A Surface Integral Equation-Based Simulator for Resistance and Inductance Extraction

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***Abstract*—This study proposes a surface integral equation–based simulator for accurate and efficient extraction of the resistance/inductance (RL) parameters under the magneto-quasi-static assumption. The proposed simulator employs a novel centroid-midpoint (CM) basis function and a highly accurate equivalent surface impedance (ESI) model. The utilization of CM basis functions allows for the conversion of edge-based vector potential computations to panel-based scalar potential computation. Such conversion makes the implementation of fast multipole method straightforward and allows drastically reducing the matrix-fill time, matrix-vector multiplication time, and setup time for the preconditioner. Utilization of the ESI model makes the simulator highly accurate for RL extraction at low frequencies. The numerical results show that the proposed simulator requires 93x less CPU time and 14x less memory than Ansys Q3D for the same level of accuracy while extracting the RL parameters of a 50-port ball grid array package.**

I. Introduction

The resistance/inductance (RL) parameters extracted under the magneto-quasi-static (MQS) assumption are of utmost significance for the power/signal integrity studies. To extract RL parameters, many simulators have been developed so far. Among those, FastHenry [1], VoxHenry [2], and TetraHenry [3] simulators solve the volume integral equation (VIE) on filament, voxel, and tetrahedral discretized structures, respectively. However, these simulators leverage volumetric discretization, rendering them inefficient when applied to the RL extraction of complicated packages at (high-) frequencies at which the currents tend to flow on surfaces. Apart from VIE simulators, various surface integral equation (SIE) simulators have been developed for RL extraction. FastImp [4] is a popular wide-band simulator leveraging pre-corrected fast Fourier transform to accelerate matrix-vector-multiplications (MVMs). However, it requires a large number of unknowns (around seven times the number of panels used to discretize the structures) to be solved during the solution of SIEs. Recently, a loop analysis-based SIE solver for RL extraction was proposed [5]. However, its equivalent surface impedance (ESI) model is not sufficiently accurate to extract RL at low-frequencies.

This study proposes an SIE simulator, called XRL, for broadband RL extraction. The proposed XRL solves the SIE after discretizing the conductor surfaces via triangular and rectangular elements. The surface currents on these elements are discretized by novel centroid-midpoint (CM) basis functions. Such basis functions allows mapping the current vectors on edges to the panel centers. Doing so converts the integrations on the traditional edge-based basis functions to those on the panel-based pulse basis functions. Such conversion allows reducing the computational requirements of the fast MVMs of fast multipole method (FMM) by a factor of 1.5~2. Furthermore, it drastically reduces the matrix-fill time (the execution time of integrations), makes the implementation of matrix-fill routines straightforward, and yields remarkably less setup time for the preconditioner. Furthermore, the XRL employs a highly accurate ESI model that ensures the accuracy of RL extraction at the low-frequency regime. Preliminary results show that the proposed XRL simulator outperforms the commercial Ansys Q3D for the RL extraction of complicated packaging structures.

II. Formulation

Let f , $\omega = 2\pi f$, and μ_0 denote the frequency, angular frequency, and the free-space permeability. $\mathbf{r}$ and $\mathbf{r}'$ represent the observation and source locations on surface S', enclosing the conductors with conductivity σ . The proposed XRL simulator solves the combined surface integral and current continuity equations, which reads

$$Z_s \mathbf{J}(\mathbf{r}) + \frac{j\omega\mu_0}{4\pi} \int_{S'} \frac{\mathbf{J}(\mathbf{r}')}{\|\mathbf{r}-\mathbf{r}'\|} d\mathbf{r}' = -\nabla\Phi,\ \nabla\cdot\mathbf{J}(\mathbf{r}) = 0 . \quad (1)$$

Here Z_s is the ESI, which is implemented as described in for the panels included by rings [6]. For the rest, double-plane model described in [7] is implemented. The surface(s) of conductor(s) is/are discretized by a mixture of triangles and rectangles, which provides flexibility for modelling complicated 3-D geometries. To map the edge current interactions to panels, three sparse matrices $\bar{\mathbf{A}}_1 \in \mathbb{R}^{N_e \times N_p}$, $\bar{\mathbf{A}}_2 \in \mathbb{R}^{N_e \times N_p}$, and $\bar{\mathbf{A}}_3 \in \mathbb{R}^{N_e \times N_p}$ are introduced, where N_e and N_p are the numbers of edges and panels, respectively. Loop analysis [1, 5] is implemented to ensure the current continuity. To do that, a sparse loop matrix $\bar{\mathbf{M}} \in \mathbb{R}^{N_l \times N_e}$ is introduced, where N_l is the number of loops. Under the loop/mesh analysis, the linear system of equations obtained after discretizing with CM basis functions is written as

