# A New FDTD Formulation for Long Distance Wave Computations

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*Abstract*— A new method is presented to compute radar returns scattered by a wind mill farm. This method will enhance the capability of the radar systems to locate low flying aircraft near the wind farm. This uses a previously developed method, Wave Confinement, to propagate the details of radar returns into the far field. A simple demonstration to compute the scattered signal from a single wind turbine is presented.

## I. INTRODUCTION

Windmill farms present a difficult challenge to radar systems, especially when they are located near Air Force bases or airports. The problem that we are address concerns aircraft approaching an airport located near a windmill farm. This problem would be simple if the individual windmills were placed far apart, or were all in air flowing at the same speed and direction. However, windmill farms often have several hundred turbines spaced only a few diameters apart, each of which is exposed to varying winds because they are at different locations in a complex topography. The scattered signals will then undergo refraction and multiple reflections.

We present a new method that uses our previously developed technique, Wave Confinement (WC), to accurately compute radar returns. This involves propagation in a realistic environment that has index of refraction variations and complex terrain. Currently, the only alternative method—"Ray Tracing" [1]—can take some of these effects into account, but involves an incoherent collection of "rays", from which it is difficult to extract information. Ray tracing can even become chaotic, or neglect caustics altogether. Details of the basic method are included in [2] and some new developments are briefly discussed in this paper.

## II. METHODOLOGY

The above problem as shown in Figure 1 is solved in 4 steps: a) Far field propagation, b) Near Field generation on a "source" surface surrounding known emitters or targets such as windmills and aircraft. c) Computation of small scale return signals by applying the arrival time, attenuation factor and propagation vector. Eric Michielssen, Abdulkadir Yucel Electrical Engineering and Computer Science University of Michigan Ann Arbor, MI, USA emichiel@umich.edu



Fig.1: Problem Description

## A. Far Field Propagation

A new, simple, computationally efficient wave simulation method that overcomes most of the problems of conventional methods for long distance propagation in inhomogeneous media, including multiple reflections is used for far field propagation. In its initial form, it involves the scalar wave equation with non-dispersive and non-diffusive media, but these limitations can be removed in future versions with perturbation terms. This new method is termed the "Generalized Eikonal Method" (GEM).

The solution at any far field receiver is assumed to be represented by smooth variables (away from caustics): attenuation factor (  $A_k^{\rm rec}$  ), propagation vector (  $\vec{S}_k^{\rm rec}$  ) and arrival time  $(\tau_k^{rec})$ . It is assumed that the wave equation is accurately computed (with conventional methods) in a small region surrounding windmills and aircraft, with dimensions comparable to the wavelength of interest ( $\lambda$ ). In general, because of the reflections and refractions in realistic media, the wave paths, or Eikonal phase will be multi-valued in some regions of space, which represent multiple passes of the wave front. Then, at each grid node, an array of phase is stored, one for each wave front passage. This is easily accomplished using a counter, which serves as an array index (k). For each k, or "trajectory", the recently developed method - "Wave Confinement" - (WC), is used to solve a modified wave equation [2]. This method generates values of the "computational wave"- $\Psi_k$  at grids nodes when equation (1) is discretized.

$$\partial_t^2 \psi = c^2 \partial_x^2 \psi + F \tag{1}$$

where F is a combination of positive and negative (stable) dissipation. The purpose of this modification is to generate short, coherent Nonlinear Solitary Waves which represent wave fronts. When equation (1) is discretized, the solitary waves persist indefinitely, remaining concentrated over only 2-3 grid cells, unlike conventional numerical schemes, where the waves dissipate. The centroids of these computational waves represent the wave fronts of the actual waves, which may be much shorter the computational waves. In this way, it is computationally feasible to solve for wave propagation over a large region, containing many waves.

# B. Near Field Generation

A new parallel fast Fourier transform and fast multipole method (FFT-FMM) -accelerated surface integral equation solver [3] has been developed which is capable of analyzing scattering from wind turbines in the high-frequency regime. The novelty of the solver lies in its parallelization scheme as well as the use of a singular value decomposition scheme for compressing near-field interactions.

We assume that the transmitters are located a few kilometers away from the wind farms and hence the EM wave is planar. So, the wind turbine is illuminated by a plane-wave (propagating along y direction). E-field with unit magnitude is polarized along z direction. The time history of the scattered signal is computed on a "source" surface surrounding the near field of a scattering object, which will later be used to reconstruct the signal at any far field point. Note: In the current paper, the time history of the electric field intensity envelope is used for simplicity because the main objective of the paper is to show the demonstration of a coupled inner field near the wind mill and the WC based far field method.

## C. Return Signal Computation

Now, we project out the waves that will intersect the receiver. So, at any far field point, the physical signal is computed as

$$E\left(\overline{x}, t^{rec}\right) = A\left(\overline{x}_{source}, t^{rec}\right) * E\left(\overline{x}_{source}, t^{rec}\right).$$
(2)

where  $t^{rec} = \tau^{rec} + t^{source}$ , and  $t^{source}$  is the time of the near field computation.

### III. DEMONSTRATION

The new method described in the above section is used to compute the return single from a single wind turbine. Time history of the scattered signal computed in the near field and is saved on a "source" surface around wind turbine. The near field points are selected on a sphere with the radius of 125m (3 times blade's length). In Figure 2, a snapshot of the Electric field intensity is shown on a sphere of radius 3 rotor radii with wind turbine at the center.

Nonlinear solitary wave propagation, taking into account reflection and refraction, can provide information about the region on the inner field that accounts to the signal at the receiver, which will then be used to compute the signal. For multiple arrivals, point of origin and return signals corresponding to each arrival are computed. For the above case, the return signal is computed at a receiver placed ~700m away. The computed first arrival (incident) and second arrival (reflected) signals are plotted against the exact signals in Figure 3 and Figure 4 respectively and appear to match well.



Fig.3: First Arrival for One Windmill at ~700m distance



Fig.4: Second Arrival for One Windmill at ~700m distance

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