Stochastic Electromagnetic Analysis via High Dimensional Model Representations

Abdulkadir C. Yücel*[1], Hakan Bağcı[2], and Eric Michielssen[1]
(1) Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109, USA
(2) Division of Physical Sciences and Engineering, King Abdullah University of Science and Technology, Thuwal 23955-6900, KSA

The recent literature abounds with studies aimed at statistically characterizing electromagnetic observables given uncertainty in geometry descriptions and material parameters, as well as excitations. Often, this characterization is achieved via Monte Carlo (MC) methods, which call for the execution of a deterministic electromagnetic simulator for many realizations of the uncertain/random variables sampled with respect to their (assumed/known) probability distribution functions (pdfs). While MC methods are straightforward to implement and readily generate important statistical information (e.g., the mean, variance, and pdf of the observables), their convergence often is prohibitively slow. Stochastic collocation (SC) (D. Xiu, *Commun. Comput. Phys*., 2(2), 2007, pp. 293-309, Bağcı et. al., *IEEE Trans. Electromagn. Compat.*, 51(2), 2009, pp. 301-311) methods that use generalized polynomial (gPC) expansions to represent observables are not unlike MC methods in that they only require the repeated execution of a deterministic simulator while providing a wealth of statistical information. Even though SC methods are more accurate than MC methods, they become computationally expensive (i) when the observables are rapidly varying and/or discontinuous functions of the random variables (as their gPC representations require high-order polynomials) and (ii) when the dimension of the random domain (i.e., the number of random variables) is high.

Here, an extension to SC methods that addresses the second of the above shortcomings is presented. The proposed extension leverages high dimensional model representations (HDMRs) (O. Alis and H. Rabitz, *J. Math. Chem.*, 25(2), 1999, pp. 197–233), viz. hierarchical representations of observables involving sums of so-called multinomial component functions. The lowest-order component functions reveal the "independent" contributions of the random variables while higher-order ones reveal combined contribution of random variable groups. For an observable that only weakly depends on high-order correlations of the random variables, the number of participating component functions can be kept small with little or no effect on the accuracy of the HDMR. The HDMR is constructed iteratively, starting with low-order component functions and only including those high-order ones that "significantly" enhance the accuracy of the representation. The component functions that feature in the HDMR are approximated using adaptive SC-gPC expansions. When compared to classical SC-gPC methods, the proposed hybrid HDMR-SC-gPC approach often permits more accurate expansions with far fewer terms. The efficiency and accuracy of the proposed method will be demonstrated via its application to the statistical characterization of crosstalk on interconnects.