Problem Definitions and Evaluation Criteria for the CEC 2010 Competition on Constrained Real-Parameter Optimization

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Technical Report

April, 2010

Introduction

Most real world optimization problems have constraints of different types (e.g., physical, time, geometric, etc.) which modify the shape of the search space. During the last couple of decades, a wide variety of metaheuristics have been designed and applied to solve constrained optimization problems [1]. Evolutionary algorithms and most other metaheuristics, when used for optimization, naturally operate as unconstrained search techniques. Therefore, they require an additional mechanism to incorporate constraints into their fitness function.

Historically, the most common approach to incorporate constraints (both in evolutionary algorithms and in mathematical programming) is the penalty functions, which were originally proposed in the 1940s and later expanded by many researchers. Penalty functions have, in general, several limitations. Particularly, they are not a very good choice when trying to solve problem in which the optimum is on the boundary between the feasible and the infeasible regions or when the feasible region is disjoint. Additionally, penalty functions require a careful fine-tuning to determine the most appropriate penalty factors to be used with our metaheuristics. Researchers have also proposed a number of other approaches to handle constraints such as the self-adaptive penalty, epsilon constraint handling and stochastic ranking. Additionally, the analysis of the role of the search engine has also become an interesting research topic in the last few years. For example, evolution strategies (ES), evolutionary programming (EP), differential evolution (DE) and particle swarm optimization (PSO) have been found advantageous by some researchers over other metaheuristics such as the binary genetic algorithms (GA).

In CEC06 [2], 24 benchmark functions have been presented which have 2-20 dimensions and are not easily scalable. In addition, CEC 2006 benchmark has been solved satisfactorily by several methods. Therefore, it has become impossible to demonstrate the superior performance of newly designed algorithms. CEC05 [3] presents some of the scalable bound constrained problems. In [4] author proposed a test-case generator for constrained parameter optimization problems. In [5] the authors generated some scalable constrained problems. In this report, we present 18 benchmark functions which are scalable. The mathematical formulas and properties of these functions are described in Section 1. In Section 2, the evaluation criteria are given. A suggested format to present the results is given in Section 3.

1. Definitions of the Function Suite

In this section, 18 optimization problems with constraints are described. They are all transformed into the following format:

Minimize: $f(X), \quad X = (x_1, x_2, ..., x_n)$ and $X \in S$ … (1)

subject to: $$\begin{aligned} g_i(X) &\le 0, & i &= 1,...,p \\ h_j(X) &= 0, & j &= p+1,...,m \end{aligned}$$ … (2)

Usually equality constraints are transformed into inequalities of the form

$$\left|h_j(X)\right| - \varepsilon \le 0, \text{ for } \quad j = p+1,...,m$$ … (3)

A solution X is regarded as feasible if $g_i(X) \le 0$, $for\ i = 1, ..., p$ and $\left|h_j(X)\right| - \varepsilon \le 0$, for $j = p+1,...,m$. In this special session ϵ is set to 0.0001.

A constrained problem, in which the feasible patches are parallel to the axes (Figure 1), can be solved better by algorithms employing line search or difference of two or more solution vectors (such as DE). Therefore, to avoid the test problems from being biased to a particular class of algorithms, we rotate the constraints in most of the test problems. The effect of constraints can be observed in Figures 1 & 2. In Figure 2, it can be observed that the points A, B, C, and D have been rotated in the clockwise direction.

