An FFT-Accelerated and Tucker-Enhanced Inductance Extraction for Voxelized Superconducting Structures

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***Abstract*—An integral equation simulator is proposed to compute the inductance of the superconducting structures discretized by voxels. The proposed simulator extends the VoxHenry inductance extractor by taking into account the superconductivity in the computations. In addition to possessing all distinctive features of the VoxHenry simulator, the proposed simulator makes use of Tucker decompositions to compress its large data structures involving the block circulant tensors. By doing so, it reduces the memory requirement of these tensors from gigabytes to megabytes and CPU time required to obtain these tensors from hours/minutes to seconds for the large-scale problems.**

Keywords—Fast Fourier Transform (FFT), fast simulators, inductance extraction, superconducting structures, Tucker decomposition, volume integral equation (VIE), voxelized structures

I. Introduction

The design and verification of complex superconducting integrated circuits has been of interest for the development of energy-efficient supercomputing facilities as well as processors/computer boards. Such tasks often require efficient and accurate inductance extraction of the complex superconducting integrated circuits and utilization of the extracted parameters in the circuit simulators. So far, various inductance extraction simulator for superconducting structures have been proposed [1-3]. These simulators extend the popular FastHenry simulator [4] to the inductance extraction of superconducting structures. However, the FastHenry is not an efficient simulator for these structures, since it uses piecewise constant basis functions, which are not sufficient to model the currents around sharp corners and on the complicated ground planes [5]. Furthermore, FastHenry is not suitable for the inductance extraction of the voxelized structures (i.e., the circuits/interconnects discretized by voxels (i.e., cubes)), which are designed to be printed by the voxel-based 3D printers. To address these shortcomings of FastHenry, the VoxHenry simulator has recently been proposed [5].

In this study, we extend the VoxHenry simulator to the inductance extraction of superconducting structures and introduce the Tucker decompositions for reducing its memory requirement. The proposed extension simultaneously solves the current continuity equation and VIE which takes into account the superconductivity. Just like VoxHenry, the proposed extension first discretizes the currents via a set of basis functions including piecewise constant and linear basis functions; such set allows representing the currents around sharp corners and on complicated ground planes accurately. Next, it iteratively solves a linear system of equations (LSE) resulting from the substitution of the discretized currents in the equations and application of Galerkin testing. At each iteration, the proposed extension accelerates the matrix-vector multiplications (MVMs) via FFTs. Furthermore, it leverages a sparse preconditioner to reduce the number of iterations during the iterative solution.

In addition to making use of these distinctive features of the VoxHenry, the proposed extension leverages the Tucker decompositions to compress the block circulant tensors via low-rank core tensors and factor matrices. While the circulant tensors require gigabytes of memory for large-scale problems, their Tucker representations (i.e., core tensors and factor matrices) necessitate only a few megabytes. This allows storing the Tucker representations of block circulant tensors on hard-disk during the installation stage of the simulator. During the setup stage of each simulation, these representations are read from the hard-disk in seconds and tailored for the simulation. Doing so reduces the setup stage of the proposed simulator from tens of minutes (sometimes hours) to seconds for large-scale problems. During the iterative solution stage of the simulator, these representations are restored to the original block circulant tensors *one-by-one* and used in MVMs. Doing so significantly reduces the overall memory cost of the proposed simulator. The

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numerical results show that the proposed Tucker decomposition reduces the memory requirement of the block circulant tensors more than four orders of magnitude while requiring negligible computational overhead (i.e., restoration/decompression time). Due to limited space, only basic ideas of methodologies and some results are provided here; the details of the methodologies and extensive numerical results will be presented in the talk.

II. Formulation

Let V' denote the volume of the superconductor connected to a sinusoidal voltage source operated at angular frequency ω. To compute the inductance of the superconductor, the proposed simulator solves the VIE,

$$\left(\mathbf{J}(\mathbf{r})/s(\mathbf{r})\right)+j\omega\mu\int_{V'}\mathbf{J}\left(\mathbf{r}'\right)/\left(4\pi\left|\mathbf{r}-\mathbf{r}'\right|\right)dv'=-\nabla\Phi\left(\mathbf{r}\right), \quad (1)$$

