FFT-Accelerated and Tucker-Enhanced Impedance Extraction for Voxelized Structures

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***Abstract*—An impedance extraction simulator is proposed to compute the impedances of the structures discretized by voxels (i.e., voxelized structures). The proposed simulator solves the integral equations in conjunction with the current and charge conservation laws. During the iterative solution of the governing equations, the simulator accelerates the matrix-vector-multiplications via fast Fourier transforms. To ensure the rapid convergence of the iterative solution, the simulator leverages a sparse preconditioner tailored for the analysis of voxelized structures. Furthermore, the proposed simulator employs Tucker decompositions to reduce its memory requirement as well as the setup time. The proposed simulator's accuracy and efficiency are demonstrated by its application to the impedance extraction of a shorted transmission line.**

Keywords— Fast Fourier Transform (FFT), fast simulators, impedance extraction, integral equation (IEs), voxelized structures

I. Introduction

The efficient and accurate impedance extraction simulators facilitate the analysis, design, and verification of integrated circuits and printed circuit boards. These simulators are especially needed to expedite the iterative design explorations performed using the voxel-based virtual fabrication environments. Unfortunately, there is no efficient impedance extractor that can be used in conjunction with the voxel-based virtual fabrication environments. Various impedance extraction simulators developed so far, based on node analysis [1] and mesh analysis [2], require excessive computational resources when applied to the impedance extraction of the voxelized structures. Thereby, an efficient impedance extractor for the voxelized structures is called for. Recently, VoxHenry [3] and VoxCap [4] simulators have been developed for the inductance and capacitance extraction of the voxelized structures, respectively. These simulators perform the quasi-magneto-static and electrostatic analyses of the voxelized structures and outperform the traditional simulators in terms of accuracy and efficiency when applied to the analysis of voxelized structures.

In this study, an impedance extraction simulator combining the capabilities of the abovementioned VoxHenry and VoxCap simulators is proposed. The proposed simulator performs the electro-magneto-quasi-static (EMQS) analysis of voxelized structures and allows extracting their impedances. In order to do that, the proposed simulator solves integral equations (IEs) along with the current and charge conservation laws. It discretizes the currents via piecewise constant and linear basis functions as well as the charges via piecewise constant basis functions. After discretization and testing, it obtains a linear system of equations (LSE) and solves this LSE iteratively. During the iterative solution, the proposed simulator accelerates the matrix-vector-multiplications (MVMs) via fast Fourier transforms (FFTs). Moreover, it leverages a sparse preconditioner to reduce the number of iterations during the iterative solution stage. In addition, the proposed simulator utilizes Tucker decompositions to reduce the setup time of the simulator from *hours* to *minutes* for large-scale problems. Such decompositions are also used to compress the largest data structures in the simulator and reduce their memory requirements from *gigabytes* to *megabytes* for large-scale problems.

II. Formulation

Let V' and S' denote the volume and surface of conductors residing in vacuum with the permittivity ε_0 and permeability μ_0. The conductors with conductivity σ are separately connected to a sinusoidal current source operated at an angular frequency ω. The computational domain enclosing the conductors is discretized by voxels of size Δx and conductors occupy K number of non-empty voxels. Each non-empty voxel has one node on each of its six surfaces (panels); there are totally M unique nodes on M panels. The currents, charges, and potentials on/within the conductors are obtained via solving the volume and surface IEs, which are

$$\left(\mathbf{J}(\mathbf{r})/\sigma\right)+j\omega\mu_0\int_{V'}\mathbf{J}\left(\mathbf{r}'\right)/\left(4\pi\left|\mathbf{r}-\mathbf{r}'\right|\right)dv'=-\nabla\Phi\left(\mathbf{r}\right), \quad (1)$$

$$\int_{S'}q\left(\mathbf{r}'\right)/\left(4\pi\varepsilon_0\left|\mathbf{r}-\mathbf{r}'\right|\right)ds'=\Phi\left(\mathbf{r}\right), \quad (2)$$

along with the current and charge conservation laws, $\nabla\cdot\mathbf{J}(\mathbf{r})=0$ and $\hat{\mathbf{n}}(\mathbf{r})\cdot\mathbf{J}(\mathbf{r})=j\omega q(\mathbf{r})$. Here, $\mathbf{J}(\mathbf{r})$ and $q(\mathbf{r})$ represent the current and charge density, $\hat{\mathbf{n}}(\cdot)$ is the unit outward normal vector. In (1) and (2), $\Phi(\mathbf{r})$ represents the potential while $\mathbf{r}$ and $\mathbf{r}'$ denote the locations of observation and source points, respectively. The proposed simulator

