

Statistical Characterization of Wave Propagation in Mine Environments

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Abstract— A computational framework for statistically characterizing electromagnetic (EM) wave propagation through mine tunnels and galleries is presented. The framework combines a multi-element probabilistic collocation (ME-PC) method with a novel domain-decomposition (DD) integral equation-based EM simulator to obtain statistics of electric fields due to wireless transmitters in realistic mine environments.

I. INTRODUCTION

The design of wireless communication systems for mine environments calls for simulation frameworks capable of analyzing EM wave propagation through electromagnetically extended tunnels and galleries occupied by miners and loaded with mining equipment, trolleys, and rails. Ideally, these frameworks should provide the statistics of key observables (e.g., the probability density function (pdf) of the electric field at a receiver), given uncertainty in the mine geometry, configuration, and excitation (e.g., the positions of transmitters, receivers, and obstacles, and the material properties of mine walls). This paper presents a computational framework that achieves the above. The proposed framework leverages an ME-PC method to approximate observables (and thereby, their statistics) that potentially vary rapidly [1, 2] over the “random domain”, i.e. the domain of variables that characterize the above uncertainties. The ME-PC method uses a novel DD and integral equation-based EM simulator to compute observable values for mine configurations described by collocation/integration points in the random domain. The DD-based EM simulator divides the physical domain into smaller subdomains and characterizes EM wave propagation in each subdomain independently prior to obtaining a global inter-domain solution by assembling the subdomain solutions. The DD approach significantly expedites the computation of observable values at each ME-PC collocation point as it only requires re-characterization of the propagation properties of those subdomains that host uncertainties in the mine environment.

II. DETERMINISTIC EM SIMULATOR

A typical mine environment comprises tunnels and galleries extending hundreds of wavelengths at the operating frequencies of modern communication systems (e.g., 900MHz band for IEEE 802.15.4 standard). The rock, earth, and mine ore surrounding the tunnels and rooms act as lossy dielectrics, and the roughness on the walls are significant at these frequencies.

Characterization of EM wave propagation in such an environment is a challenging task due to the electrical size of the physical domain and the intricate geometric details that must be accounted for.

The aforementioned DD-based EM simulator tackles these challenges by decomposing the mine tunnel/environment D into smaller subdomains D_i , $i = 1, \dots, N_d$ [Fig. 1(a)]. EM wave interaction in each subdomain D_i is characterized by introducing equivalent electric and magnetic currents on surfaces S_k^e , where k are indices of the surfaces interfacing D_i [Fig. 1(b)]. Here the S_k^e , $k = 1, \dots, N_p$ are chosen to cover the air-interfaces between neighboring domains, while slightly extending into the lossy background to capture the decaying fields. A surface integral equation (SIE) approach is employed to characterize EM fields inside each D_i , $i = 1, \dots, N_d$ for arbitrary port excitations. Next, an inter-domain system of equations for the unknown equivalent currents on S_k^e is assembled and solved when D is excited with wireless transmitters. In this inter-domain system, the equivalent surfaces only interact if they bound the same domain since the background medium is assumed highly lossy. Once the unknown currents have been solved for, electric fields at the receivers (observable values) are obtained and output to the ME-PC method.

The ME-PC method requires repetitive runs of the EM simulator for many different realizations of the uncertainties in the mine environment. Typically, one mine realization only differs from another in just a few subdomains while others remain unaltered. Hence, only the subdomains hosting the uncertainties must be re-characterized, which reduces the computational cost of the DD-EM simulator significantly when compared to that of its non-DD counterparts.

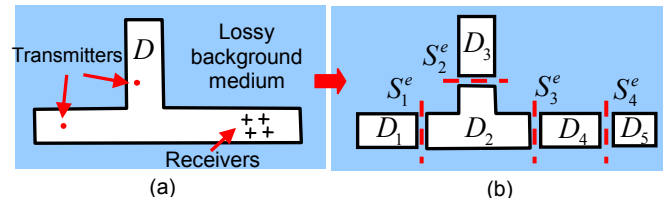


Figure 1. (a) Layout of a simplified L-shape tunnel. (b) Domain decomposition and the equivalent surfaces.

III. STATISTICAL FRAMEWORK: ME-PC METHOD

Let the N_{dof} -dimensional vector $\mathbf{x} = [x^1, x^2, \dots, x^{N_{\text{dof}}}]$ and $V(\mathbf{x})$ represent the vector of uncertain variables in the mine environment defined over a random domain Ω and the observable, in this case the E-field at a receiver, to be characterized, respectively. Each random variable x^i , $i = 1, \dots, N_{\text{dof}}$, is assumed uniformly distributed within the range $[a^i, b^i]$. The generalized polynomial chaos (gPC) expansion enables the approximation of $V(\mathbf{x})$ via multivariate orthogonal Legendre polynomials $\Psi(\mathbf{x})$ as [2]

$$V(\mathbf{x}) \approx \sum_{m=0}^{N_p} v_m \Psi_m(\mathbf{x}). \quad (1)$$

Here, $N_p = (N_{\text{dof}} + p)! / (N_{\text{dof}}! p!) - 1$, p is the order of the expansion, and v_m is the m^{th} gPC expansion coefficient which is computed as

$$v_m = \int_{\Omega} V(\mathbf{x}) \Psi_m(\mathbf{x}) d\mathbf{x}. \quad (2)$$

The N_{dof} -dimensional integral in (2) is evaluated numerically using tensor product or sparse grid integration rules. The observable values at collocation points dictated by integration rules are computed via DD-based EM simulator described above.

