$, $\Gamma^{(2)}$, $\cdots$, $\Gamma^{(N_0-1)}$ and $\Gamma^{(N_0)}$. The $n$-th planar crack $\Gamma^{(n)}$ (as sketched in Figure 1) lies in the region

$$-\ell^{(n)} < a_{j1}^{(n)}(x_j - c_j^{(n)}) < \ell^{(n)},\ a_{j2}^{(n)}(x_j - c_j^{(n)}) = 0,\ \ -\infty < x_3 < \infty, \qquad (1)$$

where

$$[a_{ij}^{(n)}] = \begin{pmatrix} \sin(\theta^{(n)}) & \cos(\theta^{(n)}) & 0 \\ -\cos(\theta^{(n)}) & \sin(\theta^{(n)}) & 0 \\ 0 & 0 & 1 \end{pmatrix}. \qquad (2)$$

Note that the Einstenian convention of summing over a repeated index holds here for lowercase Latin subscripts from 1 to 3.

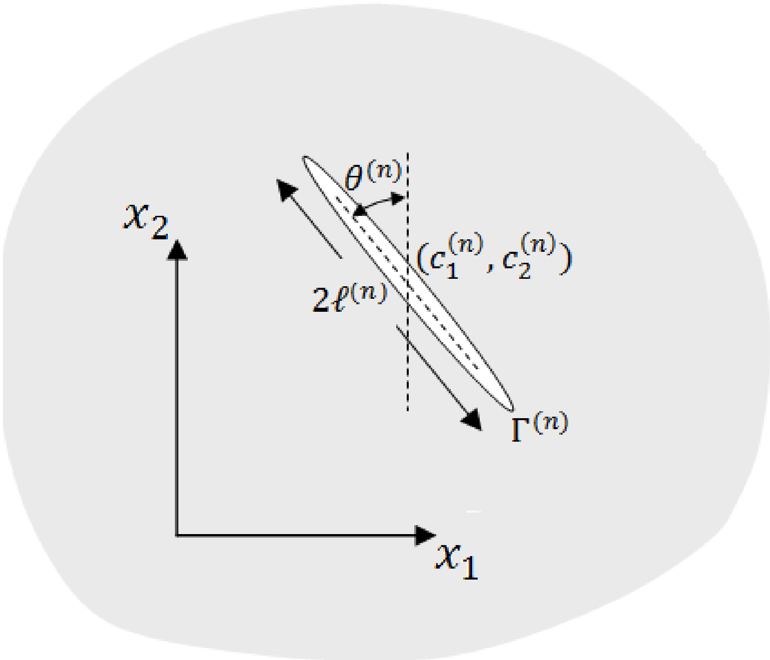



Figure 1: Parameters defining the $n$-th crack $\Gamma^{(n)}$.

The displacements $u_i$, electric potential $\phi$ and magnetic potential $\varphi$ and the corresponding stresses $\sigma_{ij}$, electric displacements $d_i$ and magnetic inductions $b_i$ in the magnetoelectroelastic space are assumed to be independent of the $x_3$ coordinate.

The cracks are assumed to open up under the action of suitably prescribed internal tractions, that is,

$$\sigma_{kj}(x_1, x_2, t)m_j^{(n)} \rightarrow -P_k^{(n)}(\xi_1, \xi_2, t) \ (k = 1, 2, 3)$$
$$\text{as } (x_1, x_2) \rightarrow (\xi_1, \xi_2) \in \Gamma^{(n)}(n = 1, 2, \cdots, N_0), \quad (3)$$

The electric and magnetic conditions on the cracks are given by either

$$\begin{aligned} d_j(x_1,x_2,t)m_j^{(n)} &\to -P_4^{(n)}(\xi_1,\xi_2,t) \\ &\quad \text{as } (x_1,x_2)\to(\xi_1,\xi_2)\in\Gamma^{(n)}(n=1,2,\cdots,N_0) \\ &\quad \text{if the cracks are electrically impermeable,} \end{aligned} \tag{4}$$

or

$$\begin{aligned} \Delta\phi(x_1,x_2,t)\to 0 \text{ as } (x_1,x_2)\to(\xi_1,\xi_2)\in\Gamma^{(n)}\ (n=1,2,\cdots,N_0) \\ \text{if the cracks are electrically permeable,} \end{aligned} \tag{5}$$

and either

$$\begin{aligned} b_j(x_1,x_2,t)m_j^{(n)} &\to -P_5^{(n)}(\xi_1,\xi_2,t) \\ &\quad \text{as } (x_1,x_2)\to(\xi_1,\xi_2)\in\Gamma^{(n)}(n=1,2,\cdots,N_0) \\ &\quad \text{if the cracks are magnetically impermeable,} \end{aligned} \tag{6}$$

or

$$\begin{aligned} \Delta\varphi(x_1,x_2,t)\to 0 \text{ as } (x_1,x_2)\to(\xi_1,\xi_2)\in\Gamma^{(n)}\ (n=1,2,\cdots,N_0) \\ \text{if the cracks are magnetically permeable,} \end{aligned} \tag{7}$$

where $P_1^{(n)}(\xi_1,\xi_2,t)$, $P_2^{(n)}(\xi_1,\xi_2,t)$, $P_3^{(n)}(\xi_1,\xi_2,t)$, $P_4^{(n)}(\xi_1,\xi_2,t)$, $P_5^{(n)}(\xi_1,\xi_2,t)$ are suitably prescribed functions for $(\xi_1,\xi_2)\in\Gamma^{(n)}$, $m_i^{(n)}=-a_{i2}^{(n)}$ are the components of a unit magnitude normal vector to the crack $\Gamma^{(n)}$ and $\Delta\phi(x_1,x_2)$ and $\Delta\varphi(x_1,x_2)$ respectively denote the jump in the electrical potential $\phi$ and magnetic potential $\varphi$ across the crack $\Gamma^{(n)}$ as defined by

$$\begin{aligned} \Delta\phi(x_1,x_2,t) &= \lim_{\varepsilon\to 0}[\phi(x_1-|\varepsilon|m_1^{(n)},x_2-|\varepsilon|m_2^{(n)},t) \\ &\quad -\phi(x_1+|\varepsilon|m_1^{(n)},x_2+|\varepsilon|m_2^{(n)},t)] \\ &\qquad \text{for } (x_1,x_2)\in\Gamma^{(n)}. \end{aligned}$$

$$\begin{aligned}\Delta\varphi(x_1,x_2,t) &= \lim_{\varepsilon\to 0}[\varphi(x_1-|\varepsilon|m_1^{(n)},x_2-|\varepsilon|m_2^{(n)},t) \\ &\quad -\varphi(x_1+|\varepsilon|m_1^{(n)},x_2+|\varepsilon|m_2^{(n)},t)] \\ &\qquad \text{for } (x_1,x_2)\in\Gamma^{(n)}. \end{aligned} \tag{8}$$

Furthermore, it is required that the displacements $u_k$ and its first order partial derivative with respect to time are both zero at time $t = 0$ and that $\sigma_{ij}(x_1, x_2, t)$, $d_i(x_1, x_2, t)$ and $b_i(x_1, x_2, t)$ vanish as $x_1^2 + x_2^2$ tends to infinity.

The governing partial differential equations for plane magnetoelectroelastostatic problems involving a homogeneous solid are given (in compact form) by

$$C_{IjK\ell}\frac{\partial^2 U_K}{\partial x_j \partial x_\ell} = B_{IK}\frac{\partial^2 U_K}{\partial t^2} \ (I = 1,2,3,4,5), \tag{9}$$

with

$$U_J = \begin{cases} u_j & \text{for } J = j = 1,2,3, \\ \phi & \text{for } J = 4, \\ \varphi & \text{for } J = 5, \end{cases}$$

$$C_{IjK\ell} = \begin{cases} c_{ijk\ell} & \text{for } I = i = 1,2,3 \text{ and } K = k = 1,2,3, \\ e_{\ell ij} & \text{for } I = i = 1,2,3 \text{ and } K = 4, \\ h_{\ell ij} & \text{for } I = i = 1,2,3 \text{ and } K = 5, \\ e_{jk\ell} & \text{for } I = 4 \text{ and } K = k = 1,2,3, \\ -\kappa_{jp} & \text{for } I = 4 \text{ and } K = 4, \\ -\beta_{jp} & \text{for } I = 4 \text{ and } K = 5, \\ h_{jk\ell} & \text{for } I = 5 \text{ and } K = k = 1,2,3, \\ -\beta_{jp} & \text{for } I = 5 \text{ and } K = 4, \\ -\gamma_{jp} & \text{for } I = 5 \text{ and } K = 5, \end{cases}$$

$$B_{IK} = \begin{cases} \rho & \text{if } I = K \text{ and } I \neq 4,5 \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

where $c_{ijk\ell}$, $e_{\ell ij}$,$\kappa_{i\ell}$, $h_{\ell ij}$, $\beta_{i\ell}$, and $\gamma_{i\ell}$ are the constant elastic moduli (N/m$^2$), piezoelectric coefficient (C/m$^2$), dielectric coefficient (C$^2$/Nm$^2$), piezomagnetic coefficient (N/Am), magnetoelectric coefficient (Ns/VC) and magnetic-permeability coefficient (Ns$^2$/C$^2$) respectively and $\rho$ is the density (kg/m$^3$), uppercase Latin subscripts (such as $I$ and $K$) have values 1, 2, 3, 4 and 5, and the Einsteinian convention of summing over a repeated is also assumed for uppercase Latin subscripts.