and current conservation law ($\nabla\cdot\mathbf{J}(\mathbf{r})=0$) for the vector current density $\mathbf{J}(\mathbf{r})$ and scalar potential $\Phi(\mathbf{r})$. In (1), $\mathbf{r}$ and $\mathbf{r}'$ denote the locations of observation and source points in V', respectively, $s(\mathbf{r})=\sigma(\mathbf{r})+(j\omega\mu\lambda(\mathbf{r})^2)^{-1}$; $\lambda(\mathbf{r})$ is the London penetration depth from the surface which the magnetic field can penetrate. $\sigma(\mathbf{r})$ is the conductivity of the normal channel in the "two-fluid" superconductor model and $\mu=\mu_0$ is the permeability. The proposed simulator discretizes the current density $\mathbf{J}(\mathbf{r})$ via piecewise constant and linear basis functions [5] and obtains an LSE after substituting the discretized currents in the equations, enforcing the current continuity on the voxel surfaces, and applying Galerkin testing. While the LSE is solved iteratively, a sparse preconditioner is applied at each iteration to reduce the number of iterations [5]. At each iteration, the MVMs involving the convolutions of the circulant tensors with the tensors storing current coefficients, are accelerated by the FFTs. Before performing each convolution via FFTs, each circulant tensor $\mathcal{C}_j$, $j=1,\ldots,7$, is restored from its Tucker representation via

$$\mathcal{C}_j \approx \mathcal{T}_j \times_1 \bar{\mathbf{U}}_j^1 \times_2 \bar{\mathbf{U}}_j^2 \times_3 \bar{\mathbf{U}}_j^3, \quad (2)$$

where $\mathcal{T}_j$ and $\bar{\mathbf{U}}_j^{\{1,2,3\}}$ denote the core tensor and factor matrices, respectively, and $\times_i$, $i=1,\ldots,3$, denotes the mode $-i$ matrix product of a tensor (see [6] for details).

III. Numerical Examples

First, the proposed simulator is validated by comparing its results with the results obtained by other simulators and measurements. To this end, a superconducting stripline (proposed in [7] and depicted in Fig. 1) with 10 µm length, varying width, and 0.2 µm thickness is positioned between ground planes with size 5x10x0.2 µm (length x width x thickness). Its unit inductance at 2.5 GHz by considering $\lambda(\mathbf{r})=90\,\text{nm}$ ($\sigma(\mathbf{r})=0$) is computed using the proposed simulator, FastHenry for superconductors [1], and InductEx [3] for various stripline widths; the computed results are compared with the measurement results [Fig. 1]. The results obtained by the proposed simulator perfectly match with the measurement ones especially for the line widths larger than 1 µm.

Next, the memory saving (or compression ratio) achieved by the Tucker decompositions is demonstrated for the inductance extraction of a superconducting cube [Fig. 2]. In this test, the cube with varying edge length from 150 µm to 500 µm is discretized by voxels with edge length of 1 µm. The compression ratio is the ratio of the memory requirement of original circulant tensors to the memory requirement of their Tucker representations (i.e., core tensors and factor matrices). Fig. 2 shows that the compression ratio increases with increasing number of voxels in voxelized structure and decreases with increasing tolerance (*tol*). The Tucker decomposition achieves four orders of magnitude memory reduction for a cube discretized by 125 million voxels (for *tol*=1e-8).

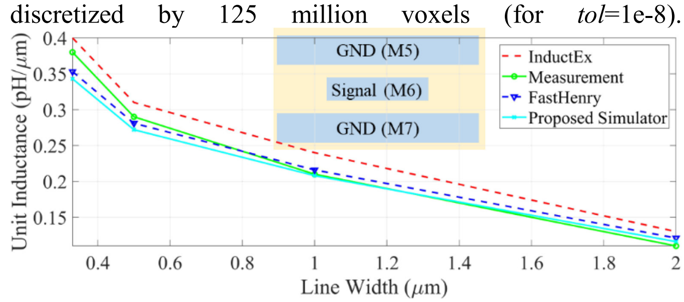

Fig. 1. Unit inductances with respect to line width obtained by the proposed simulator, FastHenry, InductEx, and measurement.

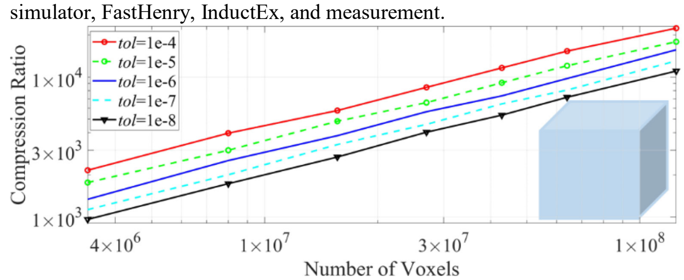

Fig. 2. The compression ratio achieved by Tucker decompositions versus the number of voxels in the voxelized structure.

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