Unfortunately, when the observable varies rapidly in the random variables, expansion (1) becomes inefficient as it calls for polynomials of very high order to yield sufficient accuracy. The ME-PC method circumvents this bottleneck by recursively and adaptively dividing the initial random domain Ω into subdomains $\tilde{\Omega}$ using the decay rates of $V(\mathbf{x})$'s local variances as a guide [2], and using the gPC expansion in (1) within each subdomain to locally approximate $V(\mathbf{x})$ using low-order polynomials. The local variance of $V(\mathbf{x})$ in $\tilde{\Omega}$ is approximated by the p^{th} -order gPC expansion as

$$\text{var}_p = \sum_{m=1}^{N_p} \tilde{v}_m^2. \quad (3)$$

Here, \tilde{v}_m is the m^{th} coefficient of the gPC expansion constructed in $\tilde{\Omega}$ and is computed using (2). The decay rate of the local gPC expansion's relative error is defined as $\gamma = (\text{var}_p - \text{var}_{p-1}) / \text{var}_p$. If γ exceeds a specified tolerance ϵ_1 , the subdomain $\tilde{\Omega}$ is selected for adaptive refinement. The refinement is performed along dimension(s) in which $V(\mathbf{x})$ varies rapidly. The sensitivity of each dimension in $\tilde{\Omega}$ is defined as $\alpha^i = \tilde{v}^i / (\text{var}_p - \text{var}_{p-1})$, where \tilde{v}^i stands for the coefficient of the p^{th} -order gPC expansion, which is only pertinent to the i^{th} dimension. Refinement is performed along the i^{th} dimension if α_j^i satisfies a certain criterion. This procedure is recursively applied to generate non-overlapping subdomains which do not need refinement. Using the gPC

coefficients of such subdomains, accurate approximations to $V(\mathbf{x})$ and its statistics are obtained [2].

IV. NUMERICAL RESULTS

The proposed framework is applied to the statistical characterization of E-fields in a realistic mine tunnel configuration [Fig. 2(a)]. The relative permittivity of tunnel walls is $5 - j$. A unit source operating at 900 MHz is located at $(x, y) = (2, 2)$ m. The uncertain variable in the configuration is the position of source along y axis, which is uniformly distributed in the range $[a^1, b^1] = [1.5, 2.5]$ m. The observables are the magnitudes of the z -component of the E-field, $|E_z|$, sampled on a grid (with 0.5 m spacing along each dimension) in "Observer domains". Mean and standard deviation of $|E_z|$ on grid points are computed using the ME-PC method, which required 91 deterministic simulations for $\epsilon_1 = 10^{-2}$ [Fig. 2 (b)-(c)]. Next, the pdf of $|E_z|$ on all grid points is estimated via a Monte Carlo simulation (with 10,000 samples) performed using approximate $|E_z|$ values [Fig. 2 (d)]. Apparently, the empirical pdf of $|E_z|$ matches with a gamma distribution.

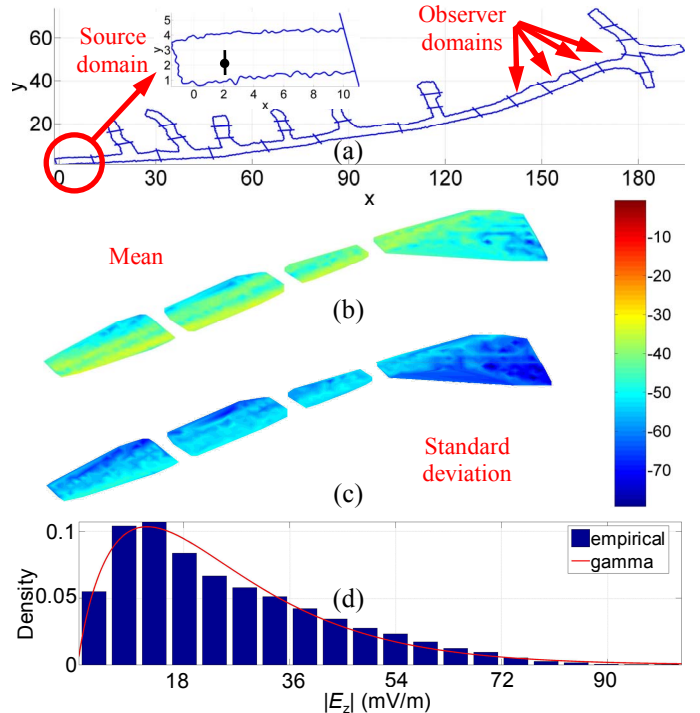


Figure 2. (a) Layout of a realistic mine tunnel. (b)-(c) Mean and standard deviation of $|E_z|$ on grid points in observer domains (in dB scale). (d) Empirical pdf of $|E_z|$ computed on all grid points and a fitting gamma distribution.

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