A Multi-Region Internally Combined Volume Surface Integral Equation for EM Analysis of Inhomogeneous Negative and Positive Permittivity Scatterers

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I. Introduction

Electromagnetic (EM) analyses of inhomogeneous scatterers with positive and negative permittivities enable the development and deployment of many emerging technologies. For example, such analyses are necessary for the characterization of exciton-plasmon interactions in the design of composite nanostructures as well as the radar signature characterization of hypersonic re-entry space vehicles engulfed by plasma. When the traditional volume integral equation (VIE) is used for these analyses, it yields an ill-conditioned linear system of equations (LSE); the iterative solution of this ill-conditioned LSE converges very slowly or fails to converge to the desired accuracy. Recently, an internally combined volume-surface integral equation (ICVSIE) technique has been proposed for the EM analyses of inhomogeneous negative permittivity plasma scatterers, which reside in a positive permittivity background medium [1]. This technique yields a well-conditioned LSE by artificially changing the sign of the permittivity of the background medium in which the polarization currents radiate. That said, the standard ICVSIE technique is valid for a scatterer containing only a negative permittivity region; it was not formulated for EM analysis of a scatterer containing both regions of negative and positive permittivity.

In this study, we propose a multi-region ICVSIE technique for EM analysis of inhomogeneous positive and negative permittivity scatterers. In this technique, the regions with strictly positive and negative permittivities are wrapped by equivalent surfaces. The sign of the permittivity of the background media is artificially changed for the wrapped negative permittivity regions. To solve for the unknown fields, surface and volume equivalence principles are invoked and the equivalent surface currents and volume currents are obtained by solving the combined system of equations, which are guaranteed to be well-conditioned (when discretized) regardless of the difference in the permittivity of regions.

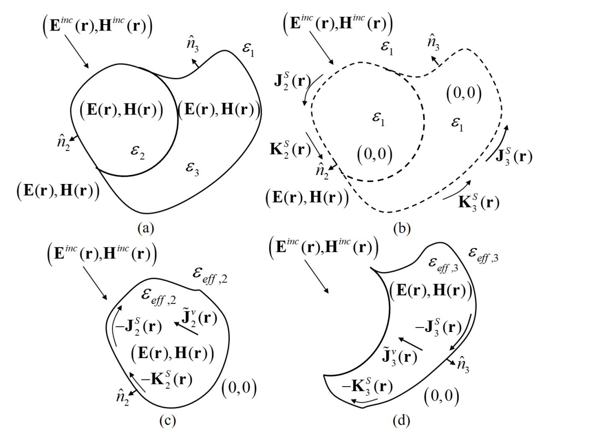

Fig. 1. (a) The original problem. The equivalent (b) exterior problem and interior problem for (c) region 2 and (d) region 3.

II. Formulation

Consider a time-harmonic field $(\mathbf{E}^{inc}(\mathbf{r}), \mathbf{H}^{inc}(\mathbf{r}))$ exciting inhomogeneous scatterers with positive and negative permittivities [Fig. 1(a)]. Here the scatterers with position-dependent positive permittivities $\varepsilon_2(\mathbf{r})$ and negative permittivities $\varepsilon_3(\mathbf{r})$ are residing in the background medium with permittivity ε_1. (Note: For the sake of simplicity, the multi-region method is explained here for two regions.) To solve for the unknown fields $(\mathbf{E}(\mathbf{r}), \mathbf{H}(\mathbf{r}))$, the original problem is converted into one equivalent exterior [Fig. 1(b)] and two equivalent interior problems [Figs. 1(c) and (d)]. The region enclosed by the surface S_p resides in a medium with permittivity $\varepsilon_{eff,p}$, for $p \in \{2, 3\}$. The background media in the exterior and interior scenarios have the intrinsic impedance of η_1 and $\eta_{eff,p}$, respectively.

