Parallel Self-Tuning MLFMA Library

***Abdulkadir C. YUCEL*, Francesco P. Andriulli, and Eric Michielssen**

Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI, USA

E-mail: acyucel@umich.edu, fandri@umich.edu, emichiel@umich.edu

The Multilevel Fast Multipole Algorithm (MLFMA) has gained considerable attention for its ability to reduce the computational complexity of matrix-vector multiplies arising in the integral equation-based iterative solution of electromagnetic scattering problems. The MLFMA evaluates fields due to known source constellations comprising *N* discrete sources in a hierarchical framework that requires (i) breaking up the source into groups, (ii) characterizing each group's far field signatures, and (iii) translating these far-field signatures between group centers to arrive at observer fields. Steps (ii) and (iii) are often accomplished with the aid of spherical scalar and vector interpolators and filters. The MLFMA reduces the cost of evaluating fields from $O(N^2)$ to $O(NLogN)$. However, the multiplicative constant inherent in these complexity estimates, as well as the MLFMA's accuracy, heavily depends on the choice of some key parameters. First, there is the number of multipoles *L* used to compute the MLFMA's translation operators. Second, there is the oversampling ratio *s* used to sample far-field signatures. And third, there is the number of interpolation points *p* used when locally interpolating fields during the upward traversal of the MLFMA tree. (Other parameters exist, but these are the three most important ones).

Here, robust techniques for optimally selecting *L*, *s*, and *p* to achieve a desired error level are presented. These algorithms enable the MLFMA to automatically self-tune for any source-test configuration without sacrificing accuracy. The well-known excess bandwidth formula to determine *L* parameter (S. Koc et al., *SIAM J. Numer. Anal.*, 36(3), 906-921, 1999) is semi-heuristic in nature and in many cases not accurate/optimal. In contrast, the algorithm elucidated here finds the minimum truncation number *L* for any source-test configuration due to given error rate. In addition, the method to obtain optimal (*p*,*s*) pairs for Lagrange interpolators described in (O. Ergul et al., *IEEE Trans. Antennas Propag.*, 54(12), 3822-3826, 2006) is based upon tests conducted on several box sizes (or truncation numbers *L*) yielding 10^{-3} error levels. In contrast, the algorithms proposed here directly locate the optimum *p* or *s* parameter for any desired error level, truncation number *L*, and *s* or *p* without performing any tests. The algorithms presented rely on an error analysis of approximate prolate spheroidal functions (APS) that can be considered optimal local interpolators for massively parallel MLFMA implementations (S. Velamparambil et al., *IEEE Antennas Propag. Mag.*, 45(2), 43-58, 2003). The proposed algorithms have been implemented in a library, to be executed as a preprocessor to the actual MLFMA call. In practice, its execution requires negligible CPU time and storage, while resulting in significant MLFMA savings.