Advantages and Challenges of FMCW Radar Imaging with Shifting Sub-bands

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***Abstract*—High-resolution imaging is an ever-popular research topic in the radar community. However, achieving higher resolution generally necessitates larger bandwidths, posing significant engineering challenges. To overcome these challenges, synthesizing a large bandwidth using shifting sub-bands across different center frequencies has emerged as a promising technique. This study explores the advantages and challenges of frequency modulated continuous wave (FMCW) radar imaging with shifting sub-bands. Methods to compensate for phase errors have been investigated to synthesize baseband signal. The imaging performance has been evaluated through both simulations and outdoor experiments using universal software radio peripheral (USRP) X410.**

***Index Terms*—High-resolution imaging, phase and amplitude discontinuities, frequency modulated continuous wave (FMCW), shifting sub-bands, software defined radar (SDR).**

I. Introduction

High-resolution imaging has always been a critical aspect of radar technology, playing a pivotal role in applications ranging from military surveillance to weather forecasting and automotive safety. The ability to resolve fine details in radar images significantly enhances the performance and reliability of radar systems. As the demand for higher resolution grows, achieving this capability has become a major focus in radar engineering. High-resolution radar imaging typically requires large bandwidths, which presents significant engineering challenges due to the constraints on hardware, signal processing, and the electromagnetic spectrum.

Over the years, various techniques have been developed to address these challenges. One fundamental approach is the use of generalized sampling expansions. As demonstrated by Papoulis in his seminal work, a band-limited function can be uniquely determined using samples from multiple linear systems, sampled at a fraction of the Nyquist rate [1]. This approach provides a theoretical foundation for many advanced sampling techniques used in radar signal processing today.

Building on this foundation, Brown introduced a multichannel sampling technique that allows for the reconstruction of a band-limited signal from samples of multiple channel outputs, each sampled at a reduced rate [2]. This method has been shown to be effective in reducing the overall sampling rate while maintaining the integrity of the signal, thereby addressing some of the bandwidth limitations in radar systems.

Another significant advancement in high-resolution radar imaging is the use of narrowband linear chirps offset in frequency, as proposed by Lord and Inggs. Their method combines several narrow-bandwidth chirp pulses, each stepped in frequency, to synthesize a wideband signal that achieves high range resolution [3]. This technique has been particularly useful in synthetic aperture radar (SAR) imaging, where high resolution is essential for detailed ground mapping.

More recently, innovative methods such as chirp subpulse stepped frequency radar signal processing have been developed to overcome specific issues like "ghost images" caused by multiple scatters within a distance unit. Wei et al. proposed a technique that interfaces sub-pulses in the timeline and applies frequency shifting to create a high-resolution FM signal [4]. This method enhances the range resolution and mitigates artifacts in the radar image.

Additionally, Wang introduced a bandwidth synthesis algorithm for stepped chirp signals based on multichannel sampling theory. This approach views sub-band signals as responses of linear time-invariant filters, enabling effective reconstruction of the wideband signal. The algorithm improves the integrated sidelobe ratio and suppresses paired echoes, leading to clearer high-resolution range profiles [5].

The development of ultrawideband (UWB) software-defined radar (SDRadar) has further pushed the boundaries of high-resolution radar imaging. Prager et al. presented a method for

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synthesizing a wideband waveform using frequency stacking of stepped frequency signals. This technique leverages the flexibility of SDR technology to create low-cost, reconfigurable radar systems with high-range resolution [6].

While significant progress has been made in high-resolution radar imaging, challenges remain in achieving large bandwidths without compromising performance or increasing costs. Synthesizing large bandwidths using shifting sub-bands across different center frequencies offers a promising solution. This study aims to investigate the advantages and challenges of frequency modulated continuous wave (FMCW) radar imaging with shifting sub-bands.

II. FMCW Imaging via shifting sub-bands

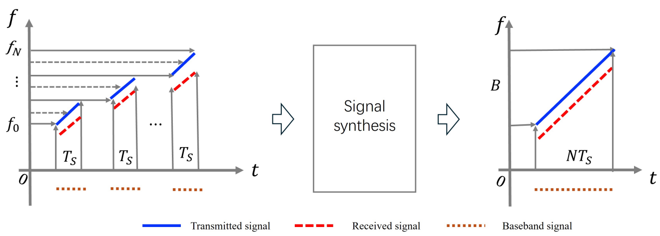


Fig. 1. Conceptual diagram of shifting sub-bands.