If the generalized stresses $S_{Ij}$ are defined by

$$S_{Ij} = \begin{cases} \sigma_{ij} & \text{for } I = i = 1, 2, 3, \\ d_j & \text{for } I = 4, \\ b_j & \text{for } I = 5, \end{cases} \tag{11}$$

then the linear constitutive equations for magnetoelectroelasticity are given by

$$S_{Ij} = C_{IjK\ell} \frac{\partial U_K}{\partial x_\ell}. \tag{12}$$

Application of the Laplace transformation on (9) together with the initial conditions stated below (8) gives

$$C_{IjK\ell} \frac{\partial^2 \widehat{U}_K}{\partial x_j \partial x_\ell} - s^2 B_{IK} \widehat{U}_K(x_1, x_2, s) = 0 \ \ (I = 1, 2, 3, 4, 5), \tag{13}$$

where $\widehat{U}_K(x_1, x_2, s)$ denotes the Laplace transform of $U_K(x_1, x_2, t)$ and $s$ is the Laplace transform parameter (assumed to be real here).

The problem of interest is to solve the partial differential equations (13) subject to the Laplace transforms of the boundary conditions (3)-(7) and the far-field conditions stated below (8).

# 3 Solution in Laplace transform domain

Extending the analysis in Ang and Athanasius [2] for the electroelastodynamic analysis of multiple planar cracks in a piezoelectric space to the magnetoelectroelastic crack problem in Section 2, we find that the magnetoelectroelastic fields in the Laplace transform domain may be expressed in terms of exponential Fourier integrals as given by

$$\begin{aligned}
&\widehat{U}_K(x_1,x_2,s)\\
=&\sum_{n=1}^{N_0}\mathrm{Re}\{\sum_{\alpha=1}^{5}\int_0^\infty A_{K\alpha}^{(n)}(\xi,s)[H(a_{r2}^{(n)}(x_r-c_r^{(n)}))M_{\alpha P}^{(n)}(\xi,s)\psi_P^{(n)}(\xi,s)\\
&\times\exp(i\xi(a_{j1}^{(n)}+\tau_\alpha^{(n)}(\xi,s)a_{j2}^{(n)})(x_j-c_j^{(n)}))\\
&+H(-a_{r2}^{(n)}(x_r-c_r^{(n)}))M_{\alpha P}^{(n)}(\xi,s)\overline{\psi}_P^{(n)}(\xi,s)\\
&\times\exp(-i\xi(a_{j1}^{(n)}+\tau_\alpha^{(n)}(\xi,s)a_{j2}^{(n)})(x_j-c_j^{(n)}))]d\xi\},
\end{aligned}\tag{14}$$

and

$$\begin{aligned}
&\widehat{S}_{Ij}(x_1,x_2,s)\\
=&\sum_{n=1}^{N_0}\mathrm{Re}\{\sum_{\alpha=1}^{5}\int_0^\infty i\xi L_{Ij\alpha}^{(n)}(\xi,s)[H(a_{r2}^{(n)}(x_r-c_r^{(n)}))M_{\alpha P}^{(n)}(\xi,s)\psi_P^{(n)}(\xi,s)\\
&\times\exp(i\xi(a_{j1}^{(n)}+\tau_\alpha^{(n)}(\xi,s)a_{j2}^{(n)})(x_j-c_j^{(n)}))\\
&-H(-a_{r2}^{(n)}(x_r-c_r^{(n)}))M_{\alpha P}^{(n)}(\xi,s)\overline{\psi}_P^{(n)}(\xi,s)\\
&\times\exp(-i\xi(a_{j1}^{(n)}+\tau_\alpha^{(n)}(\xi,s)a_{j2}^{(n)})(x_j-c_j^{(n)}))]d\xi\}.
\end{aligned}\tag{15}$$

where $i=\sqrt{-1}$, the overhead bar denotes the complex conjugate of a complex number, $\widehat{S}_{Ij}(x_1,x_2,s)$ is the Laplace transform of $S_{Ij}(x_1,x_2,t)$, $H(x)$ is the unit-step Heaviside function, $\tau_\alpha^{(n)}(\xi,s)$ $(n=1,\ 2,\ \cdots,\ N_0)$ are roots, with positive imaginary parts, of the 10-th order polynomial equation (in $\tau$) given by

$$\begin{aligned}
&\det[\frac{s^2}{\xi^2}B_{IK}+(a_{11}^{(n)}+\tau a_{12}^{(n)})^2C_{I1K1}\\
&+(a_{21}^{(n)}+\tau a_{22}^{(n)})(a_{11}^{(n)}+\tau a_{12}^{(n)})(C_{I1K2}+C_{I2K1})\\
&\qquad+(a_{21}^{(n)}+\tau a_{22}^{(n)})^2C_{I2K2}]=0,
\end{aligned}\tag{16}$$

$A_{K\alpha}^{(n)}(\xi,s)$ $(n=1,\,2,\,\cdots,\,N_0)$ are non-trivial solutions of the system

$$
\begin{aligned}
&[\frac{s^2}{\xi^2}B_{IK}+(a_{11}^{(n)}+\tau_\alpha^{(n)}(\xi,s)a_{12}^{(n)})^2C_{I1K1}\\
&+(a_{21}^{(n)}+\tau_\alpha^{(n)}(\xi,s)a_{22}^{(n)})(a_{11}^{(n)}+\tau_\alpha^{(n)}(\xi,s)a_{12}^{(n)})(C_{I1K2}+C_{I2K1})\\
&+(a_{21}^{(n)}+\tau_\alpha^{(n)}(\xi,s)a_{22}^{(n)})^2C_{I2K2}]A_{K\alpha}^{(n)}=0, \qquad (17)
\end{aligned}
$$

$L_{Ij\alpha}^{(n)}(\xi,s)$ are given by

$$
\begin{aligned}
L_{Ij\alpha}^{(n)}(\xi,s) \;&=\; [(a_{11}^{(n)}+\tau_\alpha^{(n)}(\xi,s)a_{12}^{(n)})C_{IjK1}\\
&\quad+(a_{21}^{(n)}+\tau_\alpha^{(n)}(\xi,s)a_{22}^{(n)})C_{IjK2}]A_{K\alpha}^{(n)}, \qquad (18)
\end{aligned}
$$

$M_{\alpha P}^{(n)}(\xi,s)$ are defined by

$$
\sum_{\alpha=1}^{5}m_j^{(n)}L_{Ij\alpha}^{(n)}(\xi,s)M_{\alpha P}^{(n)}(\xi,s)=\delta_{IP}\;(n=1,2,\cdots,N_0), \qquad (19)
$$

$\delta_{IP}$ is the kronecker-delta and $\psi_P^{(n)}(\xi,s)$ are given by

$$
\psi_P^{(n)}(\xi,s)=iT_{PJ}^{(n)}(\xi,s)\int_{-\ell^{(n)}}^{\ell^{(n)}}r_J^{(n)}(v,s)\exp(-i\xi v)dv, \qquad (20)
$$

with $T_{PJ}^{(n)}(\xi,s)$ being implictly defined by

$$
i\sum_{\alpha=1}^{5}[A_{K\alpha}^{(n)}(\xi,s)M_{\alpha P}^{(n)}(\xi,s)-\overline{A}_{K\alpha}^{(n)}(\xi,s)\overline{M}_{\alpha P}^{(n)}(\xi,s)]T_{PJ}^{(n)}(\xi,s)=\delta_{KJ}, \qquad (21)
$$

and $r_J^{(n)}(v,s)$ being related to the Laplace transform of the generalized crack opening displacement $\Delta U_K(x_1,x_2,t)$ by

$$
\begin{aligned}
r_K^{(n)}(a_{j1}^{(n)}(x_j-c_j^{(n)}),s) \;&=\; \frac{1}{\pi}\Delta\widehat{U}_K(x_1,x_2,s)\\
\text{for }-\ell^{(n)} \;&<\; a_{j1}^{(n)}(x_j-c_j^{(n)})<\ell^{(n)},\; a_{j2}^{(n)}(x_j-c_j^{(n)})=0. \qquad (22)
\end{aligned}
$$

