

## Tucker-Enhanced VoxHenry Simulator for Inductance Extraction of Voxelized Conducting/Superconducting Structures

Mingyu Wang<sup>(1)</sup>, Cheng Qian<sup>(1)</sup>, Zhuotong Chen<sup>(1)</sup>, Enrico di Lorenzo<sup>(2)</sup>, Luis J. Gomez<sup>(3)</sup>, Shucheng Zheng<sup>(4)</sup>, Vladimir Okhmatovski<sup>(4)</sup>, and Abdulkadir C. Yucel<sup>(1)</sup>

(1) School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798, Singapore

(2) FastFieldSolvers S.R.L., Vimercate 20871, Italy

(3) Department of Psychiatry & Behavioral Sciences, Duke University School of Medicine, Durham, NC 27701 USA

(4) Electrical and Computer Engineering, University of Manitoba, Winnipeg, MB R3T 2N2, Canada  
Email: acyucel@ntu.edu.sg

Inductance extraction tools are of paramount importance for the analysis and design of integrated circuits. These tools are particularly needed for accurately quantifying the inductive effects of interconnects on the circuits' normal operations as well as operating margins. Among a plethora of inductance extractors, the ones that permit easy and accurate modeling of the currents on sharp bends, vias, and complicated ground planes are needed by the designers. Furthermore, a parasitic extractor that can be incorporated into modern (voxel-based) virtual fabrication environments is highly desirable for the designers to expedite their iterative design explorations. To meet these demands, VoxHenry simulator has been recently developed for inductance extraction of voxelized structures (Yucel et. al, *IEEE Trans. Microw. Theory Techn.*, 66(4), 2018, pp. 1723-1735). The VoxHenry is capable of accurately modeling the currents via piecewise constant and linear basis functions. It is accelerated by exploiting fast Fourier transforms (FFTs) and employing a sparse preconditioner. Just like other parasitic extractors, the applicability of the VoxHenry to many practical problems is limited by its memory footprint, which primarily stems from the memory requirements of the sparse preconditioner and block circulant tensors.

In this study, Tucker decompositions are proposed to dramatically reduce the memory requirement of block circulant tensors and hence to boost the applicability of VoxHenry simulator. The proposed decompositions allow representing the block circulant tensors in terms of low-rank core tensors and factor matrices, which are constructed for each tensor during the simulator's setup stage and then used to restore the original tensor during the simulator's matrix-vector multiplication stage. For large-scale examples, the proposed Tucker enhancement achieves more than three or four orders of magnitude reduction in the memory required for storing block circulant tensors while imposing negligible computational overhead (i.e., restoration/decompression time). Preliminary results show that the memory requirement of circulant tensors in Tucker-enhanced VoxHenry simulator scales with  $O(N^{1/4} \log(N))$ , where  $N$  is the number of voxels used to discretize the structure. Furthermore, for an example structure discretized on a 320x320x320 voxel grid, it reduces the memory requirement of block circulant tensors by a factor of 4360 for a given accuracy of  $10^{-8}$  and required negligible decompression time, which is less than one third of the computational time needed to perform one FFT operation with the original tensor.

In addition, the VoxHenry simulator is extended to superconductors in the magneto-quasi-static regime, using the two-fluid model London equations to directly support superconductivity. Further improvements to VoxHenry are presented to accelerate tensor element calculation via leveraging quasi-static Green's kernel properties in the integration of the terms and symmetry, leading also to a reduction in the overall number of tensor blocks required altogether. The accuracy of, memory savings achieved by, and (negligible) computational overhead imposed by the proposed Tucker enhancement in VoxHenry simulator, plus the pre-calculations speed and memory improvements of the proposed optimizations, will be demonstrated through the inductance extraction of various voxelized superconducting/conducting structures.