Fig. 1 illustrates the concept of shifting sub-bands in an FMCW radar system. The diagram depicts the transmitted signal, received signal, and the process of signal synthesis leading to the baseband signal. The technique of shifting sub-bands involves breaking down a wideband signal into several narrowband sub-bands, each transmitted and received separately. By stepping through different center frequencies, the system effectively covers a large overall bandwidth. The transmitted signal, represented by a blue solid line, consists of multiple sub-bands that are stepped in frequency over time. The frequency increases linearly within each sub-band, covering the range from f_0 to f_N. The sub-bandwidth is given by $B_s = K_r T_S$, where K_r is the chirp rate, and T_S is the sub-sweep duration. The k-th sub-band signal, with a stepped center frequency $(f_0 + (k - 0.5)B_s)$, is transmitted sequentially, with each occupying a duration T_S. The received signal, shown as a red dashed line, is a time-delayed version of the transmitted signal due to the round-trip delay. Each received sub-band follows the same frequency pattern as the transmitted sub-band but is shifted in time.

In the signal synthesis block, the received N sub-bands are processed to reconstruct a continuous wideband signal. This involves combining the frequency-stepped sub-bands to form a signal that spans the entire bandwidth $B = NB_s$. The synthesized baseband signal, depicted as a brown dotted line, is the result of combining all the received sub-bands and spans a total duration of NT_S. After receiving all sub-bands, they are equivalently synthesized to form a single, wideband received signal.

A. Advantages

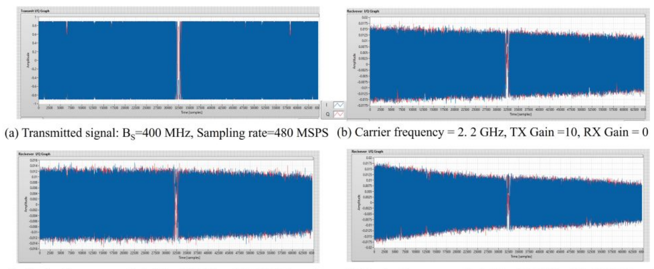
FMCW radar utilizing shifting sub-bands not only effectively controls and improves signal linearity but also significantly reduces sampling rates and enhances imaging resolution.

- **Ensuring linearity of FMCW**
  Achieving linearity over a wide bandwidth is challenging for FMCW radar signals. As the bandwidth increases, maintaining signal linearity becomes more difficult. By employing the shifting sub-bands technique, it is possible to transmit narrower sub-band signals over shorter durations, thereby ensuring the linearity of each sub-band.
- **Reducing sampling rate**
  For pulsed compression FMCW radar systems, the sampling rate of the signal must be at least 1.2 times the bandwidth. High sampling rates increase the complexity of data processing and storage requirements. By using the shifting sub-bands technique, the overall bandwidth is divided into multiple narrower sub-bands, each with a relatively lower sampling rate. Specifically, if the bandwidth is divided into N sub-bands, the sampling rate for each sub-band is reduced by a factor of N.
- **Enhancing imaging resolution**
  In radar imaging, successfully synthesizing an equivalent wideband received signal from multiple narrow sub-bands can significantly improve imaging resolution. Through the shifting sub-bands technique, multiple narrow sub-band signals can be combined after reception to achieve the equivalent of a wideband signal, thereby enhancing resolution.

B. Challenges

FMCW radar systems using the shifting subband technique to achieve high-resolution imaging face challenges such as amplitude discontinuity, phase discontinuity, and delay synchronization issues.

- **Amplitude discontinuity**
  The amplitude of radio frequency signals at different frequencies may vary in hardware implementation, leading to discontinuities in each subband signal's amplitude, as shown in Fig. 2.
- **Phase discontinuity**
  Each subband has a different initial phase, leading to phase discontinuities in the synthesized signal. Consequently, when sweeping frequencies, the phases of the subbands do not align seamlessly, resulting in phase discontinuities in the synthesized baseband signal, as shown in Fig.3.
  Compensating for phase and amplitude discontinuities is crucial to overcoming the disadvantages caused by shifting subbands. For amplitude compensation, referring to the amplitude of the dominant target in the range frequency domain for the first subband, the amplitude of the other subbands can be adjusted accordingly. For



(a) Transmitted signal: B_s=400 MHz, Sampling rate=480 MSPS (b) Carrier frequency = 2. 2 GHz, TX Gain =10, RX Gain = 0

(c) Carrier frequency = 2. 6 GHz, TX Gain =10, RX Gain = 0 (d) Carrier frequency = 3. 0 GHz, TX Gain =**30**, RX Gain = 0

Fig. 2. Amplitude response with different shifting sub-bands implemented on universal software radio peripheral (USRP) X410. (a) Transmitted sub-band signal; (b) Received signals with center frequency of 2.2 GHz, Tx Gain = 10, Rx Gain = 0; (c) Received signals with center frequency of 2.6 GHz, Tx Gain = 10, Rx Gain = 0; (d) Received signals with center frequency of 3.0 GHz, Tx Gain = 30, Rx Gain = 0.

