Tucker-Accelerated Volume Integral Equation Solver for Magneto-Quasi-Static Analysis of Multiscale Voxelized Structures

Xiaofan Jia(1), Mingyu Wang(2), Yang Liu(3), Chao-Fu Wang(4), and Abdulkadir C. Yucel (1)
(1) Nanyang Technological University, 639798, Singapore
(2) Xpeedic Co., LTD., Shanghai, 201210, China
(3) Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA
(4) National University of Singapore, 117411, Singapore

Volume integral equation (VIE) solvers have been widely used for magneto-quasi-static (MQS) analysis required for many applications, ranging from low-frequency bioelectromagnetic analysis and chip design to geophysical exploration. Among a plethora of VIE solvers developed so far, VoxHenry (A. C. Yucel, et al., IEEE Trans. Microw. Theory Techn., 66(4), 1723-1735, 2018) has recently received significant attention as it offers the MQS analysis of structures uniformly discretized by voxels (i.e., cubes) and ultra-fast matrix-vector multiplications (MVMs) required during the iterative solution of VIEs via fast Fourier transforms (FFTs). However, its uniform voxel-based discretization requirement hinders its applicability to structures with multiscale features. To this end, various techniques can be adopted in lieu of FFT acceleration on uniform voxels to expedite the MVMs. These techniques include H-matrix, singular value decomposition (SVD), adaptive cross approximation, and butterfly compression. As all these techniques leverage low-rank matrix decomposition-based approximations, they cannot effectively exploit the low-rank nature of the tensors arising in VIE solvers. Recently, tensor decompositions, specifically Tucker decompositions, have been used to compress the low-rank tensors in various low- and high-frequency integral equation solvers. These tensor decompositions effectively reduced the memory requirement and setup time in electrostatic and magneto-quasi-static solvers (M. Wang, et al., IEEE Trans. Microw. Theory Techn., 68(12), 5154-5168, 2020) (M. Wang, et al., IEEE Trans. Appl. Supercond., 31(7), 1-11, 2021) while those lessen the computational time requirement of aggregation and memory requirement of translation operator tensors in fast multipole method - fast Fourier transform (FMM-FFT) accelerated high-frequency solvers (C. Qian, M. Wang, and A. C. Yucel, IEEE Trans. Antennas Propag., 69(10), 6660-6668, 2021) (C. Qian and A. C. Yucel, IEEE Trans. Antennas Propag., 69(6), 3359-3370, 2021). That said, there exists no study that leverages Tucker decompositions to accelerate the MVMs for the VIE solvers so far.

In this study, Tucker decompositions are leveraged to compress the blocks of the system matrix arising in the VIE solver for MQS analysis and accelerate MVMs during the iterative solution of the VIE for the first time. In particular, the system matrix is hierarchically partitioned into admissible and inadmissible blocks. The admissible blocks are then transformed into 6-D arrays or tensors and compressed via Tucker decompositions. The compressed tensors are used to perform fast tensor-vector multiplications (TVMs) in the compressed format and the resulting vectors of MVMs are obtained from the results of TVMs. Preliminary results on the admissible blocks of well-separated source and observer constellations show that the Tucker compression achieves at least 2 and 3 orders of compression ratio enhancement compared to SVD compression for the tolerances of 10^{-3} and 10^{-6}, respectively. Furthermore, the computational time required for one TVM performed via Tucker-compressed tensors is always less than that required for one MVM performed via SVD-compressed blocks. In the talk, various computational studies performed to demonstrate the memory and time efficiency of the proposed Tucker-accelerated VIE solver will be presented.