For electrically and magnetically impermeable cracks with conditions given by (4) and (6), the functions $r_J^{(n)}(v,s)$ are determined by solving the hypersingular integral equations

$$
\begin{aligned}
&\frac{1}{\ell^{(q)}} \,\,=\!\!\!\!\!\!\int_{-1}^{1} \frac{D_{IK}^{(q)} r_K^{(q)}(\ell^{(q)}u,s)}{(v-u)^2} du + \ell^{(q)} \int_{-1}^{1} r_K^{(q)}(\ell^{(q)}u,s)\Omega_{IK}^{(q)}(u,v,s)du \\
&+ \ \ell^{(q)} \,-\!\!\!\!\!\!\int_{-1}^{1} s^2 G_{IK}^{(q)} r_K^{(q)}(\ell^{(q)}u,s)\cosh(\ell^{(q)}\eta|v-u|)\ln(\ell^{(q)}\eta|v-u|)du \\
&+ \sum_{\substack{n=1\\ n\neq q}}^{N_0} \ell^{(n)} \int_{-1}^{1} r_K^{(n)}(\ell^{(n)}u,s)\Theta_{IK}^{(nq)}(u,v,s)du \\
= \ & -\widehat{P}_I^{(q)}(X_1^{(q)}(v), X_2^{(q)}(v), s) \ (I=1,2,3,4,5) \\
&\qquad \text{for } -1<v<1 \ (q=1,2,\cdots,N_0),
\end{aligned} \tag{23}
$$

where $X_1^{(q)}(v) = c_1^{(q)} + \ell^{(q)} v \sin(\theta^{(q)})$, $X_2^{(q)}(v) = c_2^{(q)} - \ell^{(q)} v \cos(\theta^{(q)})$, $-\!\!\!\!\!\int$ denotes that the integral is to be interpreted in the Cauchy principal sense and $=\!\!\!\!\!\!\int$ denotes that the integral is to be interpreted in the Hadamard finite-part sense, $D_{IK}^{(q)}$ and $G_{IK}^{(q)}$ are given by

$$
\begin{aligned}
D_{IK}^{(q)} &= \lim_{(\xi/s)\to\infty} T_{IK}^{(q)}(\xi,s), \\
G_{IK}^{(q)} &= \lim_{(\xi/s)\to\infty} (\frac{\xi}{s})^2 [T_{IK}^{(q)}(\xi,s) - D_{IK}^{(q)}] \ ,
\end{aligned} \tag{24}
$$

and $\Omega_{IK}^{(q)}(u,v,s)$ and $\Theta_{IK}^{(nq)}(u,v,s)$ are respectively defined by

$$
\begin{aligned}
\Omega_{IK}^{(q)}(u,v,s) \ = \ & -\int_0^\infty \xi W_{IK}^{(q)}(\xi,s)\cos(\ell^{(q)}\xi[v-u])d\xi \\
& - s^2 G_{IK}^{(q)} [\mathrm{Shi}(\ell^{(q)}\eta|v-u|)\sinh(\ell^{(q)}\eta|v-u|) \\
& - \frac{1}{2}\cosh(\ell^{(q)}\eta|v-u|)(\mathrm{Ei}(\ell^{(q)}\eta|v-u|) - E_1(\ell^{(q)}\eta|v-u|)) \\
& + \cosh(\ell^{(q)}\eta|v-u|)\ln(\ell^{(q)}\eta|v-u|)],
\end{aligned} \tag{25}
$$

and

$$
\begin{aligned}
\Theta_{IK}^{(nq)}(u,v,s) &= -\operatorname{Re}\{\sum_{\alpha=1}^{5}\int_{0}^{\infty}\xi m_j^{(q)}[H(Y_2^{(nq)}(u,v)) \\
&\quad \times L_{Ij\alpha}^{(n)}(\xi,s)M_{\alpha P}^{(n)}(\xi,s)\exp(i\xi\tau_\alpha^{(n)}(\xi,s)Y_2^{(nq)}(u,v)) \\
&\quad + H(-Y_2^{(nq)}(u,v))\overline{L}_{Ij\alpha}^{(n)}(\xi,s)\overline{M}_{\alpha P}^{(n)}(\xi,s) \\
&\quad \times \exp(i\xi\overline{\tau}_\alpha^{(n)}(\xi,s)Y_2^{(nq)}(u,v))] \\
&\quad \times T_{PK}^{(n)}(\xi,s)\exp(i\xi Y_1^{(nq)}(u,v))d\xi\} \\
&\qquad\qquad \text{if it is assumed that } Y_2^{(nq)}(u,v)\neq 0, \qquad (26)
\end{aligned}
$$

with $Y_p^{(nq)}(u,v) = a_{kp}^{(n)}(X_k^{(q)}(v) - c_k^{(n)}) - \ell^{(n)}\delta_{p1}u$, $W_{IK}^{(q)}(\xi,s)$ given by

$$
W_{IK}^{(q)}(\xi,s) = T_{IK}^{(q)}(\xi,s) - D_{IK}^{(q)} - \frac{s^2 G_{IK}^{(q)}}{\xi^2+\eta^2}\ (\eta>0), \qquad (27)
$$

and $\operatorname{Shi}(u)$, $\operatorname{Ei}(u)$ and $E_1(u)$ being special functions defined by

$$
\operatorname{Shi}(u) = \int_0^u \frac{\sinh(x)}{x}dx,\ \ \operatorname{Ei}(u) = -\int_{-u}^{\infty}\frac{\exp(-x)}{x}dx,\ \ E_1(u) = \int_u^{\infty}\frac{\exp(-x)}{x}dx. \qquad (28)
$$

The expressions for $\Theta_{IK}^{(nq)}(u,v,s)$as given in (26) are valid for $Y_2^{(nq)}(u,v)\neq 0$. If $Y_2^{(nq)}(u,v)=0$ then (26) has to be modified accordingly, that is,

$$
\begin{aligned}
\Theta_{IK}^{(nq)}(u,v,s) &= \operatorname{Re}\{\frac{\widetilde{D}_{IK}^{(nq)}}{[Y_1^{(nq)}(u,v)]^2} - \int_0^{\infty}\xi\widetilde{W}_{IK}^{(nq)}(\xi,s)\exp(i\xi Y_1^{(nq)}(u,v))d\xi \\
&\quad - s^2\widetilde{G}_{IK}^{(nq)}[\operatorname{Shi}(\eta|Y_1^{(nq)}(u,v)|)\sinh(\eta|Y_1^{(nq)}(u,v)|) \\
&\quad - \frac{1}{2}\cosh(\eta|Y_1^{(nq)}(u,v)|) \\
&\quad \times(\operatorname{Ei}(\eta|Y_1^{(nq)}(u,v)|) - E_1(\eta|Y_1^{(nq)}(u,v)|)) \\
&\quad + i\frac{\pi}{2}\operatorname{sgn}(Y_1^{(nq)}(u,v)) \\
&\quad \times(\cosh(\eta|Y_1^{(nq)}(u,v)|) - \sinh(\eta|Y_1^{(nq)}(u,v)|))]\ \} \\
&\qquad\qquad \text{if } Y_2^{(nq)}(u,v)=0, \qquad (29)
\end{aligned}
$$

where $\text{sgn}(x)$ denotes the sign of $x$ and

$$
\begin{aligned}
\widetilde{D}_{IK}^{(nq)} &= \lim_{(\xi/s)\to\infty} \widetilde{T}_{IK}^{(nq)}(\xi, s), \\
\widetilde{T}_{IK}^{(nq)}(\xi, s) &= \sum_{\alpha=1}^{4} m_j^{(q)} L_{Ij\alpha}^{(n)}(\xi, s) M_{\alpha P}^{(n)}(\xi, s) T_{IK}^{(n)}(\xi, s), \\
\widetilde{G}_{IK}^{(nq)} &= \lim_{(\xi/s)\to\infty} (\frac{\xi}{s})^2 [\widetilde{T}_{IK}^{(nq)}(\xi, s) - \widetilde{D}_{IK}^{(nq)}] \\
\widetilde{W}_{IK}^{(nq)}(\xi, s) &= \widetilde{T}_{IK}^{(nq)}(\xi, s) - \widetilde{D}_{IK}^{(nq)} - \frac{s^2 \widetilde{G}_{IK}^{(nq)}}{\xi^2 + \eta^2} \ (\eta > 0). \qquad (30)
\end{aligned}
$$

If all the cracks are electrically permeable then (5) implies that $r_4^{(q)}(w) = 0$ and the hypersingular integral equation in (23) given by $I = 4$ may be discarded. Similarly, if all the cracks are magnetically permeable, we may take $r_5^{(q)} = 0$ and disregard the hypersingular integral equation in (23) given by $I = 5$. Thus, if all the cracks are both electrically and magnetically permeable, we only have to determine $r_1^{(q)}(v)$, $r_2^{(q)}(v)$ and $r_3^{(q)}(v)$ by solving (23) for $I$ given by 1, 2 and 3 only.

