

# A Domain Decomposition based Surface Integral Equation Solver for Characterizing Electromagnetic Wave Propagation in Mine Environments

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**Abstract**— A 3D domain decomposition (DD) based surface integral equation (SIE) solver for analyzing electromagnetic (EM) wave propagation in 3D mine environments is proposed. In addition to the fast characterization of deterministic propagation scenarios the solver permits the efficient statistical characterization of EM wave propagation scenarios as well as the fast design, deployment, and reconfiguration of wireless sensing, communication, and tracking systems in realistic tunnel and gallery environments.

**Keywords**—domain decomposition; integral equation; mine tunnels.

## I. INTRODUCTION

In addition to their critical role in planning and conducting routine mining operations, wireless sensing, communication, and tracking systems are essential for safeguarding miners' health and planning life-saving actions following catastrophic events. The design of such systems as well as their optimum placement in mine tunnels and galleries greatly benefits from EM simulators, either approximate or full wave in nature. Approximate EM solvers based on ray-tracing, waveguide models, and cascaded impedance techniques are typically useful for only limited frequency bands and do not readily take into account the presence of miners, mining equipment (e.g., cables, carts, rails), and a possible (partial) tunnel cave-in [1]. Full wave EM solvers based on finite-difference time-domain, finite element, and surface integral equation (IE) methods are free from any such restrictions and in principle can be used to simulate EM wave propagation in any realistic mine tunnel and gallery [1]. That said, full-wave solvers are computationally expensive, especially when used to statistically characterize the environment or to optimally (re)configure wireless networks. This is because such statistical analysis or synthesis require the repetitive execution of the full-wave solver for different mine configurations and/or excitations, each of which requires significant computational time [2,3].

To alleviate this computational burden, a domain decomposition (DD)-based SIE full-wave solver was recently proposed and applied to statistical characterization of  $TM_z$  wave propagation for 2D models of mine environments [2]. The DD-based SIE solver divides the physical mine tunnel or gallery into subdomains and characterizes wave propagation in

each subdomain separately. It next obtains a global inter-domain solution by assembling the solutions of subdomains. The DD-based SIE approach is well suited for stochastic EM analysis and optimum wireless network (re)configuration since it only requires re-characterization of subdomains in which the excitation and/or configuration of mine tunnel change for each of the mine configurations requiring the execution of the full-wave solver.

Here, we extend this methodology to 3D and demonstrate its applicability for characterizing EM wave propagation in realistic mine environments. Numerical result shows that the proposed solver is roughly four times faster than the traditional SIE solvers when applied to statistical characterization of electric fields in a 400-meter tunnel at 455 MHz. The speedup achieved by the DD-based SIE solver over traditional SIE solvers increases with frequency and the electrical dimensions of the environment.

## II. FORMULATION

Let  $\Omega$  denote the dielectric walls of a closed mine tunnel or gallery that is surrounded by unbounded ore with relative permittivity  $\epsilon_r$ , relative permeability  $\mu_r$ , and conductivity  $\sigma$ . The proposed DD strategy decomposes the physical domain of a mine environment into subdomains. For the sake of illustration, consider a straight rectangular mine tunnel split into four subdomains, each of which contains a portion of  $\Omega$  denoted  $\Omega_i$ ,  $i=1,\dots,4$  [Fig. 1]. By invoking Huygens' equivalence principle, equivalent electric currents  $\mathbf{J}_k^e$  and magnetic currents  $\mathbf{M}_k^e$  are defined on the equivalent surfaces  $S_k^e$ ,  $k=1,\dots,3$  between neighboring subdomains [Fig. 1]. The equivalent surfaces only cover the air-interface region of two neighboring subdomains plus a short extension into the typically highly lossy ore, which suffices to capture decaying fields. Next, the equivalent currents on each  $S_k^e$ ,  $k=1,\dots,3$  are expanded in terms of  $N_k$  Rao-Wilton-Glisson (RWG) basis functions  $\mathbf{f}_{k,j}$  [4] as

$$\mathbf{J}_k^e = \sum_{j=1}^{N_k} \{\mathbf{I}_k^J\}_j \mathbf{f}_{k,j}, \quad \mathbf{M}_k^e = \sum_{j=1}^{N_k} \{\mathbf{I}_k^M\}_j \mathbf{f}_{k,j} \quad (1)$$

where  $\mathbf{I}_k^J$  and  $\mathbf{I}_k^M$  are vectors of unknown current expansion coefficients on  $S_k^e$ . Utilizing the discrete current representations, enforcing field continuity conditions on each of the  $S_k^e$ , and applying a Galerkin testing procedure yields the inter-domain system of equations

$$\begin{bmatrix} \mathbf{Z}_{11}^1 + \mathbf{Z}_{11}^2 & \mathbf{Z}_{12}^2 & 0 \\ \mathbf{Z}_{21}^2 & \mathbf{Z}_{22}^2 + \mathbf{Z}_{22}^3 & \mathbf{Z}_{23}^3 \\ 0 & \mathbf{Z}_{32}^3 & \mathbf{Z}_{33}^3 + \mathbf{Z}_{33}^4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1^{inc} \\ \mathbf{V}_2^{inc} \\ \mathbf{V}_3^{inc} \end{bmatrix} \quad (2)$$