A. The Exterior Problem

By enforcing boundary conditions on the surface between the object and the background medium [Fig. 1(b)], we obtain the electric field integral equations (EFIE) and magnetic field integral equation (MFIE) for the equivalent currents as [1]-[3]

$$\begin{aligned}\hat{n}_p \times \left(\eta_1 \mathcal{L}_1 \left(\mathbf{J}_2^S(\mathbf{r}) + \mathbf{J}_3^S(\mathbf{r})\right) - \mathcal{K}_1 \left(\mathbf{K}_2^S(\mathbf{r}) + \mathbf{K}_3^S(\mathbf{r})\right)\right) \\ +0.5\mathbf{K}_p^S(\mathbf{r}) = -\hat{n}_p \times \mathbf{E}^{inc}(\mathbf{r});\ \mathbf{r} \in S_p\end{aligned} \tag{1}$$

$$\begin{aligned}-0.5\mathbf{J}_p^S(\mathbf{r}) + \hat{n}_p \times \frac{1}{\eta_1}\mathcal{L}_1 \left(\mathbf{K}_2^S(\mathbf{r}) + \mathbf{K}_3^S(\mathbf{r})\right) \\ +\hat{n}_p \times \mathcal{K}_1 \left(\mathbf{J}_2^S(\mathbf{r}) + \mathbf{J}_3^S(\mathbf{r})\right) = -\hat{n}_p \times \mathbf{H}^{inc}(\mathbf{r});\ \mathbf{r} \in S_p\end{aligned} \tag{2}$$

where $\mathcal{L}_q$ is the electric source field operator and $\mathcal{K}_q$ is the magnetic source field operator defined for the medium q; here q could be 1; $eff, 2$; or $eff, 3$ (See the definitions of operators in [1]-[3]). Similarly, by invoking the volume equivalence principle for the volume V_p enclosed by S_p, $p \in \{2, 3\}$, we obtain the corresponding VIE as

$$\begin{aligned}-\mathcal{K}_1 \left(\mathbf{K}_2^S(\mathbf{r}) + \mathbf{K}_3^S(\mathbf{r})\right) + \eta_1 \mathcal{L}_1 \left(\mathbf{J}_2^S(\mathbf{r}) + \mathbf{J}_3^S(\mathbf{r})\right) \\ = -\mathbf{E}^{inc}(\mathbf{r});\ \mathbf{r} \in V_p\end{aligned} \tag{3}$$

B. The Interior Problems

Similar to the exterior problem, the interior EFIE and MFIE are obtained by enforcing the boundary conditions on the interior surface of the domains 2 and 3 [Figs. 1(c) and (d)] as

$$\begin{aligned}-\hat{n}_p \times \mathcal{K}_{eff,p} \left(\mathbf{K}_p^S(\mathbf{r})\right) + \hat{n}_p \times \eta_{eff,p}\mathcal{L}_{eff,p} \left(\mathbf{J}_p^S(\mathbf{r})\right) \\ -0.5\mathbf{K}_p^S(\mathbf{r}) - \hat{n}_p \times \eta_{eff,p}\mathcal{L}_{eff,p} \left(\tilde{\mathbf{J}}_p^v(\mathbf{r})\right) = 0;\ \mathbf{r} \in S_p\end{aligned} \tag{4}$$

$$\begin{aligned}\hat{n}_p \times \frac{1}{\eta_{eff,p}}\mathcal{L}_1 \left(\mathbf{K}_p^S(\mathbf{r})\right) + \hat{n}_p \times \mathcal{K}_{eff,p} \left(\mathbf{J}_p^S(\mathbf{r})\right) \\ +0.5\mathbf{J}_p^S(\mathbf{r}) - \hat{n}_p \times \mathcal{K}_{eff,p} \left(\tilde{\mathbf{J}}_p^v(\mathbf{r})\right) = 0;\ \mathbf{r} \in S_p\end{aligned} \tag{5}$$