The collocation technique in Kaya and Erdogan [7] may be used to solve approximately the hypersingular integral equations in (23) by making the approximations

$$
r_K^{(q)}(u\ell^{(q)}, s) \simeq \frac{1}{\pi}\sqrt{1-u^2} \sum_{j=1}^{J} \omega_K^{(qj)}(s) U^{(j-1)}(u) \text{ for } -1 < u < 1, \qquad (31)
$$

where $U^{(j)}(x) = \sin([j+1]\arccos(x))/\sin(\arccos(x))$ is the $j^{th}$ order Chebyshev polynomial of the second kind and $\omega_P^{(nj)}(s)$ are unknown coefficients. Substitution of (31) into (23) yields a system of linear algebraic equations which can be used to determine $\omega_P^{(nj)}(s)$ for any fixed value of $s$. Details on how the linear algebraic equations may be set up may be found in Athanasius, Ang and Sridhar [3].

## 4 Generalized stress intensity factors

The dynamic stress, electric displacement, magnetic induction intensity factors at the tips $(X_1^{(n)}(\pm 1), X_2^{(n)}(\pm 1))$ of the $n$-th crack $\Gamma^{(n)}$ are defined by

$$\begin{aligned}
& K_I(X_1^{(n)}(\pm 1), X_2^{(n)}(\pm 1), t) \\
= & \lim_{u\to\pm 1^{\pm}} \sqrt{\pm 2\ell^{(n)}(u \mp 1)}(S_{1j}(X_1^{(n)}(u), X_2^{(n)}(u), t)m_1^{(n)} \\
& + S_{2j}(X_1^{(n)}(u), X_2^{(n)}(u), t)m_2^{(n)})m_j^{(n)},
\end{aligned}$$

$$\begin{aligned}
& K_{II}(X_1^{(n)}(\pm 1), X_2^{(n)}(\pm 1), t) \\
= & \lim_{u\to\pm 1^{\pm}} \sqrt{\pm 2\ell^{(n)}(u \mp 1)}(S_{1j}(X_1^{(n)}(u), X_2^{(n)}(u), t)m_2^{(n)} \\
& - S_{2j}(X_1^{(n)}(u), X_2^{(n)}(u), t)m_1^{(n)})m_j^{(n)},
\end{aligned}$$

$$\begin{aligned}
& K_{III}(X_1^{(n)}(\pm 1), X_2^{(n)}(\pm 1), t) \\
= & \lim_{u\to\pm 1^{\pm}} \sqrt{\pm 2\ell^{(n)}(u \mp 1)}S_{3j}(X_1^{(n)}(u), X_2^{(n)}(u), t)m_j^{(n)},
\end{aligned}$$

$$\begin{aligned}
& K_{IV}(X_1^{(n)}(\pm 1), X_2^{(n)}(\pm 1), t) \\
= & \lim_{u\to 1^{\pm}} \sqrt{\pm 2\ell^{(n)}(u \mp 1)}S_{4j}(X_1^{(n)}(u), X_2^{(n)}(u), t)m_j^{(n)},
\end{aligned}$$

$$\begin{aligned}
& K_V(X_1^{(n)}(\pm 1), X_2^{(n)}(\pm 1), t) \\
= & \lim_{u\to\pm 1^{\pm}} \sqrt{\pm 2\ell^{(n)}(u \mp 1)}S_{5j}(X_1^{(n)}(u), X_2^{(n)}(u), t)m_j^{(n)}. \qquad (32)
\end{aligned}$$

Once the coefficients $\omega_P^{(nj)}(s)$ in (31) are determined, the above intensity factors may be approximately calculated in the Laplace transform domain by using

$$\begin{aligned}
\widehat{K}_I(X_1^{(n)}(\pm 1), X_2^{(n)}(\pm 1), s) \simeq & \frac{1}{\sqrt{\ell^{(n)}}}(D_{1P}^{(n)}m_1^{(n)} + D_{2P}^{(n)}m_2^{(n)}) \\
& \times \sum_{j=1}^{J} \omega_P^{(nj)}(s)U^{(j-1)}(\pm 1),
\end{aligned}$$

$$\begin{aligned}\widehat{K}_{II}(X_1^{(n)}(\pm 1), X_2^{(n)}(\pm 1), s) &\simeq \frac{1}{\sqrt{\ell^{(n)}}}(D_{1P}^{(n)} m_2^{(n)} - D_{2P}^{(n)} m_1^{(n)}) \\ &\quad \times \sum_{j=1}^{J} \omega_P^{(nj)}(s) U^{(j-1)}(\pm 1),\end{aligned}$$

$$\widehat{K}_{III}(X_1^{(n)}(\pm 1), X_2^{(n)}(\pm 1), s) \simeq -\frac{1}{\sqrt{\ell^{(n)}}} D_{3P}^{(n)} \sum_{j=1}^{J} \omega_P^{(nj)}(s) U^{(j-1)}(\pm 1),$$

$$\widehat{K}_{IV}(X_1^{(n)}(\pm 1), X_2^{(n)}(\pm 1), s) \simeq -\frac{1}{\sqrt{\ell^{(n)}}} D_{4P}^{(n)} \sum_{j=1}^{J} \omega_P^{(nj)}(s) U^{(j-1)}(\pm 1),$$

$$\widehat{K}_{V}(X_1^{(n)}(\pm 1), X_2^{(n)}(\pm 1), s) \simeq -\frac{1}{\sqrt{\ell^{(n)}}} D_{5P}^{(n)} \sum_{j=1}^{J} \omega_P^{(nj)}(s) U^{(j-1)}(\pm 1). \quad (33)$$

As in Ang [1], the intensity factors at any time $t > 0$ may be recovered by using the numerical Laplace transform algorithm in Stehfest [11], that is, by using the formula

$$F(t) \simeq \frac{\ln(2)}{t} \sum_{n=1}^{2M} V_n \widehat{F}(\frac{n \ln(2)}{t}), \quad (34)$$

where $\widehat{F}(s)$ denotes the Laplace transform of $F(t)$, $M$ is a positive integer and

$$V_n = (-1)^{n+M} \sum_{m=[(n+1)/2]}^{\min(n,M)} \frac{m^M (2m)!}{(M-m)! m! (m-1)! (n-m)! (2m-n)!}, \quad (35)$$

with $[r]$ denoting the integer part of the real number $r$.

# 5 Specific problems

In this section, the dynamic generalized stress intensity factors are computed for specific cases of the magnetoelectroelastic crack problem in Section 2. For the cases of a single planar crack and a pair of coplanar cracks, the stress intensity factors computed here are compared with those published in the literature.