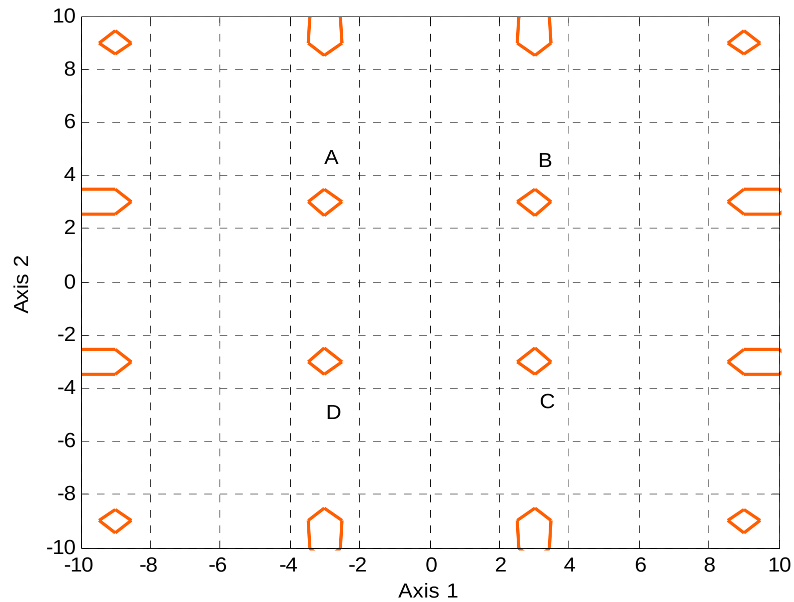

Figure 1: Contour plot without rotation

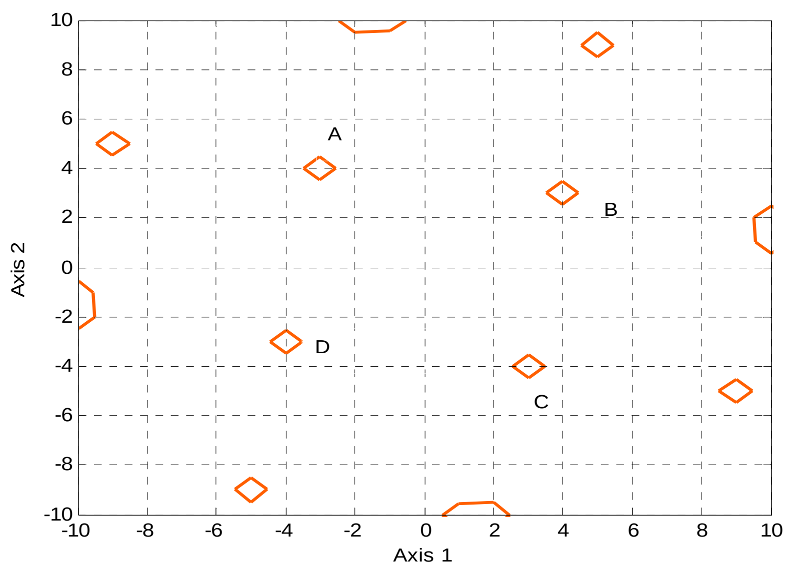

Figure 2: Contour plot with rotation

C01

$$\text{Min } f(x) = -\left| \frac{\sum_{i=1}^{D} \cos^4(z_i) - 2\prod_{i=1}^{D} \cos^2(z_i)}{\sqrt{\sum_{i=1}^{D} i z_i^2}} \right| \quad z = x - o$$

$$g_1(x) = 0.75 - \prod_{i=1}^{D} z_i \le 0$$

$$g_2(x) = \sum_{i=1}^{D} z_i - 7.5D \le 0$$

$$x \in [0,10]^D$$

C02

$$\text{Min } f(x) = \max(z) \qquad z = x - o,\ y = z - 0.5$$

$$g_1(x) = 10 - \frac{1}{D}\sum_{i=1}^{D}\left[z_i^2 - 10\cos(2\pi z_i) + 10\right] \le 0$$

$$g_2(x) = \frac{1}{D}\sum_{i=1}^{D}\left[z_i^2 - 10\cos(2\pi z_i) + 10\right] - 15 \le 0$$

$$h(x) = \frac{1}{D}\sum_{i=1}^{D}\left[y_i^2 - 10\cos(2\pi y_i) + 10\right] - 20 = 0$$

$$x \in [-5.12, 5.12]^{\mathrm{D}}$$

C03

$$\text{Min } f(x) = \sum_{i=1}^{D-1}(100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2) \quad z = x - o$$