where  $\mathbf{I}_k = [\mathbf{I}_k^J, \mathbf{I}_k^M]^T$ ,  $\mathbf{V}_k^{inc}$  is the vector of tested incident electric and magnetic fields on  $S_k^e$  due to impressed sources in the subdomains touching to  $S_k^e$ , and  $\mathbf{Z}_{nk}^i$  maps currents on  $S_k^e$  to fields on  $S_n^e$  by accounting for the propagation characteristics of  $i$ th subdomain. The procedure to obtain  $\mathbf{Z}_{nk}^i$  submatrices is as follows: (i) each basis function on  $S_k^e$  is excited by one current source at a time and the fields generated by this current source are tested on RWG basis functions defined on  $\Omega_i$  of  $i$ th subdomain. (ii) The electric and magnetic currents on  $\Omega_i$ , which give rise to the tested fields obtained in previous step, are computed using a parallel SIE-based, full-wave solver [3]. (iii) The fields generated by the currents on  $\Omega_i$  are tested on each basis function on  $S_n^e$ . Once the  $\mathbf{Z}_{nk}^i$  submatrices are computed and stored, the linear system of equations in (2) is solved to obtain the unknown coefficients of the currents on  $S_k^e$ . The currents on  $S_k^e$ ,  $k=1, \dots, 3$ , are used to compute the currents on  $\Omega_i$ ,  $i=1, \dots, 4$ , (and hence fields anywhere inside the tunnel) by performing the abovementioned steps (i) and (ii). While repetitively executing the proposed solver for different mine excitations, the solver only updates the right hand side (RHS) of (2) and re-solves the reduced system. In case the configuration of mine tunnel changes, the solver re-computes the  $\mathbf{Z}_{nk}^i$  submatrices for the changed subdomains and re-solves the reduced system in (2).

### III. NUMERICAL RESULTS

The proposed DD-based SIE solver is used to statistical characterization of electric fields in a 400-meter long rectangular mine tunnel at 455 MHz [Fig. 2(a)]. The tunnel is excited by a  $z$ -directed electric dipole (vertical polarization) that is positioned at  $(50.0, 0.9, z)$  m, where  $z$  is the uncertainty variable uniformly distributed in the range  $[0.6, 1.6]$  m. The proposed solver decomposed the tunnel into 80 subdomains and computed the normalized magnitudes of electric fields on a line connecting  $(51.0, 0.9, 1.2)$  m and  $(225, 0.9, 1.2)$  m. In Fig. 2(b), the mean and standard deviation of electric fields obtained by the proposed solver via a ME-PC method [3] are presented. Next, a pdf of electric field at a selected receiver point is obtained using the estimated field values from the ME-PC [Fig. 2(c)]; 50 simulations are required by the ME-PC in this case. The computational time required for one execution by the proposed solver is compared with that required by the

(traditional) SIE solver [3] after both solvers were executed on 16 dual hexacore X5650 Intel processors. While the traditional SIE solver requires 4.2 hours, the computational times to obtain  $\mathbf{Z}_{nk}^i$  submatrices and to solve (2) in DD-based SIE solver are 5.0 and 1.0 hours, respectively. As the proposed solver only requires updating RHS of (2) and re-solution of (2) during repetitive executions for the statistical characterization, the proposed DD-SIE solver is roughly four times faster than the traditional SIE full wave solver.

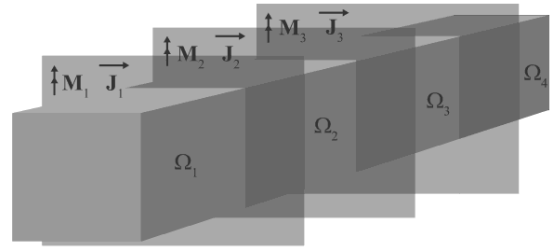


Fig. 1. A rectangular mine tunnel decomposed into four subdomains and associated equivalent surfaces and currents

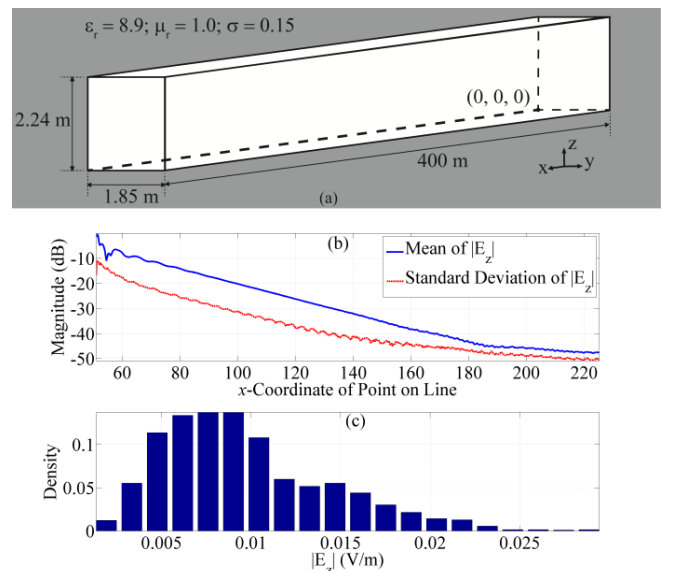


Fig. 2. (a) The geometry of the rectangular mine tunnel. (b) Mean and standard deviation of normalized electric fields on all receiver points and (c) pdf for selected receiver point.

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