For all the specific problems considered below, the constitutive equations for the magnetoelectroelastic space are given by

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{32} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} A & F & N & 0 & 0 & 0 \\ F & C & F & 0 & 0 & 0 \\ N & F & A & 0 & 0 & 0 \\ 0 & 0 & 0 & L & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(A-N) & 0 \\ 0 & 0 & 0 & 0 & 0 & L \end{pmatrix} \begin{pmatrix} \gamma_{11} \\ \gamma_{22} \\ \gamma_{33} \\ 2\gamma_{32} \\ 2\gamma_{31} \\ 2\gamma_{12} \end{pmatrix}$$
$$- \begin{pmatrix} 0 & e_2 & 0 \\ 0 & e_3 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_1 \\ 0 & 0 & 0 \\ e_1 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} - \begin{pmatrix} 0 & h_2 & 0 \\ 0 & h_3 & 0 \\ 0 & h_2 & 0 \\ 0 & 0 & h_1 \\ 0 & 0 & 0 \\ h_1 & 0 & 0 \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix},$$

$$\begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & e_1 \\ e_2 & e_3 & e_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & e_1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \gamma_{11} \\ \gamma_{22} \\ \gamma_{33} \\ 2\gamma_{32} \\ 2\gamma_{31} \\ 2\gamma_{12} \end{pmatrix}$$
$$+ \begin{pmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_1 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} + \begin{pmatrix} \beta_1 & 0 & 0 \\ 0 & \beta_2 & 0 \\ 0 & 0 & \beta_1 \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix},$$

$$
\begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & h_1 \\ h_2 & h_3 & h_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & h_1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \gamma_{11} \\ \gamma_{22} \\ \gamma_{33} \\ 2\gamma_{32} \\ 2\gamma_{31} \\ 2\gamma_{12} \end{pmatrix}
+ \begin{pmatrix} \beta_1 & 0 & 0 \\ 0 & \beta_2 & 0 \\ 0 & 0 & \beta_1 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} + \begin{pmatrix} \gamma_1 & 0 & 0 \\ 0 & \gamma_2 & 0 \\ 0 & 0 & \gamma_1 \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix},
$$

where $2\gamma_{kj} = \partial u_k/\partial x_j + \partial u_j/\partial x_k$, $E_k = -\partial\phi/\partial x_k$ and $H_k = -\partial\varphi/\partial x_k$. Note that $\gamma_{33} = 0$, $E_3 = 0$ and $H_3 = 0$ here since $u_k$, $\phi$ and $\varphi$ are independent of $x_3$.

The constitutive equations above are for magnetoelectroelastic materials with particular symmetries and may be recovered as special cases from the more general equations in (12) by taking non-zero coefficients $C_{IjKp}$ to be given by

$$
\begin{aligned}
& C_{1111} = C_{3333} = A,\ C_{1133} = C_{3311} = N,\ C_{2222} = C, \\
& C_{1122} = C_{2211} = C_{2233} = C_{3322} = F, \\
& C_{1212} = C_{2112} = C_{2121} = C_{1221} = C_{2323} = C_{3223} = C_{3232} = C_{2332} = L, \\
& C_{1313} = C_{3113} = C_{3131} = C_{1331} = \frac{1}{2}(A - N), \\
& C_{2141} = C_{1241} = C_{3243} = C_{2343} = C_{4121} = C_{4112} = C_{4332} = C_{4323} = e_1, \\
& C_{1142} = C_{3342} = C_{4211} = C_{4233} = e_2, \\
& C_{2242} = C_{4222} = e_3,\ C_{4141} = C_{4343} = -\epsilon_1,\ C_{4242} = -\epsilon_2, \\
& C_{2151} = C_{1251} = C_{3253} = C_{2353} = C_{5121} = C_{5112} = C_{5332} = C_{5323} = h_1, \\
& C_{1152} = C_{3352} = C_{5211} = C_{5233} = h_2, \\
& C_{2252} = C_{5222} = h_3,\ C_{4151} = C_{4353} = C_{5141} = C_{5343} = -\beta_1, \\
& C_{4252} = C_{5242} = -\beta_2, C_{5151} = C_{5353} = -\gamma_1,\ C_{5252} = -\gamma_2.
\end{aligned}
$$

For the purpose of obtaining numerical values of the generalized stress intensity factors, we use the magnetoelectroelastic coefficients for the material

$BaTiO_3 - CoFe_2O_4$, given in SI units by

$$A = 22.6 \times 10^{10},\ N = 11.7 \times 10^{10},\ F = 12.5 \times 10^{10},\ C = 21.6 \times 10^{10},\ L = 4.4 \times 10^{10},$$
$$e_1 = 5.8,\ e_2 = -2.2,\ e_3 = 9.3, h_1 = 275,\ h_2 = 290.2,\ h_3 = 350,$$
$$\epsilon_1 = 56.4 \times 10^{-10},\ \epsilon_2 = 63.5 \times 10^{-10}, \beta_1 = 5.367 \times 10^{-12},\ \beta_2 = 2737.5 \times 10^{-12},$$
$$\gamma_1 = 297 \times 10^{-6},\ \gamma_2 = 83.5 \times 10^{-6}.$$

**Problem 1.** Take a single horizontal crack with crack tips $(-a, 0)$ and $(a, 0)$. The crack is electrically and magnetically impermeable. The only non-zero load on the crack faces are given by $S_{22} = -H(t)\sigma_0$, where $\sigma_0$ is a given positive constant.

In Figures 2, 3 and 4, the non-dimensionalized intensity factors $K_I/(\sigma_0\sqrt{a})$, $e_3 K_{IV}/(\epsilon_2\sigma_0\sqrt{a})$ and $h_3 K_V/(\gamma_2\sigma_0\sqrt{a})$, calculated by using $M = 4$ in (34) and $J = 10$ in (31), are plotted against the non-dimensionalized time $t\sqrt{L/(\rho a^2)}$ and compared with the values given in Rojas-Díaz *et al* [10]. Although there is a discernible difference between the plots for the generalized stress intensity factors computed here and those in [10], they exhibit the same general trends and reach the peak values at about the same time.

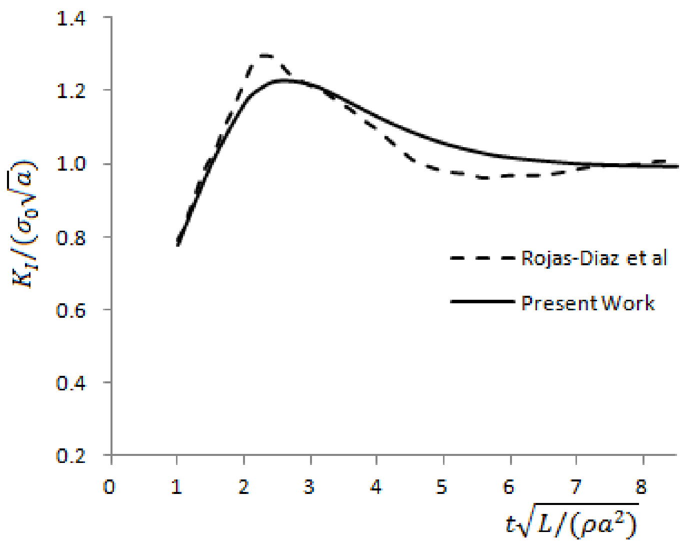



Figure 2: Plots of $K_I/(\sigma_0\sqrt{a})$ against the non-dimensionalized time $t\sqrt{L/(\rho a^2)}$ .

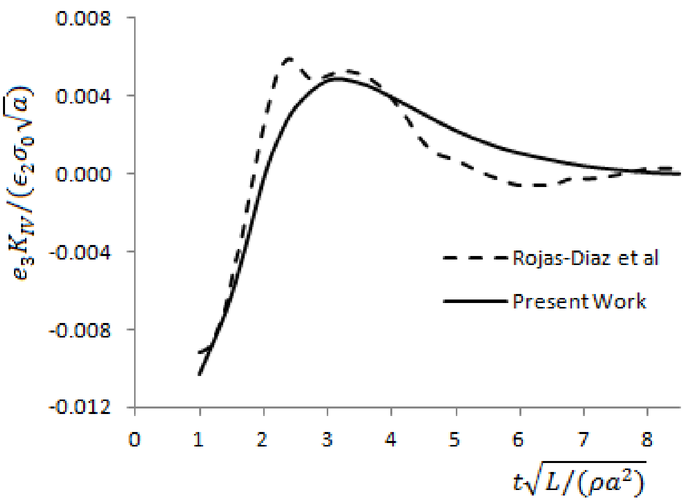



Figure 3: Plots of $e_3K_{IV}/(\epsilon_2\sigma_0\sqrt{a})$ against the non-dimensionalized time $t\sqrt{L/(\rho a^2)}$ .

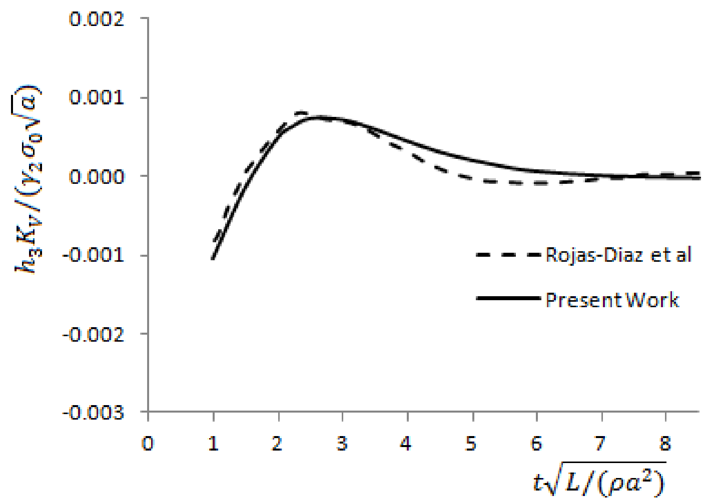



Figure 4: Plots of $h_3K_V/(\gamma_2\sigma_0\sqrt{a})$ against the non-dimensionalized time $t\sqrt{L/(\rho a^2)}$ .