$$h(x) = \sum_{i=1}^{D-1}(z_i - z_{i+1})^2 = 0$$

$$x \in [-1000, 1000]^{\mathrm{D}}$$

C04

$$\text{Min } f(x) = \max(z) \qquad z = x - o$$

$$h_1(x) = \frac{1}{D}\sum_{i=1}^{D}(z_i \cos(\sqrt{|z_i|})) = 0$$

$$h_2(x) = \sum_{i=1}^{D/2-1}(z_i - z_{i+1})^2 = 0$$

$$h_3(x) = \sum_{i=D/2+1}^{D-1}(z_i^2 - z_{i+1})^2 = 0$$

$$h_4(x) = \sum_{i=1}^{D} z = 0$$

$$x \in [-50, 50]^{\mathrm{D}}$$

C05

$$\text{Min } f(x) = \max(z) \quad z = x - o$$

$$h_1(x) = \frac{1}{D}\sum_{i=1}^{D}(-z_i \sin(\sqrt{|z_i|})) = 0$$

$$h_2(x) = \frac{1}{D}\sum_{i=1}^{D}(-z_i \cos(0.5\sqrt{|z_i|})) = 0$$

$$x \in [-600, 600]^{\mathrm{D}}$$

C06

$$\text{Min } f(x) = \max(z)$$

$$z = x - o,\ y = (x + 483.6106156535 - o)M - 483.6106156535$$

$$h_1(x) = \frac{1}{D}\sum_{i=1}^{D}(-y_i \sin(\sqrt{|y_i|})) = 0$$

$$h_2(x) = \frac{1}{D}\sum_{i=1}^{D}(-y_i \cos(0.5\sqrt{|y_i|})) = 0$$

$$x \in [-600, 600]^{\mathrm{D}}$$

C07

$$\text{Min } f(x) = \sum_{i=1}^{D-1}(100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2)$$

$$z = x + 1 - o,\ y = x - o$$

$$g(x) = 0.5 - \exp(-0.1\sqrt{\frac{1}{D}\sum_{i=1}^{D} y_i^2}) - 3\exp(\frac{1}{D}\sum_{i=1}^{D}\cos(0.1y)) + \exp(1) \le 0$$

$$x \in [-140, 140]^{\mathrm{D}}$$

C08

$$\text{Min } f(x) = \sum_{i=1}^{D-1}(100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2)$$

$$z = x + 1 - o,\ y = (x - o)M$$

$$g(x) = 0.5 - \exp(-0.1\sqrt{\frac{1}{D}\sum_{i=1}^{D} y_i^2}) - 3\exp(\frac{1}{D}\sum_{i=1}^{D}\cos(0.1y)) + \exp(1) \le 0$$

$$x \in [-140, 140]^{\mathrm{D}}$$

C09

$$\text{Min } f(x) = \sum_{i=1}^{D-1}(100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2)$$

$$z = x + 1 - o,\ y = x - o$$

$$h(x) = \sum_{\mathrm{i}=1}^{\mathrm{D}}(y \sin(\sqrt{|y_i|})) = 0$$

$$x \in [-500, 500]^{\mathrm{D}}$$

C10

$$\text{Min } f(x)=\sum_{i=1}^{D-1}(100(z_i^2-z_{i+1})^2+(z_i-1)^2)$$

$$z = x+1-o, \; y = (x\text{-}o)M$$

$$h(x)=\sum_{\mathrm{i=1}}^{\mathrm{D}}(y\sin(\sqrt{|y_i|}))=0$$

$$x\in[-500,500]^{\mathrm{D}}$$

C11

$$\text{Min } f(x)=\frac{1}{D}\sum_{i=1}^{D}(-z_i\cos(2\sqrt{|z_i|}))$$

$$z=(x\text{-}o)M, \; y=x+1-o$$

$$h(x)=\sum_{i}^{D-1}(100(y_i^2-y_{i+1})^2+(y_i-1)^2)=0$$

$$x\in[-100,100]^{\mathrm{D}}$$

C12

$$\text{Min } f(x)=\sum_{i=1}^{D}(z_i\sin(\sqrt{|z_i|})) \qquad z=x-o$$

$$h(x)=\sum_{i=1}^{D-1}(z_i^2-z_{i+1})^2=0$$

$$g(x)=\sum_{i=1}^{D}(z-100\cos(0.1z)+10)\le 0$$

$$x\in[-1000,1000]^{\mathrm{D}}$$

C13

$$\text{Min } f(x)=\frac{1}{D}\sum_{i=1}^{D}(-z_i\sin(\sqrt{|z_i|})) \qquad z=x-o$$

$$g_1(x)=-50+\frac{1}{100D}\sum_{i}^{D}z_i^2\le 0$$

$$g_2(x)=\frac{50}{D}\sum_{i=1}^{D}\sin(\frac{1}{50}\pi z)\le 0$$

$$g_3(x)=75-50(\sum_{i=1}^{D}\frac{z_i^2}{4000}-\prod_{i=1}^{D}\cos(\frac{z_i}{\sqrt{i}})+1)\le 0$$