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discretizes the current density $\mathbf{J}(\mathbf{r})$ via three piecewise constant and two linear basis functions [3] in voxels and the charge density $q(\mathbf{r})$ via piecewise constant basis functions [4] on panels. After substituting the discretized currents and charges in the equations, enforcing the current and charge continuity on the unique nodes, and applying Galerkin testing, the LSE with dimensions $(5K+2M)\times(5K+2M)$ is obtained as [1]

$$\begin{bmatrix} \bar{\mathbf{Z}} & 0 & -\bar{\mathbf{A}}^T \\ 0 & \bar{\mathbf{P}}/(j\omega) & -\bar{\mathbf{A}}_q^T \\ \bar{\mathbf{A}} & \bar{\mathbf{A}}_q & 0 \end{bmatrix} \begin{bmatrix} \mathbf{I}_c^v \\ \mathbf{I}_c^p \\ \mathbf{\Phi} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mathbf{I}_t \end{bmatrix}. \tag{3}$$

The entries of the full matrix $\bar{\mathbf{Z}}$ with dimensions $5K\times 5K$, and the sparse incidence matrix $\bar{\mathbf{A}}$ with dimensions $M\times 5K$ are provided in [3]. The entries of the full matrix $\bar{\mathbf{P}}$ with dimensions $M\times M$ are given in [4]. The identity matrix $\bar{\mathbf{A}}_q$ with dimensions $M\times M$ sums the charges at each node. $\mathbf{I}_c^v$, $\mathbf{I}_c^p$, and $\mathbf{\Phi}$ are the vectors storing the unknown coefficients of voxel currents, panel currents, and potentials on nodes, respectively. Note that the charges are replaced with the panel currents normal to the panel surfaces. $\mathbf{I}_t$ is the excitation vector. While solving LSE in (3) iteratively, MVMs between $\bar{\mathbf{Z}}$ and $\mathbf{I}_c^v$ as well as $\bar{\mathbf{P}}$ and $\mathbf{I}_c^p$ are accelerated by FFTs. To do that, the Toeplitz tensors corresponding to the blocks in $\bar{\mathbf{Z}}$ and $\bar{\mathbf{P}}$ are obtained and embedded in the circulant tensors. These circulant tensors and the tensors storing current and potential coefficients are used to perform convolutions via FFTs, as detailed in [3, 4].

The Tucker decompositions are utilized to reduce the setup time of the simulator. For this purpose, during the installation stage of the simulator, the Toeplitz tensors are generated for a large computational domain with $\Delta x = 1$. These tensors are then compressed via Tucker decomposition and stored on a hard disk. During the setup stage of the simulator's each execution, the stored compressed Toeplitz tensors are read from hard disk and restored to their original format. Then these tensors are tailored according to the size of the computational domain required for the analysis and multiplied by scaling factors [4, 5]. The generated Toeplitz tensors are used to obtain the circulant tensors by embedding. Doing so reduces the CPU time required for the simulator's setup stage from *hours* to *seconds* for large-scale structures.

Furthermore, the Tucker decompositions are leveraged to compress the circulant tensors via low-rank core tensors and factor matrices, which reduces the memory requirement of these tensors from *gigabytes* to *megabytes* for the large-scale structures. These compressed circulant tensors are then restored one-by-one and used in the MVMs during the iterative solution stage.

III. Numerical Example

The proposed simulator is used to extract the impedance of a shorted transmission line with the cross-section of $50\,\mu\text{m}\times 50\,\mu\text{m}$ and $\sigma = 7.4\times 10^5$ S/m [Fig. 1]. The structure is discretized by voxels of size $\Delta x = 10\,\mu\text{m}$, leading to 1,180,425 unknown coefficients. The impedance of the transmission line is obtained by a frequency sweep from 1 GHz to 10 GHz. The impedance values obtained by the proposed simulator are compared with those obtained by FastImp [6] operated in EMQS mode [Fig. 1]. Clearly, the impedance values obtained by the proposed simulator show a very good match with the ones obtained by the FastImp; the average relative difference between the values is 6.87%. Even for this small-scale validation example, the Tucker decomposition reduces the CPU time required to obtain Toeplitz tensors from 138.83 s to 4.01 s, while it reduces the memory requirement of circulant tensors from 506.7 MB to 9.8 MB (for the decomposition tolerance of 10^{-6}). As shown in [4, 5], Tucker decomposition yields very high performance for large-scale structures.

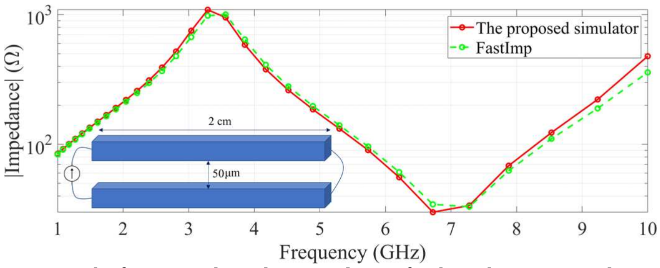

Fig. 1. The frequency-dependent impedance of a shorted transmission line

IV. Conclusion

An FFT-accelerated impedance extraction simulator for the voxelized structure is proposed. Its memory and CPU time requirements are reduced by the Tucker decompositions. The details and performance of the sparse preconditioner used in the proposed simulator will be presented in the talk.

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