**Problem 2.** Consider a pair of electrically and magnetically permeable coplanar cracks, each of length $2a$, as shown in Figure 5. The distance between the inner tips of the cracks is $2d$. The uniform tractions acting on the crack faces are given by $S_{22} = -H(t)\sigma_0$,where $\sigma_0$ is a given positive constant.

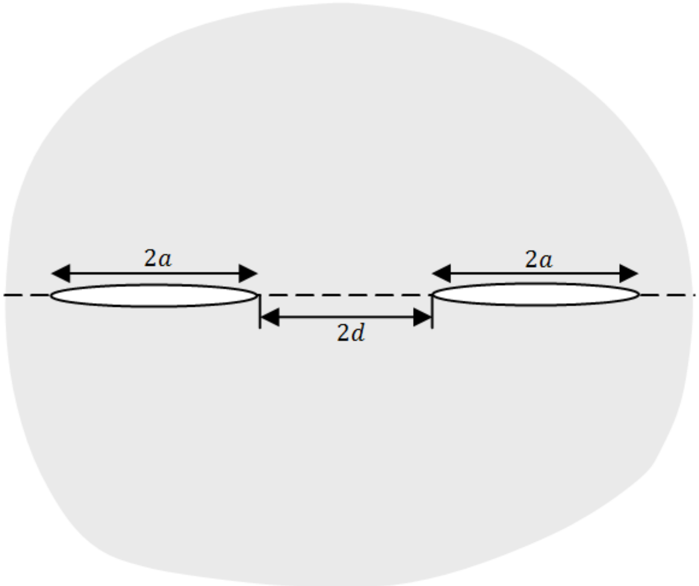



Figure 5: A pair of coplanar cracks.

For $d/a = 0.2$, the non-dimensionalized stress intensity factor $K_I/(\sigma_0\sqrt{a})$ at the inner and outer tips of the cracks calculated here are plotted against $t\sqrt{L/(\rho a^2)}$ and compared with the results in Zhong *et al* [17] in Figure 6. Over a significantly large interval of time, there is a close agreement between the values computed here and the ones given in [17].

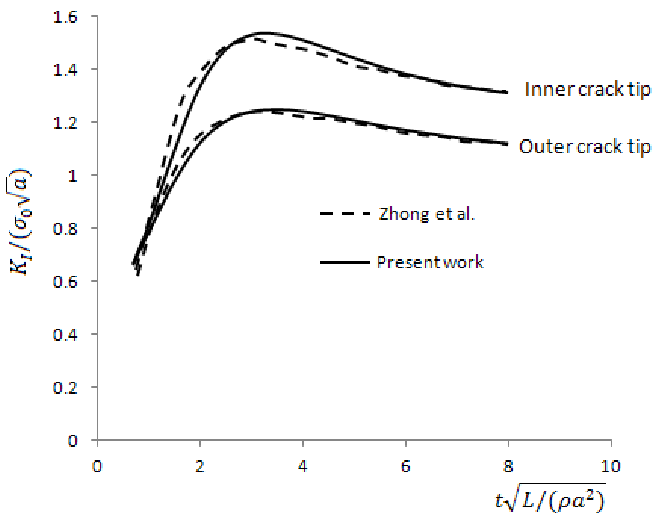



Figure 6: Plots of $K_I/(\sigma_0\sqrt{a})$ against the non-dimensionalized time $t\sqrt{L/(\rho a^2)}$ .

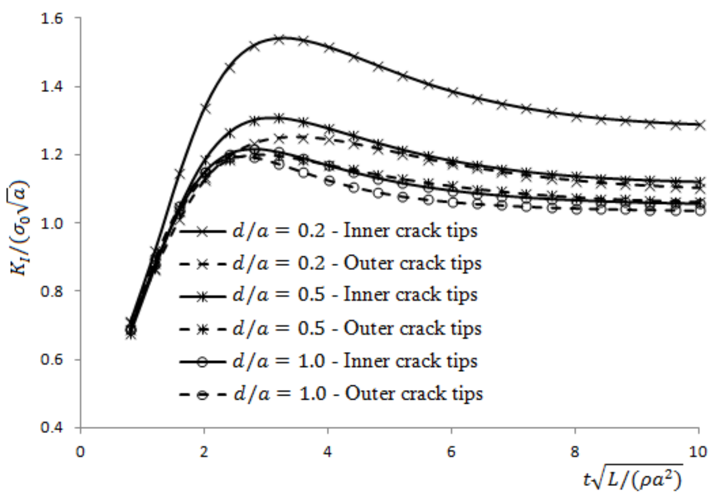



Figure 7: Plots of $K_I/(\sigma_0\sqrt{a})$ against the non-dimensionalized time $t\sqrt{L/(\rho a^2)}$at inner and outer crack tips for selected values of $d/a$.

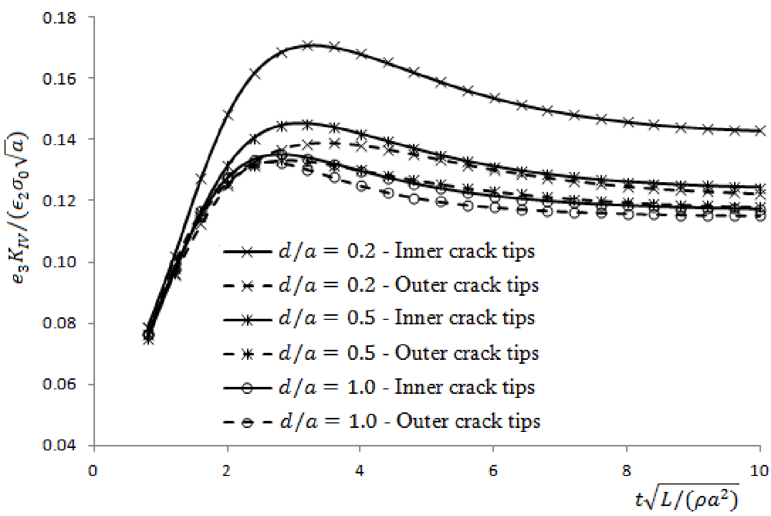



Figure 8: Plots of $e_3K_{IV}/(\epsilon_2\sigma_0\sqrt{a})$ against the non-dimensionalized time $t\sqrt{L/(\rho a^2)}$at inner and outer crack tips for selected values of $d/a$.

To examine how the distance separating the inner crack tips may affect the behaviors of the cracks, plots of the non-dimensionalized generalized stress intensity factors $K_I/(\sigma_0\sqrt{a})$, $e_3K_{IV}/(\epsilon_2\sigma_0\sqrt{a})$ and $h_3K_V/(\gamma_2\sigma_0\sqrt{a})$ at the inner and outer crack tips against the non-dimensionalized time $t\sqrt{L/(\rho a^2)}$ are given for $d/a = 0.2$, 0.50 and 1.0 in Figures 7, 8 and 9. In each of the plots, the generalized stress intensity factor increases rapidly to a peak value before settling down to the corresponding static value. The peak value of the intensity factor at the inner crack tip is larger than that at the outer tip. It appears that decreasing $d/a$ has the effect of increasing the intensity factors.

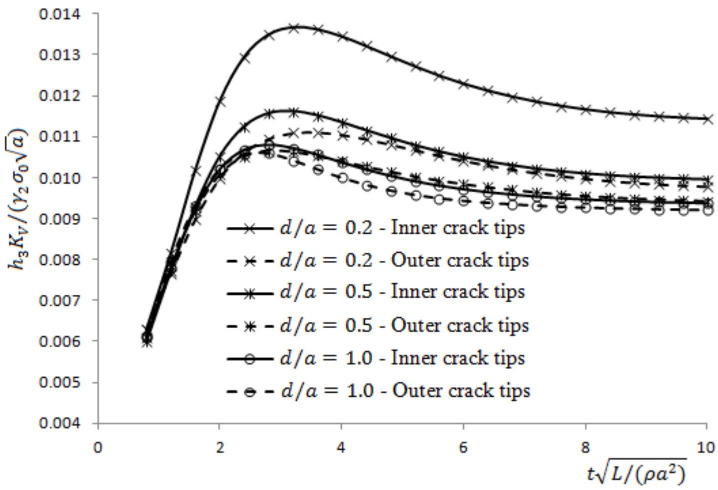



Figure 9: Plots of $h_3K_V/(\gamma_2\sigma_0\sqrt{a})$ against the non-dimensionalized time $t\sqrt{L/(\rho a^2)}$at inner and outer crack tips for selected values of $d/a$.

