An FFT-Accelerated Inductance Extractor for Voxelized Structures

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***Abstract*—A fast Fourier transform (FFT)-accelerated integral equation solver for efficiently and accurately extracting inductance of structures discretized by voxels is described. This new solver called VoxHenry, unlike its predecessor FastHenry, can accurately account for the current flow around corners of structures using a carefully selected set of basis functions. VoxHenry solves the matrix system obtained upon discretization of the volume integral and current continuity equations iteratively. Matrix-vector multiplications required by the iterative solver are carried out using FFTs and the number of iterations is reduced using a sparse preconditioner. The accuracy and efficiency of VoxHenry are demonstrated via its application to inductance extraction of a circular coil.**

Keywords*—*Fast Fourier transform, inductance extraction, interconnects.

I. Introduction

The design of high-speed integrated circuits calls for efficient and accurate inductance extractors to produce circuit representations that can be used for signal timing, signal integrity, and cross-talk analyses. Inductance extractors that have been developed so far require structures to be discretized with volume filaments [1], tetrahedrons [2], or surface elements [3]. That said, there is a need for inductance extractors capable of working on structures discretized with voxels. Such an extractor would be extremely easy to incorporate into voxel-based virtual fabrication environments and suitable for modeling the structures during unit process steps of iterative design explorations. Additionally, its acceleration using fast Fourier transform (FFT) techniques [4]-[9] is straightforward.

In this work, an FFT-accelerated inductance extractor, named VoxHenry, is developed for solving a coupled system of volume integral and current continuity equations on voxelized structures. This new inductance extractor is more accurate and efficient than its predecessor FastHenry [1] thanks to algorithmic developments listed next. (i) VoxHenry expands the current density in terms of piecewise constant and linear basis functions permitting its accurate representation around corners. (ii) During the iterative solution of the matrix system obtained upon discretization, VoxHenry uses FFTs to expedite the matrix-vector multiplications. (iii) The number of iterations is reduced using a sparse preconditioner. Numerical results demonstrating the accuracy, efficiency, applicability of VoxHenry are provided.

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II. Formulation

Let V represent the volume of a non-magnetic structure with conductivity σ, which resides in an unbounded background medium with permittivity ε_0 and permeability μ_0. The structure is excited by source(s) operated at frequency ω. The structure is discretized by K_t voxels of size Δd, $K_t = K_x \times K_y \times K_z$, where K_x, K_y, and K_z are the numbers of voxels along x, y, and z directions; Δd is selected to be equal to or less than the skin depth at ω. Let K denote the number of non-empty voxels (with non-zero σ), each of which has one node on each of its six surfaces; let M denote the total number of these nodes.

VoxHenry solves the coupled system of volume integral and current continuity equations:

$$\frac{\mathbf{J}(\mathbf{r})}{\sigma} + j\omega\frac{\mu_0}{4\pi}\int_V \frac{\mathbf{J}(\mathbf{r}')}{\left|\mathbf{r}-\mathbf{r}'\right|}dv' = -\nabla\phi(\mathbf{r}),\ \{\mathbf{r},\mathbf{r}'\}\in V \tag{1}$$

$$\nabla\cdot\mathbf{J}(\mathbf{r}) = 0\,, \tag{2}$$

where $\mathbf{J}(\mathbf{r})$ and $\phi(\mathbf{r})$ represent the current density and scalar potential. To solve (1) and (2), $\mathbf{J}(\mathbf{r})$ is expanded in terms of divergence-free basis functions as:

$$\mathbf{J}(\mathbf{r}) \approx \sum_{k=1}^{K}\sum_{i=1}^{5} I_k^i \mathbf{b}_k^i(\mathbf{r}) \tag{3}$$

where I_k^i is the unknown current coefficient,

$$\mathbf{b}_k^1(\mathbf{r}) = \hat{\mathbf{x}},\ \mathbf{b}_k^2(\mathbf{r}) = \hat{\mathbf{y}},\ \mathbf{b}_k^3(\mathbf{r}) = \hat{\mathbf{z}}\,, \tag{4}$$

$$\mathbf{b}_k^4(\mathbf{r}) = \Delta d^{-1}\left(\left(x-x_k\right)\hat{\mathbf{x}} - \left(y-y_k\right)\hat{\mathbf{y}}\right), \tag{5}$$

$$\mathbf{b}_k^5(\mathbf{r}) = \Delta d^{-1}\left(\left(x-x_k\right)\hat{\mathbf{x}} + \left(y-y_k\right)\hat{\mathbf{y}} - 2\left(z-z_k\right)\hat{\mathbf{z}}\right), \tag{6}$$

and (x_k, y_k, z_k) are the coordinates of center of k^{th} voxel. Substituting (3) into (1), testing the resulting equation with $\mathbf{b}_l^j$, $j = 1,\ldots,5$, and enforcing (2) (in the form of normal current density continuity) across the voxel surfaces, yield a linear system of equations with dimensions $(5K+M)\times(5K+M)$:

$$\begin{bmatrix}\mathbf{V}\\ 0\end{bmatrix}=\begin{bmatrix}\bar{\mathbf{R}}+j\omega\bar{\mathbf{L}} & -\bar{\mathbf{A}}^T\\ \bar{\mathbf{A}} & 0\end{bmatrix}\begin{bmatrix}\mathbf{I}\\ \mathbf{\Phi}\end{bmatrix}. \tag{7}$$