$$x\in[-500,500]^{\mathrm{D}}$$

C14

$$\text{Min } f(x)=\sum_{i=1}^{D-1}(100(z_i^2-z_{i+1})^2+(z_i-1)^2)$$

$$z = x+1\text{-}o,\ y = x\text{-}o$$

$$g_1(x)=\sum_{i=1}^{D}(-y_i\cos(\sqrt{|y_i|}))-D\le 0$$

$$g_2(x)=\sum_{i=1}^{D}(y_i\cos(\sqrt{|y_i|}))-D\le 0$$

$$g_3(x)=\sum_{i=1}^{D}(y_i\sin(\sqrt{|y_i|}))-10D\le 0$$

$$x\in[-1000,1000]^{\mathrm{D}}$$

C15

$$\text{Min } f(x)=\sum_{i=1}^{D-1}(100(z_i^2-z_{i+1})^2+(z_i-1)^2)$$

$$z = x+1\text{-}o,\ y = (x\text{-}o)M$$

$$g_1(x)=\sum_{i=1}^{D}(-y_i\cos(\sqrt{|y_i|}))-D\le 0$$

$$g_2(x)=\sum_{i=1}^{D}(y_i\cos(\sqrt{|y_i|}))-D\le 0$$

$$g_3(x)=\sum_{i=1}^{D}(y_i\sin(\sqrt{|y_i|}))-10D\le 0$$

$$x\in[-1000,1000]^{\mathrm{D}}$$

C16

$$\text{Min } f(x)=\sum_{i=1}^{D}\frac{z_i^2}{4000}-\prod_{i=1}^{D}\cos(\frac{z_i}{\sqrt{i}})+1 \qquad z=x-o$$

$$g_1(x)=\sum_{i=1}^{D}\left[z_i^2-100\cos(\pi z_i)+10\right]\le 0$$

$$g_2(x)=\prod_{i=1}^{D}z_i\le 0$$

$$h_1(x)=\sum_{i=1}^{D}(z_i\sin(\sqrt{|z_i|}))=0$$

$$h_2(x)=\sum_{i=1}^{D}(-z_i\sin(\sqrt{|z_i|}))=0$$

$$x\in[-10,10]^{\mathrm{D}}$$

C17

$$\text{Min } f(x) = \sum_{i=1}^{D-1} (z_i - z_{i+1})^2 \qquad z = x - o$$

$$g_1(x) = \prod_{i=1}^{D} z_i \le 0$$

$$g_2(x) = \sum_{i=1}^{D} z_i \le 0$$

$$h(x) = \sum_{i=1}^{D} (z_i \sin(4\sqrt{|z_i|})) = 0$$

$$x \in [-10, 10]^{\mathrm{D}}$$

C18

$$\text{Min } f(x) = \sum_{i=1}^{D-1} (z_i - z_{i+1})^2 \qquad z = x - o$$

$$g(x) = \frac{1}{D}\sum_{i=1}^{D} (-z_i \sin(\sqrt{|z_i|})) \le 0$$

$$h(x) = \frac{1}{D}\sum_{i=1}^{D} (z_i \sin(\sqrt{|z_i|})) = 0$$

$$x \in [-50, 50]^{\mathrm{D}}$$

Table 1: Details of 18 test problems. *D* is the number of decision variables, $\rho = {|F|}/{|S|}$ is the estimated ratio between the feasible region and the search space, *I* is the number of inequality constraints, *E* is the number of equality constraints

Problem/Search Range	Type of Objective	Number of Constraints		Feasibility Region (ρ)	
		E	*I*	10D	30D
C01 $[0,10]^D$	Non Separable	0	2 Non Separable	0.997689	1.000000
C02 $[-5.12,5.12]^D$	Separable	1 Separable	2 Separable	0.000000	0.000000
C03 $[-1000,1000]^D$	Non Separable	1 Non Separable	0	0.000000	0.000000
C04 $[-50,50]^D$	Separable	4 2 Non Separable, 2 Separable	0	0.000000	0.000000
C05 $[-600,600]^D$	Separable	2 Separable	0	0.000000	0.000000
C06 $[-600,600]^D$	Separable	2 Rotated	0	0.000000	0.000000