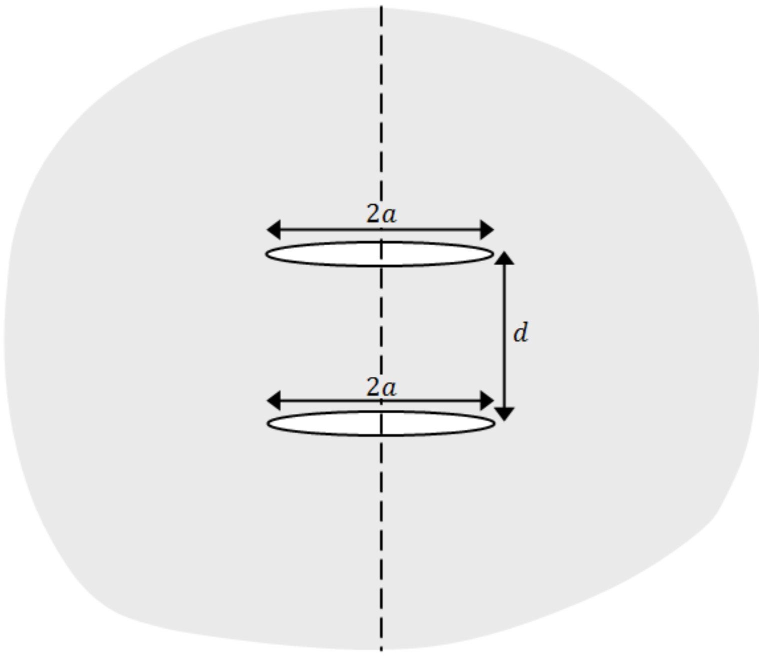



Figure 10: Two parallel cracks of equal length.

**Problem 3.** Consider two parallel equal length planar cracks as sketched in Figure 10. The half length of each crack is given by $a$. The centers of the cracks lie on a vertical line and are separated by a distance denoted by $d$. The cracks are electrically and magnetically impermeable. The non-zero constant loads acting on the crack faces are given by $S_{22} = -H(t)\sigma_0$, $S_{42} = -H(t)D_0$ and $S_{52} = -H(t)B_0$, with $\sigma_0$, $D_0$ and $B_0$ being positive constants such that $\sigma_0/D_0 = 10^{10}$ NC$^{-1}$ and $\sigma_0/B_0 = 10^8$ Am$^{-1}$.

For $d/a = 3.0$, 5.0 and 10.0, plots of the non-dimensionalized generalized stress intensity factors $K_I/(\sigma_0\sqrt{a})$, $K_{II}/(\sigma_0\sqrt{a})$, $K_{IV}/(D_0\sqrt{a})$ and $K_V/(B_0\sqrt{a})$ against the non-dimensionalized time $t\sqrt{L/(\rho a^2)}$ are given in Figures 11, 12, 13 and 14 respectively.

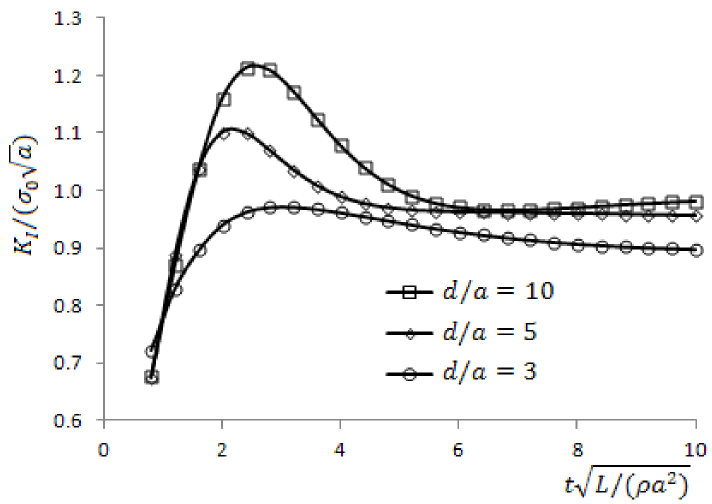



Figure 11: Plots of $K_I/(\sigma_0\sqrt{a})$ against the non- dimensionalized time for selected values of $d/a$.

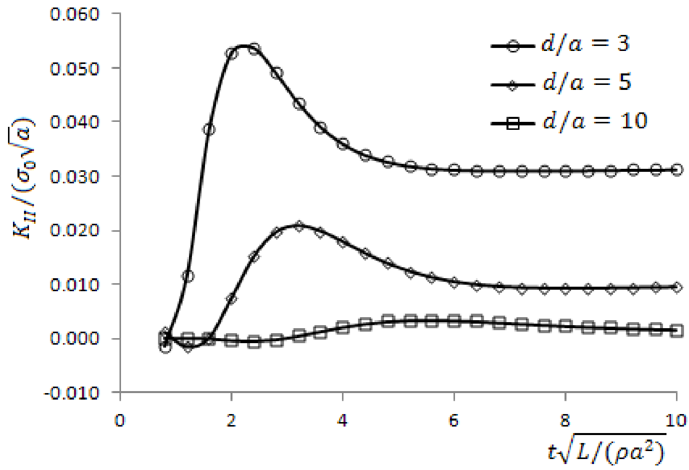
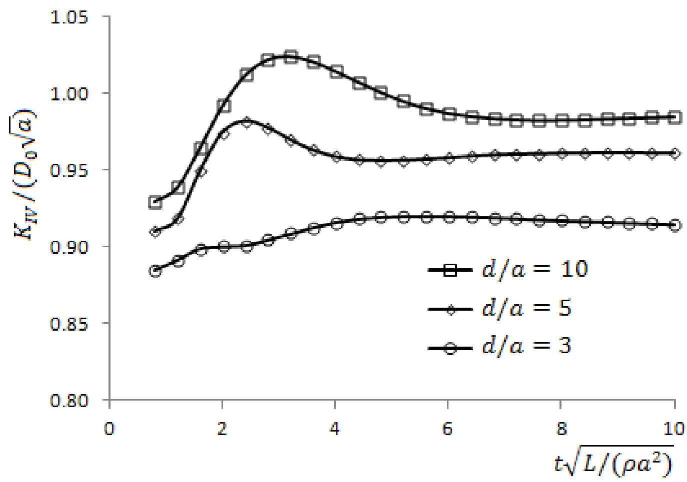



Figure 12: Plots of $K_{II}/(\sigma_0\sqrt{a})$ against the non-dimensionalized time for selected values of $d/a$.




Fiugure 13: Plots of $K_{IV}/(D_0\sqrt{a})$ against the non- dimensionalized time for selected values of $d/a$.

In Figure 11, for a given $d/a$, the non-dimensionalized stress intensity factor $K_I/(\sigma_0\sqrt{a})$ rises to a peak (maximum) value and then drops to a trough (mininum) value before gradually settling down to approach its static value. Both the trough and the peak values decrease in magnitude as $d/a$ decreases. A similar observation may be made of the non-dimensionalized electric displacement and magnetic induction intensity factors in Figures 13 and 14.

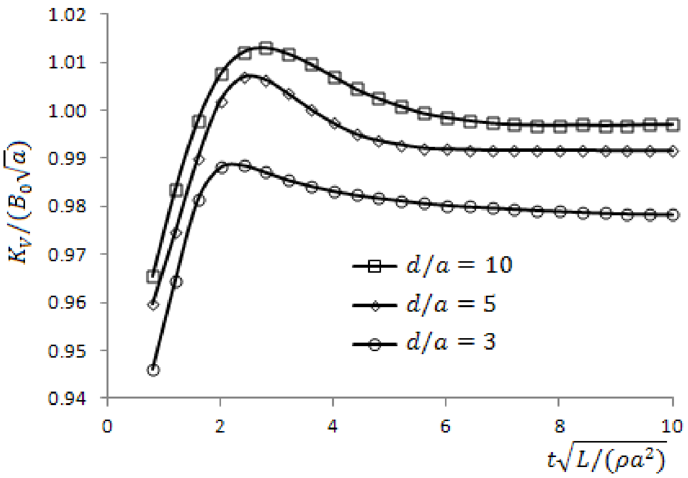



Figure 14: Plots of $K_V/(B_0\sqrt{a})$ against the non- dimensionalized time for selected values of $d/a$.