Here, $\mathbf{V}$, $\mathbf{I}=[\mathbf{I}^1;\mathbf{I}^2;\mathbf{I}^3;\mathbf{I}^4;\mathbf{I}^5]$, and $\mathbf{\Phi}$ are the excitation vector and the vectors of unknown current and potential coefficients, respectively. The entries of sparse matrix $\bar{\mathbf{A}}_{mn}$, $n=1,\ldots,5K$, $m=1,\ldots,M$, are assigned as the value of n^{th} basis function on m^{th} node, multiplied with ± 1 . The entries of $\mathbf{V}$ are obtained by summing the columns of $\bar{\mathbf{A}}^T$ corresponding to the port nodes. Matrix $\bar{\mathbf{R}}$ consists of $K\times K$ diagonal blocks $\bar{\mathbf{R}}_{lk}^{ii}$ with entries $\sigma^{-1}\left\langle \mathbf{b}_k^i(\mathbf{r}),\mathbf{b}_k^i(\mathbf{r})\right\rangle_{V_k}$, $k=1,\ldots,K$, where $\langle\cdot,\cdot\rangle_\Omega$ denotes standard inner product with support Ω and V_k is the support of k^{th} voxel. Similarly, matrix $\bar{\mathbf{L}}$ consists of $K\times K$ blocks $\bar{\mathbf{L}}_{lk}^{ji}$ with the entries:

$$\bar{\mathbf{L}}_{lk}^{ji}=\left\langle \mathbf{b}_l^j(\mathbf{r}),\frac{\mu_0}{4\pi}\int_{V_k}\frac{\mathbf{b}_k^i(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}dv'\right\rangle_{V_l} . \tag{8}$$

Blocks $\bar{\mathbf{L}}^{ji}$ are either zero or full matrices. The multiplication of full matrix $\bar{\mathbf{L}}^{ji}$ with $\mathbf{I}^i$, i.e., $\mathbf{C}^j=\bar{\mathbf{L}}^{ji}\mathbf{I}^i$, requires $O(K^2)$ operations and is reduced to $O(K_t\log K_t)$ using FFTs [4]-[9]:

$$\mathcal{C}^j = IFFT\left(\sum\nolimits_i \tilde{\mathcal{L}}^{ji} * FFT(\mathcal{I}^i)\right), \tag{9}$$

where $\tilde{\mathcal{L}}^{ji}=FFT(\mathcal{L}^{ji})$, $\mathcal{L}^{ji}$ is the circulant tensor with the entries computed using (8), $\mathcal{I}^i$ is a tensor formed by using current coefficients and zero padding, $\mathcal{C}^j$ is the resultant tensor that provides the entries of $\mathbf{C}^j$. Dimensions of all tensors are $2K_x\times 2K_y\times 2K_z$. Note that the multiplications of sparse matrices $\bar{\mathbf{A}}$ and $\bar{\mathbf{A}}^T$ with $\mathbf{I}$ and $\mathbf{\Phi}$, respectively, require only $O(K)$ operations. To ensure the rapid convergence of iterative solution of (7), a sparse preconditioner,

$$\bar{\mathbf{P}}=\begin{bmatrix}\bar{\mathbf{R}}+j\omega\bar{\mathbf{D}} & -\bar{\mathbf{A}}^T\\ \bar{\mathbf{A}} & 0\end{bmatrix}^{-1}, \tag{10}$$

is applied to matrix-vector multiplications; here $\bar{\mathbf{D}}=diag(\bar{\mathbf{L}})$.

III. Numerical Result

VoxHenry is used for extracting the inductance of a circular coil with conductivity of 5.8×10^7 S/m and loop and tube radii of $150\ \mu\text{m}$ and $5\ \mu\text{m}$. The coil positioned on the xy plane is excited from a spacing in the lower part of the coil between $x=155\ \mu\text{m}$ and $x=155.5\ \mu\text{m}$. Discretization parameters $\Delta d=0.5\ \mu\text{m}$, $5K=2\,969\,300$ and $M=1\,867\,528$. The inductance of the coil is computed at frequencies ranging from 1 Hz to 1 THz and compared to the values provided by an analytical formula [10] [Fig. 1 (a)]. An excellent match between results is observed. In addition, current distribution on the coil at 3GHz is plotted in Fig. 1 (b). For this problem, VoxHenry computes the inductance at one frequency point in 2.93 minutes.

IV. Conclusion

An FFT-accelerated inductance extractor, VoxHenry, and its application to a voxelized circular coil are presented. The filament-based approaches, such as FastHenry, can not model the 3D currents in a circular coil accurately and are not directly applicable to voxelized circular coils (without remeshing) due to reasons explained in [11].

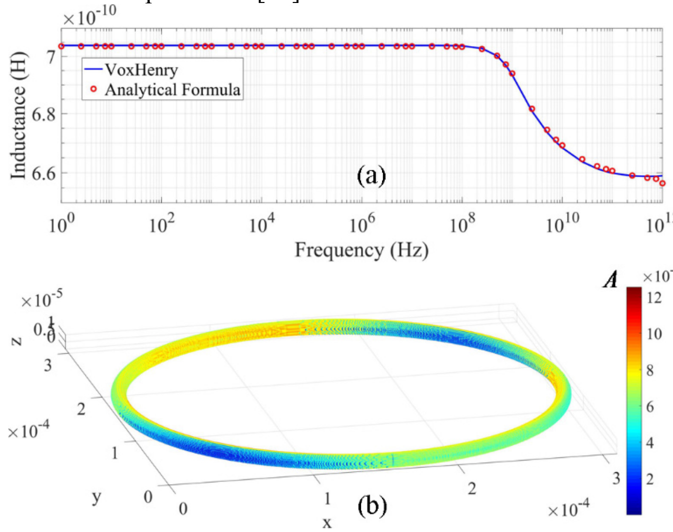

Fig. 1. (a) The frequency-dependent inductance of the circular coil. (b) The current distribution at 3 GHz.

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