Tensor Train Accelerated MoM Solution of Scattering Problems on Voxelized Grids and Unstructured Meshes

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Abstract

Tensor Train (TT) decomposition allowing for $O(\log N)$ storage of the Method of Moments (MoM) impedance matrix under fixed discretization of Volume Integral Equation (VIE) and performing its matrix-vector product (MVP) with a vector in $O(N \log N)$ operations requires that the matrix has Toeplitz structure. In this work we shown how MoM discretization of VIE can be formulated to produce such Toeplitz matrix structure in both the case of the arbitrary geometry voxelized on a regular grid and the case of the arbitrary geometries discretized with unstructured meshes.

I. Introduction

MoM discretization of VIE [1] governing solution of the full-wave scattering problems with N basis functions produced dense $N \times N$ matrix equation requiring $O(N^2)$ memory storage and $O(N^2)$ CPU time when solved iteratively. Such complexity scaling requires prohibitive computational resources when the problem size N becomes large. The Fast Multipole Method (FMM) [2], [3], FFT-based fast algorithms such as CG-FFT [4] and P-FFT [5], and methods based on Hierarchical matrices ($\mathcal{H}$-matrices) [6] have been widely used to reduce this complexity of storage and CPU time to $O(N \log N)$. Recently, an alternative approach based on the TT decomposition [7], [8] of the Toeplitz matrix has been also developed which has a capability to further reduce both the memory use and CPU time. The requirement of the MoM impedance matrix to have the Toeplitz structure however prevents its direct TT decomposition in the cases when the scatterer has an arbitrary shape and/or discretized with unstructured mesh. In this work we demonstrate two approaches which allow to factor the MoM impedance matrix into the Toeplitz matrix pre- and post-multiplied with sparse basis conversion matrices requiring $O(N)$ memory to store and $O(N)$ CPU time to perform their multiplication with a vector. The proposed methods are termed CG-TT and P-TT algorithms by analogy with the well-known FFT-based GC-FFT and P-FFT methods for the reason that the pertinent Toeplitz matrices are handled in them via TT decomposition rather than FFT while the remaining operations and sparse matrices remain same.

References

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