On the Tensor Compression of Far-Field Signatures in the Fast Multipole Method

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The last three decades have witnessed substantial research efforts on the development and application of fast multipole method (FMM)-accelerated integral equation (IE) simulators. These simulators including their multilevel and fast Fourier transform (FFT) variants have been parallelized and applied to many large-scale electromagnetic (EM) problems in various applications ranging from remote sensing and wireless channel characterization to bio-electromagnetics. These simulators' applicability to the large-scale problems are often limited by their memory requirements, which primarily stem from the data structures storing the (i) near-field interactions of basis functions, (ii) translation operator samples, and (ii) far-field signatures of the FMM groups. In the past, the matrix decomposition techniques have been used to reduce the memory requirement of (i) (Kapur and Long, *IEEE Comp. Sci. Eng.*, 5(4), 60–67, 1998), while the tensor decomposition and interpolation-based techniques have been proposed to lessen the memory requirement of (ii) (Yucel et.al., *IEEE Antennas Wireless Propag. Lett.*, 16, 2667–2670, 2017) (Song and Chew, Microw. *Opt. Technol. Lett.*, 30(2), 109-114, 2001). Moreover, a truncated singular value decomposition (tSVD)-based technique has been proposed to reduce the memory requirements of (iii) (Rodriguez et. al., *IEEE Trans. Antennas Propagat.*, 56(8), 2325-2334, 2008). In this technique, the far-field signatures of the basis functions in FMM groups are stored in a matrix format and compressed by tSVD, which yielded substantial memory reduction. The compressed representations are then used to perform aggregation and disaggregation stages of FMM and reduce the computational costs of these stages significantly.

In this study, tensor decompositions are proposed to reduce the memory requirements of the far-field signatures of groups as well as the computational costs of aggregation/disaggregation stages in FMM-accelerated IE simulators. During the setup stage of these simulators, the far-field signatures of the basis functions in FMM groups are stored in a tensor format and compressed by Tucker and tensor-train (TT) decompositions. During the iterative solution stage of these simulators, the compressed representations are directly used to perform fast tensor-vector products required in aggregation/disaggregation stages. The preliminary numerical results have shown that the proposed Tucker and TT decomposition-based methods yield 2x and 1.4x more memory reduction compared to the tSVD method, respectively, for the EM scattering analysis of a 30λ diameter sphere. Furthermore, the Tucker decomposition-based method often yields more than 2x more reduction in the computational costs of aggregation and disaggregation stages achieved by the traditional tSVD method. The proposed drop-in tensor decomposition enhancements can be easily incorporated into the existing FMM-accelerated IE simulators and boost their applicability on fixed computational resources. In the talk, the achieved reduction in memory requirement and computational cost with respect to parameters such as the decomposition tolerance, FMM box size, triangle orientations, mesh size, and constitutive parameters of the medium will be discussed.