Using the volume equivalent current $\tilde{\mathbf{J}}_p^v(\mathbf{r})$, we obtain the corresponding VIE as

$$\begin{aligned}\mathbf{E}(\mathbf{r}) - \mathcal{K}_{eff,p} \left(\mathbf{K}_p^S(\mathbf{r})\right) + \eta_{eff,p}\mathcal{L}_{eff,p} \left(\mathbf{J}_p^S(\mathbf{r})\right) \\ -\eta_{eff,p}\mathcal{L}_{eff,p} \left(\tilde{\mathbf{J}}_p^v(\mathbf{r})\right) = 0;\ \mathbf{r} \in V_p\end{aligned} \tag{6}$$

Finally, a combined ICVSIE system is obtained by coupling Muller combined field integral equations with VIEs after multiplying (3) with ε_1 and (6) with $-\varepsilon_{eff,p}$ and summing the resulting VIEs. The resulting combined equations are discretized via surface and volume basis functions and tested by Galerkin scheme to obtain a well-conditioned LSE. The LSE is iteratively solved for the unknown expansion coefficients that define the equivalent surface electric and magnetic currents as well as the volume polarization currents. From the solution, the unknown fields on the exterior and interior are computed using electric and magnetic source field operators.

III. Numerical Results

To demonstrate the accuracy and efficiency of the multi-region ICVSIE technique, the scattering from two homogeneous spheres with relative permittivities $\varepsilon_2 = -4$ and $\varepsilon_3 = 4$ (inset of Fig. 2(a)) is considered for an incident field at 300 THz. For this scenario, the radar cross section (RCS) obtained using ICVSIE is compared to that obtained by the commercial FEKO software [4] [Fig. 2(a)]. When compared with FEKO, the ICVSIE produces results with a relative L2-norm difference of 1.65%, which demonstrates the accuracy of the proposed scheme. While the iterative solution of ICVSIE system is obtained in 27 iterations (for relative residual error (RRE) of 10^{-6}), the iterative solution of VIE system fails to converge for this scenario, as shown in Fig. 2(b).

In the talk, several numerical examples demonstrating the accuracy and efficiency of the multi-region ICVSIE technique as well as the condition number of ICVSIE system will be presented. The performances of ICVSIE and VIE systems for epsilon-near-zero materials will be discussed.

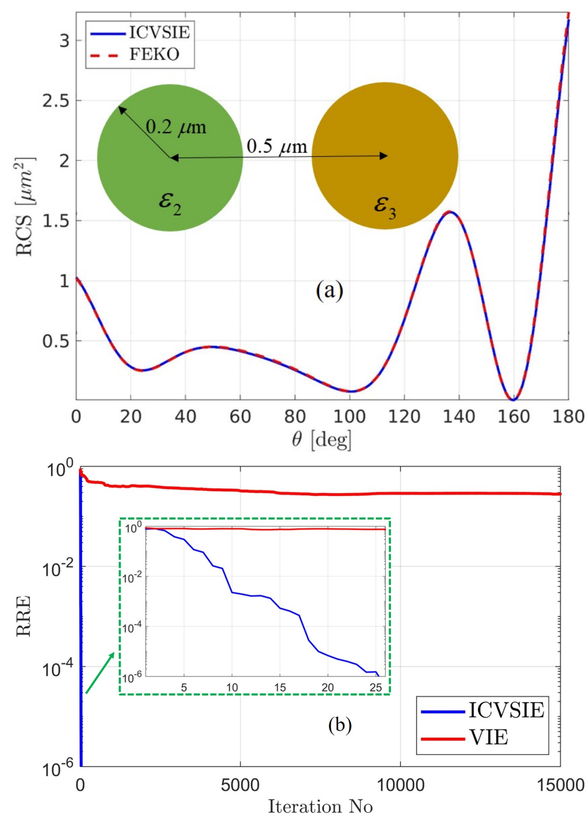

Fig. 2. (a) The RCS computed using the proposed ICVSIE method and the FEKO. (b) The number of iterations versus RREs obtained during the iterative solutions of ICVSIE and VIE systems.

References

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[3] L. J. Gomez, A. C. Yucel, and E. Michielssen, "Internally combined volume-surface integral equation for 3D electromagnetic scattering analysis of high-contrast media", *IEEE Antennas Wireless Propag. Lett.*,vol. 16, pp. 1691-1694, 2017.

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