A Beginner's Course in Boundary Element Methods

Whye Teong Ang

Preface

During the last few decades, the boundary element method, also known as the boundary integral equation method or boundary integral method, has gradually evolved to become one of the few widely used numerical techniques for solving boundary value problems in engineering and physical sciences. In implementing the method, only the boundary of the solution domain has to be discretized into elements. In the case of a two-dimensional problem, this is really easy to do. Put closely packed points on the boundary (a curve) and join up two consecutive neighboring points to form straight line elements.

In March 1985, when I started research work for a doctoral degree in the Department of Applied Mathematics at the University of Adelaide, Australia, I was introduced to the method by my thesis supervisor, David L Clements. At that time, the term "boundary element method" was relatively new. It was first used in a 1977 paper by CA Brebbia and J Dominguez*. Carlos Brebbia, editor-in-chief of the journal *Engineering Analysis with Boundary Elements*, and his co-researchers had undoubtedly played an important role in introducing the method to the engineering research community. Less than 200 journal papers whose titles contained the term "boundary element method" could be found in 1985. In 2006, there were several thousand or perhaps even more such papers.

The history of the method could be traced back to an earlier time, well before the 1970s, however. The mathematics that laid the theoretical foundation for the development of the method could be found in the works of famous mathematicians like Laplace, Gauss, Fredholm, Betti, Muskhelishvili and Mikhlin. In the 1960s, there were attempts at using electronic computers to approximate solutions of potential problems through the use of boundary integral equations, notably the pioneering works of M A Jaswon and G T Symm†. The work of Frank J Rizzo‡ was regarded by many researchers as the beginning of a novel direct boundary integral method for the numerical solution of elasticity problems.

After completing my doctoral work in mid 1987, I continued to keep myself informed on the development of the boundary integral method and related mathematical works, pick up some new ideas now and then, attend conferences, give talks and seminars, and contribute to boundary element research with applications to problems in engineering and physical sciences. Some specific research areas I had worked

*CA Brebbia and J Dominguez, "Boundary element methods for potential problems," *Applied Mathematical Modelling*, Volume 1, 1977, pp. 372-378.

†One may refer to the following papers: (a) MA Jaswon, "An integral equation methods in potential theory I," *Proceedings of the Royal Society of London Series A*, Volume 275, 1963, pp. 23-32, and (b) GT Symm, "An integral equation methods in potential theory II," *Proceedings of the Royal Society of London Series A*, Volume 275, 1963, pp. 33-46.

‡FJ Rizzo, "An integral equation approach to boundary value problems of classical elastostatics," *Quarterly of Applied Mathematics*, Volume 25, 1967, pp. 83-95. This was the work presented by FJ Rizzo in his doctoral dissertation. Much later on in 1993, it won him an ASME Warner Medal.

on using the boundary integral method include linear fracture mechanics (accurate computation of stress intensity factors using special Green's functions), analyses of nonhomogeneous media (such as functionally graded materials), diffusion with specification of mass, modeling of photonic crystal fibers, integral formulation of imperfect interfaces, and bioheat transfer.

Occasionally, I undertook the task of introducing the method to beginners, mainly advanced undergraduate and research students who were working on projects under my supervision. To do this, I had produced various notes over a period of time. The chapters in this book were written based on those notes. In writing this book, I assume that the readers have some prior basic knowledge of vector calculus (covering topics such as line, surface and volume integrals and the various integral theorems), ordinary and partial differential equations, complex variables, and computer programming.

FORTRAN 77 codes for the numerical procedures discussed are listed in the chapters. Some justifications (if any is needed at all) for using "good old FORTRAN 77" would be as follows. Firstly, in spite its seniority, it still remains a powerful "number crunching tool". Secondly, its codes are relatively easy to decipher and would be of some use even to readers who are attempting to implement the numerical procedures using newer software tools (such as C++ and MATLAB). Thirdly, free FORTRAN 77 compilers (e.g. FTN77 from Salford Software and GNU Fortran) can be downloaded from the internet.

The constant encouragement and support of my dear wife, Young Soon, had greatly motivated me to start and finish writing this book. Ean-Hin Ooi, a research student at Nanyang Technological University, had read several chapters in earlier drafts and pointed out some typographical errors.

Whye-Teong Ang, Singapore, 2007