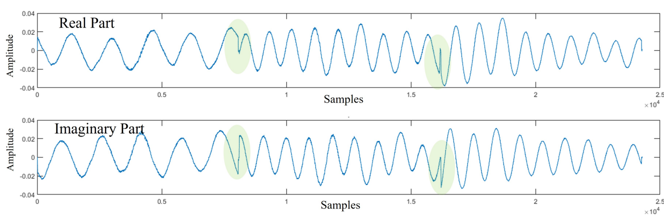


Fig. 3. Phase discontinuity in the synthesized baseband signal.

phase discontinuities, an l_1 norm-based method [7] can be applied to address this issue after amplitude compensation.

- **Delay synchronization issues**
  In Software-Defined Radio (SDR) implementations, moving different center frequencies involves different channel delays for each subband, making delay correction and synchronization difficult (See Fig. 4).

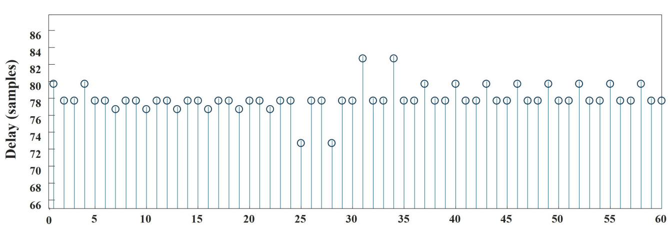


Fig. 4. Receiving delay of subbands with different center frequencies measured on the USRP X410.

C. Synthesised baseband signal with shifting sub-bands

Assuming the bandwidth for the FMCW signal is B, the sweep time is T, and the chirp rate is $K_r = \frac{B}{T}$, with a center frequency f_c, the transmitted signal can be written as

$$S_{\text{TX}}(t) = e^{j\phi(t)} = A_0 e^{j\left(2\pi f_c t + \pi K_r t^2\right)}, \quad t \in [0, T], \tag{1}$$

where A_0 is the amplitude of the transmitted signal.

Assuming a target is located at range R, the echo signal can be formulated as

$$S_{\text{RX}}(t) = e^{j\phi(t-\tau)} = A_1 e^{j\left[2\pi f_c (t-\tau) + \pi K_r (t-\tau)^2\right]}, \tag{2}$$

where A_1 is the amplitude of the received signal, $\tau = \frac{2R}{c}$ is the round-trip time delay, and c is the speed of light.

To perform the dechirp operation, the baseband signal $S_{\text{b}}(t)$ can be obtained by mixing the received signal with the conjugate of the transmitted signal

$$S_{\text{b}}(t) = S_{\text{RX}}(t) \cdot S^{*}_{\text{TX}}(t) = A_2 e^{-j2\pi f_c \frac{2R}{c} - j2\pi K_r t \frac{2R}{c} + j\pi K_r \frac{4R^2}{c^2}}, \tag{3}$$

where A_2 is the resulting amplitude after mixing.

Now consider the transmission and reception with N shifting subbands. The chirp rate remains K_r, the sub-bandwidth is defined as $B_s = \frac{B}{N}$, and the sub-sweep time is $T_s = \frac{T}{N}$. The center frequency of the k-th subband is defined as $f_c^k = f_0 + (2k-1)\frac{B}{2N}$.And the f_c can be rewritten as $f_c = f_0 + B/2$. According to equation (3), the baseband signal for the k-th subband is given by

$$S_{\text{b}}^k(t_s) = A_2^k e^{-j2\pi f_c^k \frac{2R}{c} - j2\pi K_r t_s \frac{2R}{c} + j\pi K_r \frac{4R^2}{c^2}}, \quad t_s \in [0, T_s], \tag{4}$$

where A_2^k is the amplitude after mixing, and $t_s = t - (k-1)T_s$.

According to the phase of equations (3) and (4), the phase error is given by

$$\begin{aligned}
\phi_e^k(t) &= \phi[S_{\text{b}}(t)] - \phi[S_{\text{b}}^k(t_s)], \\
&= -2\pi\left(f_c - f_c^k\right)\frac{2R}{c} - 2\pi K_r\left(t - t_s\right)\frac{2R}{c}, \\
&= -2\pi\left(N - 2k + 1\right)\frac{B}{2N}\frac{2R}{c} - 2\pi K_r (k-1) T_s \frac{2R}{c}, \\
&= -2\pi\left(N - 2k + 1\right)\frac{B}{N}\frac{R}{c} - 2\pi(2k-2)\frac{B}{N}\frac{R}{c}, \\
&= -2\pi\frac{N-1}{N}\frac{BR}{c}.
\end{aligned} \tag{5}$$

According to the above equation, assuming there are no amplitude and phase discontinuity issues, the base signal of each sub-band has the same range variant phase errors. Therefore, the baseband signal can be synthesized in ascending order of the sub-center frequencies. Then, in the imaging algorithm, the range variant phase errors can be compensated in the range frequency-slow time domain.

