An *h*-Adaptive Stochastic Collocation Method for Stochastic EMC/EMI Analysis

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Introduction

The analysis of electromagnetic compatibility and interference (EMC/EMI) phenomena is often fraught by randomness in a system's excitation (e.g., the amplitude, phase, and location of internal noise sources) or configuration (e.g., the routing of cables, the placement of electronic systems, component specifications, etc.). To bound the probability of system malfunction, fast and accurate techniques to quantify the uncertainty in system observables (e.g., voltages across mission-critical circuit elements) are called for. Recently proposed stochastic frameworks [1-2] combine deterministic electromagnetic (EM) simulators with stochastic collocation (SC) methods that approximate system observables using generalized polynomial chaos expansion (gPC) [3] (viz. orthogonal polynomials spanning the entire random domain) to estimate their statistical moments and probability density functions (pdfs). When constructing gPC expansions, the EM simulator is used solely to evaluate system observables at collocation points prescribed by the SC-gPC scheme. The frameworks in [1-2] therefore are non-intrusive and straightforward to implement. That said, they become inefficient and inaccurate for system observables that vary rapidly or are discontinuous in the random variables (as their representations may require very high-order polynomials).

In this paper, an *h*-adaptive SC-gPC method suited for characterizing rapidly varying and/or discontinuous observables is presented. The proposed extension achieves its efficiency by recursively and adaptively dividing the random domain into sub-domains based on the decay rates of observables' local variances [4], and constructing separate gPC expansions for each sub-domain. The adaptive SC-gPC method enables the stochastic characterization of electronic systems that involve high-Q resonators and highly non-linear components with discontinuous responses (e.g., rectifier circuits). Due to these systems' high sensitivity to perturbations in excitation or geometry, their stochastic characterization would be impossible using "classical" SC methods without the proposed *h*-adaptive extension.

Formulation

This section details the *h*-adaptive SC-gPC method: In Subsection A, the gPC expansion, which is used by the proposed scheme to locally approximate the observable in each sub-domain, is reviewed. Subsection B expounds the recursive and adaptive refinement scheme.

A. gPC Expansion

Let the N_{dof}-dimensional vector $\mathbf{x} = [x^1, x^2, \ldots x^{N_{\text{dof}}-1}, x^{N_{\text{dof}}}]$ and $V(\mathbf{x})$ represent the vector of random variables (uncertainties in the system excitation and configuration)

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defined over a random domain D and a system observable that depends on $\mathbf{x}$, respectively. The gPC method approximates the observable using orthogonal polynomials, $\Psi(\mathbf{x})$ [3]:

$$V(\mathbf{x}) \simeq \sum_{m=0}^{N_p} v_m \Psi_m(\mathbf{x}) . \tag{1}$$

Here, $N_p = (N_{\text{dof}} + p)!/(N_{\text{dof}}!p!) - 1$ is the total number of expansion terms, p is the order of expansion, and v_m is the m^{th} gPC expansion coefficient computed as

$$v_m = \int_D V(\mathbf{x})\Psi_m(\mathbf{x})W(\mathbf{x})d\mathbf{x} . \tag{2}$$

Here, $W(\mathbf{x})$ is the multivariate pdf of $\mathbf{x}$, which is typically expressed as a tensor product of one dimensional pdfs $w(x^i)$, $i = 1,\ldots,N_{\text{dof}}$, that are assumed either uniform or normal. The N_{dof} - dimensional integral in (2) is evaluated numerically using sparse grid, tensor product, or Stroud integration rules. The choice of orthogonal basis in the SC-gPC framework depends on the choice of $w(x^i)$, $i = 1,...,N_{\text{dof}}$ [3].

B. h-Adaptive Framework

The gist of the proposed framework is to recursively and adaptively divide the initial random domain D_0 into sub-domains D_j with $D_0 = \cup_j D_j$ using the decay rates of the observable's local variance as a guide [4]. The method uses gPC expansions within each sub-domain to locally approximate the observable. The local variance approximated by the p^{th} order gPC expansion in sub-domain j is computed via

$$\text{var}_{p,j} = \sum_{m=1}^{N_p} v_{m,j}^2 . \tag{3}$$

Here, $v_{m,j}$ is the m^{th} gPC expansion coefficient in sub-domain j and is computed using (2) (with D replaced by D_j). To compute gPC coefficients efficiently, the N_{dof} - dimensional integral in (2) is evaluated on a sparse grid constructed using the Smolyak algorithm. The decay rate of the local gPC expansion's relative error is defined as

$$\gamma_j = (\text{var}_{p,j} - \text{var}_{p-1,j})/\text{var}_{p,j} . \tag{4}$$