C07 [-140,140]^D	Non Separable	0	1 Separable	0.505123	0.503725
C08 [-140,140]^D	Non Separable	0	1 Rotated	0.379512	0.375278
C09 [-500500]^D	Non Separable	1 Separable	0	0.000000	0.000000
C10 [-500,500]^D	Non Separable	1 Rotated	0	0.000000	0.000000
C11 [-100,100]^D	Rotated	1 Non Separable	0	0.000000	0.000000
C12 [-1000,1000]^D	Separable	1 Non Separable	1 Separable	0.000000	0.000000
C13 [-500,500]^D	Separable	0	3 2 Separable, 1 Non Separable	0.000000	0.000000
C14 [-1000,1000]^D	Non Separable	0	3 Separable	0.003112	0.006123
C15 [-1000,1000]^D	Non Separable	0	3 Rotated	0.003210	0.006023
C16 [-10,10]^D	Non Separable	2 Separable	2 1 Separable, 1 Non Separable	0.000000	0.000000
C17 [-10,10]^D	Non Separable	1 Separable	2 Non Separable	0.000000	0.000000
C18 [-50,50]^D	Non Separable	1 Separable	1 Separable	0.000010	0.000000

2. Performance Evaluation Criteria

Number of Problems: 18. Number or runs/trials: 25

Maximum Function Evaluations (Max_FES) = 200000 for 10D and 600000 for 30D

Population Size: You are free to have an appropriate population size to suit your algorithm while not exceeding the Max FES.

2.1 Presentation of Statistics

Record the function value of $f(X)$ for the achieved best solution X after 20000, 100000 and 200000 for 10D and 60000, 300000, 600000 for 30D. For each function, present the following: best, median, worst result, mean value and standard deviation for the 25 runs.

Please indicate the number of violated constraints (including the number of violations by more than 1, 0.01, and 0.0001) and the mean violations $\bar{v}$ at the median solution.

$$v = \frac{\left(\sum_{i=1}^{p} G_i(X) + \sum_{j=p+1}^{m} H_j(X)\right)}{m}$$

where

$$G_i(X) = \begin{cases} g_i(X) & if\ g_i(X) > 0 \\ 0 & if\ g_i(X) \le 0 \end{cases}$$

$$H_j(X) = \begin{cases} |h_j(X)| & if\ |h_j(X)| - \epsilon > 0 \\ 0 & if\ |h_j(X)| - \epsilon \le 0 \end{cases}$$

2.2 Feasibility Rate

Feasible Run: A run during which at least one feasible solution is found in Max FES.

Feasible Rate = (# of feasible runs) / Total runs.

The above quantity is computed for each problem separately.

2.3 Algorithm Complexity

a) $T1 = (\sum_{i=1}^{18} t1i)/18$. $t1i$ is the computing time of 10000 evaluations for problem i.

b) $T2 = (\sum_{i=1}^{18} t2i)/18$. $t2i$ is the complete computing time for the algorithm with 10000 evaluations for problem i.

The complexity of the algorithm is reflected by: $T1$; $T2$; *and* $(T2-T1)/T1$

2.4 Parameters

We discourage participants searching for a distinct set of parameters for each problem/dimension/etc. Please provide details on the following whenever applicable:

a) All parameters to be adjusted.
b) Corresponding dynamic ranges.
c) Guidelines on how to adjust the parameters.
d) Estimated cost of parameter tuning in terms of number of FEs.
e) Actual parameter values used.

2.5 Encoding

If the algorithm requires encoding, then the encoding scheme should be independent of the specific problems and governed by generic factors such as the search ranges.

3. Presentation of Results

Participants are suggested to present their results in the following format:

PC Configure:
System: CPU: RAM:
Language: Algorithm:

Parameters Setting:

a) All parameters to be adjusted.
b) Corresponding dynamic ranges.
c) Guidelines on how to adjust the parameters.
d) Estimated cost of parameter tuning in terms of number of FEs.
e) Actual parameter values used.

Results Obtained

Table 2: Function Values Achieved When FES = 2×10^4 , FES = 1×10^5, FES = 2×10^5 for 10D Problems C01-C06.

FEs		C01	C02	C03	C04	C05	C06
2×10^4	Best	237.9718					
	Median	358.3837					
	worst	446.8061					
	c	2, 0, 0					
	$\bar{v}$	5.3256					
	Mean	350.3861					
	std	103.2039					
1×10^5	Best	152.1540					
	Median	291.1380					
	worst	386.3278					
	c	0, 2 ,0					
	$\bar{v}$	4.12E-05					
	Mean	28.1940					
	std	101.189					
2×10^5	Best	-158.7482					
	Median	-55.7482					
	worst	38.5729					
	c	0, 0, 0					
	$\bar{v}$	0					
	Mean	-69.0852					
	std	64.4877					

Table 3: Function Values Achieved When FES = 2×10^4 , FES = 1×10^5, FES = 2×10^5 for 10D Problems C07-C12.