III. Imaging Experiments

A. Simulations

To investigate the imaging performance of shifting subbands, the NTU shape 19-point targets are used to simulate. The radar parameters used in the simulations are summarized in Table I.

TABLE I
Simulation Parameters.

Start frequency	Sub-band number	Sub-bandwidth
2.0 GHz	3	0.4 GHz

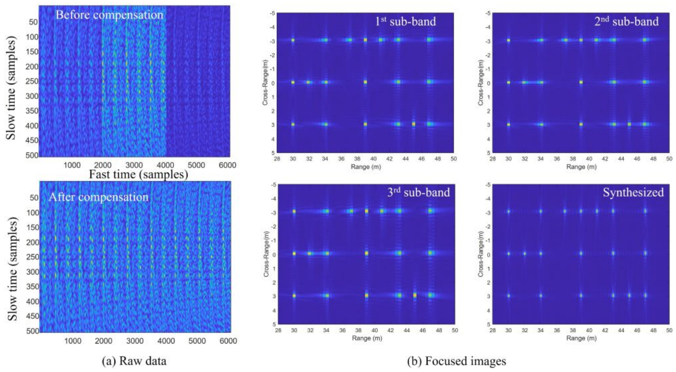


Fig. 5. Simulated imaging results.

As shown in Fig. 5(a), the amplitude discontinuities between different sub-bands are clearly visible as abrupt changes in color intensity along the fast time axis. These discontinuities result from variations in the amplitude of the received signals for different sub-bands, which have different center frequencies and bandwidths. After compensation, the discontinuities observed in the previous plot are significantly reduced, resulting in a more continuous and uniform amplitude across the different sub-bands.

According to the comparisons in Fig. 5(b), the synthesized image, obtained by combining the baseband signals of all sub-bands using the proposed method, shows significantly improved imaging resolution compared to the individual sub-band images. The targets are more clearly resolved, demonstrating the effectiveness of the proposed signal synthesis method in enhancing imaging performance.

B. Hardware experiment on USRP X410

The experimental parameters for investigating the shifting subband imaging performance are summarized in Table II. The center frequency is set to 1.8 GHz, and the sub-band number is 4. Each sub-band has a bandwidth of 0.4 GHz, resulting in a total bandwidth of 1.6 GHz. The platform movement is manually controlled by using "stop and go" mode.

TABLE II
HARDWARE EXPERIMENTAL PARAMETERS.

Start frequency	Sub-band number	Sub-bandwidth
1.0 GHz	4	0.4 GHz

Fig. 6 shows the hardware setup. The dimensions of the scenario are labeled, with the imaging length in the cross-range being 1 meter. The distance from the radar system to the wall is 6 meters. As demonstrated in Fig. 7, the fire hydrant and the two corner reflectors are not focused in the sub-band images due to random delay jumps in the different sub-bands of the hardware. These delays cause the energy to disperse in the range direction, leading to blurred targets. However, the wall is clearly imaged in the synthesized result, validating that sub-band shifting can provide improved imaging resolution. This indicates that, despite the challenges, the proposed method effectively synthesizes the sub-bands to achieve better resolution.

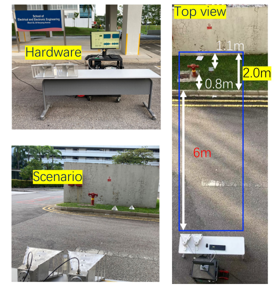


Fig. 6. Hardware imaging experiments.

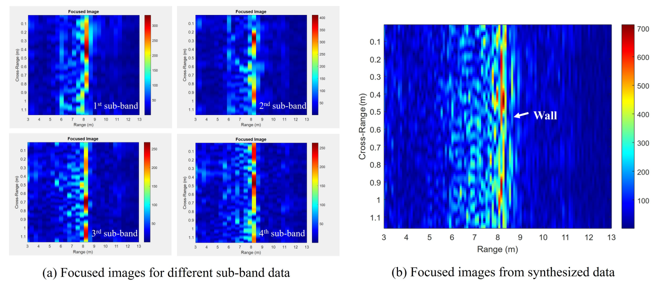


Fig. 7. Imaging results.

IV. Conclusions

Theoretically, shifting sub-bands can achieve "Small Moves, Big Gains," as verified by our theoretical simulations. However, practical implementation on the USRP X410 encounters issues such as amplitude and phase discontinuities, and varying delays across sub-bands. These issues significantly degrade imaging quality, making it challenging to focus on small targets.

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