The decay rate γ_j is a measure of how accurate the observable is represented with the gPC expansion in sub-domain j. If γ_j exceeds a specified tolerance [4], (i.e., if the gPC expansion is not accurate enough), sub-domain j is selected for adaptive refinement. The efficiency of the adaptive scheme is increased by refining only the dimension(s) along which the observable varies rapidly. The "sensitivity" of each dimension is defined as

$$\alpha_j^i = v_j^i/(\text{var}_{p,j} - \text{var}_{p-1,j}) , \tag{5}$$

where v_j^i stands for the coefficient of the p^{th}-order gPC expansion, which is only pertinent to the i^{th} dimension. Refinement is performed along the i^{th} dimension if α_j^i satisfies a certain criterion [4].

The proposed framework employs the deterministic EM simulator of [5] to compute observable values at the collocation (integration) points. The proposed adaptive framework has two advantages over its pure SC-predecessor: (i) It effectively tailors the number of collocation points to the rate of variation of the observable and (ii) the approximate observable values obtained from the resulting gPC expansions are used to replace the actual observable evaluations (i.e. the costly deterministic EM simulations) when extracting observable pdfs via Monte Carlo methods.

Numerical Results

Transmission Line (TL) Terminated by an RLC Circuit: For illustration purposes, the proposed method is used to characterize the voltage across the capacitor of an RLC circuit terminating a TL [Fig. 1(a)], which is excited by a sinusoidal voltage source with frequency $486.28\,\text{MHz}$. The inductance value L_1 and the capacitance value C_1 are the random variables, which are assumed uniformly distributed in $[5\,-\,15]\,\text{nH}$ and $[10-20]\,\text{pF}$, respectively. The observable is the real part of the voltage across the capacitor. Approximated value of observable is plotted in Fig. 1(b). The resulting refinement in the two-dimensional random domain is shown in Fig. 1(c). Clearly, near L_1 and C_1 values around the resonance region (where the observable varies very rapidly), the scheme *automatically* produces smaller sub-domains to increase the accuracy of the gPC expansion.

Microwave Amplifier: The proposed method is used to statistically characterize the voltage coupled to the output terminal of a microwave amplifier, which consists of a packaged GaAs MESFET (operated under small signal conditions) and microstrip matching networks [Fig. 2(a)]. Two EMI scenarios are investigated: (i) the amplifier is directly exposed to plane-wave excitation and (ii) it is located inside a shielding enclosure [Fig. 2(b)] and the whole structure is exposed to plane-wave excitation. In both scenarios the excitation frequency is $6\,\text{GHz}$, the plane wave propagates in $\hat{\mathbf{x}}\sin(\theta)+\hat{\mathbf{z}}\cos(\theta)$ direction, and the amplitude of the θ-polarized electric field is $1\,\text{V/m}$. The uncertainties in the small signal circuit parameters and excitation are parameterized by seven independent random variables ($N_{\text{dof}}=7$): the values of parasitic inductors and resistors at the gate (L_{g}, R_{g}), the drain (L_{d}, R_{d}), and the source terminals (L_{s}, R_{s}), and the angle of excitation θ, which are assumed to be normally distributed with means and standard deviations $(0.05, 0.001)\,\text{nH}$, $(0.5, 0.015)\,\Omega$, $(0.05, 0.001)\,\text{nH}$, $(0.5, 0.015)\,\Omega$, $(0.1, 0.002)\,\text{nH}$, $(0.7, 0.021)\,\Omega$, and $(127.5, 4.5)^\circ$, respectively. The observable is the absolute value of the voltage coupled to the output terminal. For both scenarios, the adaptive SC with sparse grid is used to obtain the gPC expansion over the random domain. Then the expansion is used to approximate the observable at 25000 random points to extract its pdf [Fig. 2(c)]. To verify the accuracy of the proposed scheme, relative L-2 norm errors between the approximate and exact observable values (for the first scenario) at 5000 random points, are compared in Table I. The approximate observable values are computed by the gPC expansions obtained using the adaptive SC method with sparse grid and SC method with tensor product. Table I also demonstrates the efficiency of the proposed scheme: Adaptive SC method requires less number of exact observable evaluations (i.e. deterministic simulations) to obtain the gPC expansion.