FEs		C07	C08	C09	C10	C11	C12
2×10^4	Best						

	Median						
	worst						
	c						
	$\bar{v}$						
	Mean						
	std						
1×10^5	Best						
	Median						
	worst						
	c						
	$\bar{v}$						
	Mean						
	std						
2×10^5	Best						
	Median						
	worst						
	c						
	$\bar{v}$						
	Mean						
	std						

Table 4: Function Values Achieved When FES = 2×10^4 , FES = 1×10^5, FES = 2×10^5 for Problems C13-C18 of 10D.

FEs		C13	C14	C15	C16	C17	C18
2×10^4	Best						
	Median						
	worst						
	c						
	$\bar{v}$						
	Mean						
	std						
1×10^5	Best						
	Median						
	worst						
	c						
	$\bar{v}$						
	Mean						
	std						
2×10^5	Best						
	Median						
	worst						
	c						

	$\bar{v}$						
	Mean						
	std						

Table 5: Function Values Achieved When FES = 6×10^4 , FES = 3×10^5, FES = 6×10^5 for Problems C01-C06 of 30D.

FEs		C01	C02	C03	C04	C05	C06
6×10^4	Best						
	Median						
	worst						
	c						
	$\bar{v}$						
	Mean						
	std						
3×10^5	Best						
	Median						
	worst						
	c						
	$\bar{v}$						
	Mean						
	std						
6×10^5	Best						
	Median						
	worst						
	c						
	$\bar{v}$						
	Mean						
	std						
	std						

Table 6: Function Values Achieved When FES = 6×10^4 , FES = 3×10^5, FES = 6×10^5 for Problems C07-C12 of 30D.

FEs		C07	C08	C09	C10	C11	C12
6×10^4	Best						
	Median						
	worst						
	c						
	$\bar{v}$						
	Mean						
	std						
3×10^5	Best						
	Median						

	worst						
	c						
	$\bar{v}$						
	Mean						
	std						
6×10^5	Best						
	Median						
	worst						
	c						
	$\bar{v}$						
	Mean						
	std						

Table 7: Function Values Achieved When FES = 6×10^4 , FES = 3×10^5, FES = 6×10^5 for Problems C13-C18 of 30D.

FEs		C13	C14	C15	C16	C17	C18
6×10^4	Best						
	Median						
	worst						
	c						
	$\bar{v}$						
	Mean						
	std						
3×10^5	Best						
	Median						
	worst						
	c						
	$\bar{v}$						
	Mean						
	std						
6×10^5	Best						
	Median						
	worst						
	c						
	$\bar{v}$						
	Mean						
	std						

c is the number of violated constraints at the median solution: the sequence of three numbers indicate the number of violations (including inequality and equalities) by more than 1.0, more than 0.01 and more than 0.0001 respectively. $\bar{v}$ is the mean value of the violations of all constraints at the median solution. The numbers in the parenthesis after the fitness value of the best, median, worst solution are the number of constraints which cannot satisfy feasible condition at the best, median and worst solutions respectively. Sorting method for the final results:

1. Sort feasible solutions in front of infeasible solutions;
2. Sort feasible solutions according to their function values *f(x*)*
3. Sort infeasible solutions according to their mean value of the violations of all constraints.

Algorithm Complexity

Table 8: Computational Complexity

*T*1	*T*2	(*T*2-*T*1)/*T*1

Convergence Graphs

The participants are expected to plot the convergence plots for the 10D and 30D problems of C09, C10, C14, C15, C17 and C18. The plot should show only feasible solutions of the best run out of the 25 runs.

Plot 1 – Convergence plot for 10D problems C09, C10, C14 and C15.
Plot 2 – Convergence plot for 30D problems C09, C10, C14 and C15.
Plot 3 – Convergence plot for 10D problems C17 and C18.
Plot 4 – Convergence plot for 30D problems C17 and C18.

Evaluation Criteria

1. The algorithms should not use explicit equations. Only the use of function calls is allowed.
2. Gradients, etc. can only be computed numerically and the function evaluations consumed in the process of gradient computations should be accumulated.
3. Evaluation of even one constraint function should be treated as one function evaluation.

References

[1] D. E. Goldberg, M. Samtani, “Engineering optimization via genetic algorithm”, Proc. of 9th Conf. on Electronic Computation, pp. 471-482, University of Alabama, 1986.

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