Fast Statistical Characterization of Rough Surface Scattering via Tensor Train Decompositions

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The analysis of electromagnetic (EM) scattering from random rough surfaces is of paramount interest to many disciplines ranging from microwave remote sensing, and communication to microelectronics and optics. Oftentimes, this analysis is performed by traditional Monte Carlo (MC) techniques. MC techniques call for the generation of random rough surfaces by probing their height probability density function (PDF) and execution of full-wave EM simulators for each realization. MC methods are straightforward to implement and readily provide the statistics (e.g. mean, standard deviation, and PDF) of the scattered EM fields. That said, they often converge slowly and require a prohibitively large number of random rough surface generations and hence full-wave EM simulations. As alternatives to traditional MC methods, surrogate-model assisted MC methods leveraging generalized polynomial chaos (gPC) expansions and sparse grid (SG) interpolation schemes have recently received significant attention due to their efficiency and accuracy. These methods construct surrogate models of observables (e.g. scattered fields) by judiciously selecting a small number of random rough surface realizations and the MC technique is applied using the surrogate model in lieu of the full-wave EM simulator. Unfortunately, when the number of random variables is large, gPC and SG generated surrogate models lack accuracy. To tackle the *curse of dimensionality* in statistical EM studies, high dimensional model representation (HDMR) techniques and anisotropic sparse grid (ASG) techniques have been proposed. These are effective when the observables can be represented in terms of the random variables' individual contributions and *low-order* combined contributions. That said, they lose their efficiency/accuracy for observables that are functions of *higher-order*, combined contributions of random variables. The latter is the case for stochastic analysis of EM scattering from rough surfaces especially when the correlation length of the random heights is small and/or their root-mean-square (rms) heights are large.

In this study, tensor train (TT) decompositions (Osedelets, *SIAM J. Sci. Comput.*, 33(5), 2295–2317, 2011) generated by density renormalization group cross approximation (DMRG-Cross) (Savostvanov and Osedelets, *NDS 2011.*, 7, 1–8, 2011) are proposed for efficient statistical characterization of EM scattering from random rough surfaces. The proposed method generates compressed representations (i.e. TT decompositions) of the high-dimensional tensor grid of EM simulation results required to generate gPC expansions. The TT decomposition is generated by adaptively executing EM simulations for a few rough surface realizations, and generating the rest of the high-dimensional tensor grid through adaptively determined interpolation rules. In accordance with the results presented in (Bigoni et al., arXiv, 1405.5713, 2014), the proposed TT decomposition is more accurate and robust than those produced via HDMR and ASG techniques. Furthermore, the number of EM simulations required to obtain an accurate TT representation increases linearly when the number of uncertain parameters increases. In other words, the TT-UQ method does not suffer from the curse of dimensionality. Our presentation will demonstrate that TT-generated surrogate models are more accurate than those generated by HDMR and ASG techniques when applied to EM scattering from rough surfaces with large rms heights and/or small correlation lengths.