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Figures

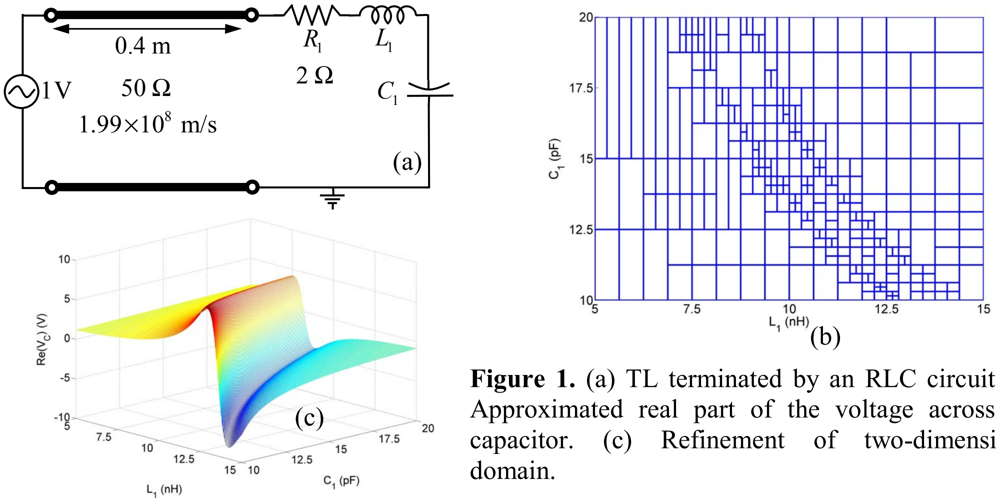

Figure 1. (a) TL terminated by an RLC circuit. (b) Approximated real part of the voltage across the capacitor. (c) Refinement of two-dimensional domain.

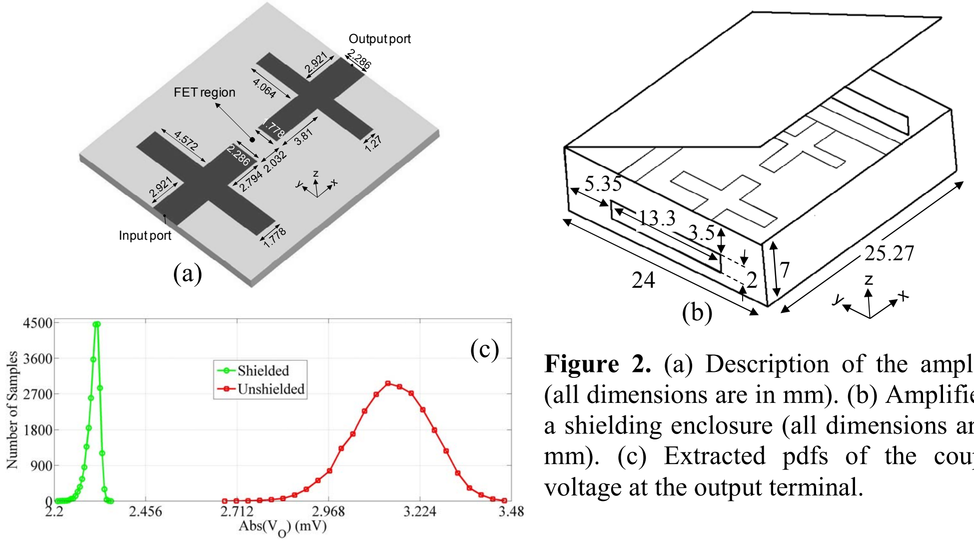

Figure 2. (a) Description of the amplifier (all dimensions are in mm). (b) Amplifier in a shielding enclosure (all dimensions are in mm). (c) Extracted pdfs of the coupled voltage at the output terminal.

Table I. Comparison of the adaptive SC and SC.

Method	Relative L-2 norm error	# of sub-domains	Total # of deterministic simulations
SC with 3-point tensor product	4.75512×10^{-3}	1	2187
Adaptive SC with 2-level sparse grid, tolerance 10^{-1}	8.83656×10^{-4}	3	423
Adaptive SC with 2-level sparse grid, tolerance 10^{-2}	9.64640×10^{-5}	13	1833