As $d/a$ tends to infinity, the non-dimensionalized stress intensity factor $K_{II}/(\sigma_0\sqrt{a})$ vanishes. In Figure 12, the magnitude of $K_{II}/(\sigma_0\sqrt{a})$ for $d/a =$ 10.0 is very small at all time. Nevertheless, as the distance $d/a$ separating the cracks becomes smaller, there is an increase in the magnitude $K_{II}/(\sigma_0\sqrt{a})$. This observation may be explained by the well known phenomenon known as Poisson effect. Due to Poisson effect, compressive stresses generated on opposite crack faces are unequal, thereby giving rise to shear stresses on each of the cracks.

**Problem 4.** Take the only non-zero load acting on the pair of parallel cracks in Figure 10 to be given by the stress $S_{22} = -H(t)\sigma_0$, where $\sigma_0$ is a given positive constant. We are interested in examining how the electric and magnetic conditions on the cracks affect the generalized stress intensity factors. For $d/a = 3.0$ and $d/a = 10.0$, the non-dimensionalized generalized stress intensity factors $K_I/(\sigma_0\sqrt{a})$, $K_{II}/(\sigma_0\sqrt{a})$, $e_3K_{IV}/(\epsilon_2\sigma_0\sqrt{a})$ and $h_3K_V/(\gamma_2\sigma_0\sqrt{a})$ for electrically and magnetically impermeable cracks are compared with the corresponding intensity factors for electrically and magnetically permeable cracks in Figures 15, 16, 17 and 18.

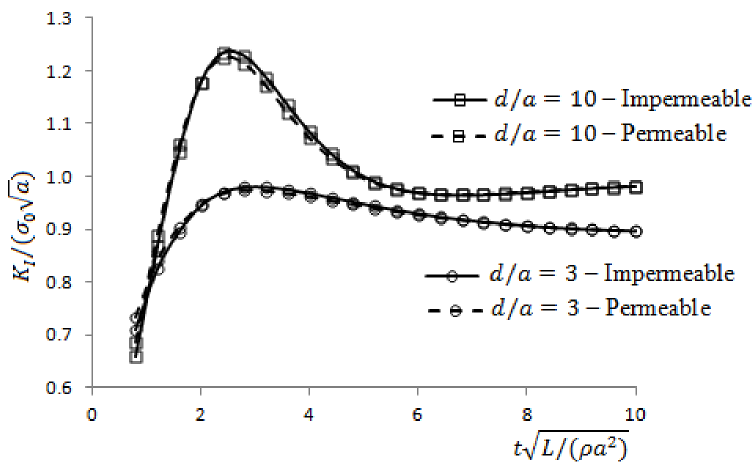



Figure 15: A comparison of $K_I/(\sigma_0\sqrt{a})$ for electrically and magnetically permeable and impermeable cracks for selected values of $d/a$.

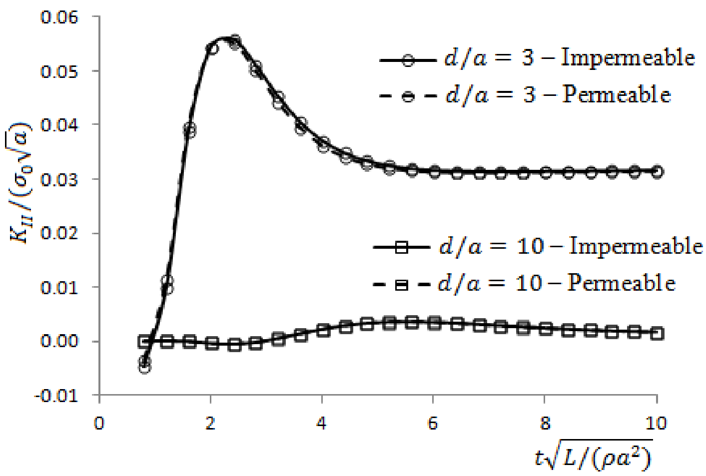



Figure 16: A comparison of $K_{II}/(\sigma_0\sqrt{a})$ for electrically and magnetically permeable and impermeable cracks for selected values of $d/a$.

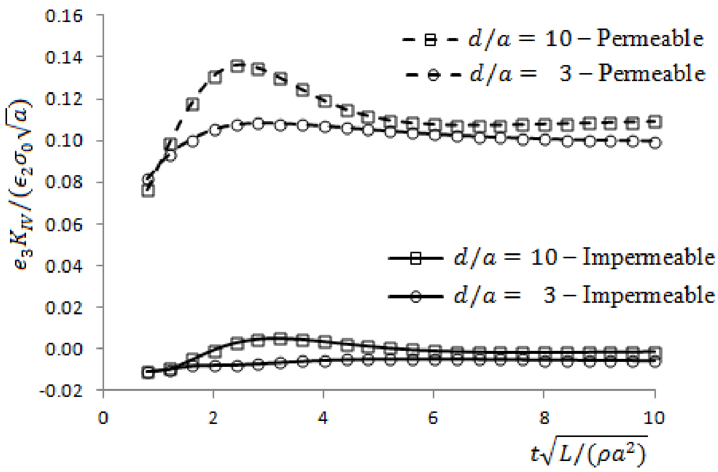



Figure 17: A comparison of $e_3K_{IV}/(\epsilon_2\sigma_0\sqrt{a})$ for electrically and magnetically permeable and impermeable cracks for selected values of $d/a$.

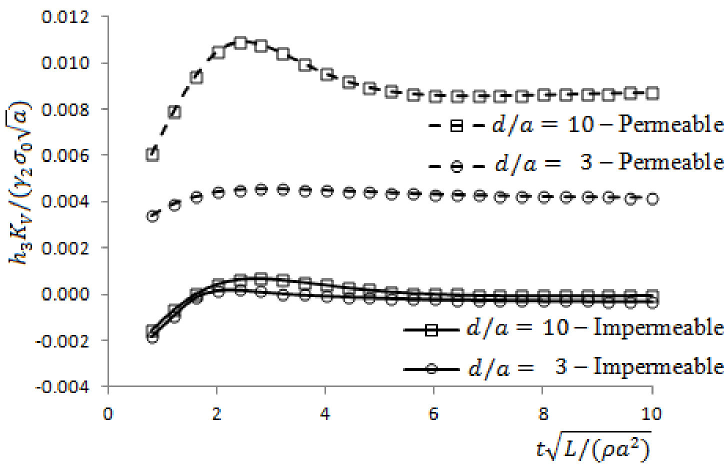



Figure 18: A comparison of $h_3K_V/(\gamma_2\sigma_0\sqrt{a})$ for electrically and magnetically permeable and impermeable cracks for selected values of $d/a$.

From Figures 15 and 16, the non-dimensionalized generalized stress intensity factors $K_I/(\sigma_0\sqrt{a})$ and $K_{II}/(\sigma_0\sqrt{a})$ for the impermeable cracks do not differ very much from the corresponding intensity factors for permeable cracks. In Figures 17 and 18, the non-dimensionalized generalized stress intensity factors $e_3K_{IV}/(\epsilon_2\sigma_0\sqrt{a})$ and $h_3K_V/(\gamma_2\sigma_0\sqrt{a})$ for the impermeable cracks are obviously different from the corresponding intensity factors for the permeable cracks. As may be expected, the magnitudes of $e_3K_{IV}/(\epsilon_2\sigma_0\sqrt{a})$ and $h_3K_V/(\gamma_2\sigma_0\sqrt{a})$ are very much smaller for the impermeable cracks than for the permeable cracks as $t\sqrt{L/(\rho a^2)}$ increases.

# 6 Summary and conclusion

An explicit solution is given in the Laplace transform domain for a magnetoelectroelastodynamic problem involving an arbitrary number of arbitrarily located planar cracks. The Laplace transform of the generalized displacement and stress fields are expressed in terms of the Laplace transform of the generalized crack opening displacements to be determined by solving a system of hypersingular integral equations. Once the generalized crack opening displacements are determined, the generalized stress intensity factors may be easily computed in the Laplace transform domain. A numerical technique for the inversion of Laplace transforms may then be used to recover the intensity factors in the physical domain.

The solution is applied to study several specific problems involving a particular magnetoelectroelastic material. For a single crack and a pair of coplanar cracks under impact loadings, the computed crack tip intensity factors are found to be in reasonably good agreement with those published in the literature.

The solution approach here does not require the difficult computation of a complicated fundamental solution for magnetoelectroelastodynamics. It may be extended to include cracks in magnetoelectroelastic solids having other idealized geometries (such as half-spaces and infinitely-long strips) and specific boundary conditions. Nevertheless, it may not be as versatile as the boundary element approach in dealing with more general geometries and boundary conditions.

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