$$\bar{\mathbf{M}}(\bar{\mathbf{A}}_1\bar{\mathbf{Z}}\bar{\mathbf{A}}_1^T + \bar{\mathbf{A}}_2\bar{\mathbf{Z}}\bar{\mathbf{A}}_2^T + \bar{\mathbf{A}}_3\bar{\mathbf{Z}}\bar{\mathbf{A}}_3^T)\bar{\mathbf{M}}^T\mathbf{I}_l = \mathbf{V}_l . \quad (2)$$

Here $\mathbf{I}_l$, $\mathbf{V}_l$, and $\bar{\mathbf{Z}}$ represent the vector storing the loop currents' coefficients, voltage vector, and system matrix, respectively. To implement FMM, (2) is split into two parts as

$$\bar{\mathbf{Z}}_s\mathbf{I}_l + \bar{\mathbf{M}}(\bar{\mathbf{A}}_1\bar{\mathbf{P}}\bar{\mathbf{A}}_1^T + \bar{\mathbf{A}}_2\bar{\mathbf{P}}\bar{\mathbf{A}}_2^T + \bar{\mathbf{A}}_3\bar{\mathbf{P}}\bar{\mathbf{A}}_3^T)\bar{\mathbf{M}}^T\mathbf{I}_l = \mathbf{V}_l\,. \quad (3)$$

Here $\bar{\mathbf{Z}}_s$ is the surface impedance matrix under loop analysis and its multiplication with $\mathbf{I}_l$ is computationally cheap. The second part in (3) is rewritten as

$$\bar{\mathbf{M}}(\bar{\mathbf{A}}_1\bar{\mathbf{P}}\mathbf{I}_{p1} + \bar{\mathbf{A}}_2\bar{\mathbf{P}}\mathbf{I}_{p2} + \bar{\mathbf{A}}_3\bar{\mathbf{P}}\mathbf{I}_{p3}) = \mathbf{V}_l\,, \quad (4)$$

where $\mathbf{I}_{pi} = \bar{\mathbf{A}}_i^T\bar{\mathbf{M}}^T\mathbf{I}_l$, $i = 1,2,3$. FMM is used to accelerate $\bar{\mathbf{P}}\mathbf{I}_{pi}$ directly and its computational complexity is $O(N_p)$. Details of the formulation and implementation will be provided during the presentation.

III. Numerical Results

The accuracy and efficiency of the proposed XRL simulator are demonstrated by RL extraction of two parallel $100\mu\text{m}\times100\mu\text{m}$ square coils with cross-sections of $5\mu\text{m}\times5\mu\text{m}$ (width x height). The distance between two coils is $10\mu\text{m}$ and the spacing between source and sink in each coil is $2\mu\text{m}$. The RL parameters are obtained by the XRL and Ansys Q3D over a frequency band of from 1 kHz to 100 GHz and compared with those obtained by VoxHenry [2], which serve as the reference. Table I shows that XRL requires 2.4x less memory and 7.6x less CPU time than Q3D for the similar number of panels, while the accuracy of R parameters (L^2-norm errors) obtained by XRL is better than those obtained by Q3D due to its accurate ESI model.

TABLE I
Accuracy and Computational Requirements of Proposed XRL and Q3D

	XRL	Ansys Q3D
Number of panels	2188	2042
Memory requirement (MB)	78.3	189.6
CPU time requirement (s)	11	84
$L^2-err\,(R_{1,1})$ (%)	4.03	8.28
$L^2-err\,(R_{1,2})$ (%)	27.84	32.24
$L^2-err\,(L_{1,1})$ (%)	1.11	0.42
$L^2-err\,(L_{1,2})$ (%)	0.11	0.58

Next, an additional numerical test on a ball grid array package with 50 source ports shows that the proposed XRL is more efficient and accurate than Ansys Q3D. Q3D adopts adaptive mesh refinement with a default accuracy 1%. Fig. 1 shows that the Q3D with the default accuracy cannot provide accurate resistance values due to its inaccurate ESI model. The resistance values obtained by Q3D with adaptive mesh refinement accuracy of 0.1% can match those obtained by the proposed XRL. Table II shows that XRL requires 93x less CPU time and 14x less memory than Q3D for relatively the same level of accuracy.

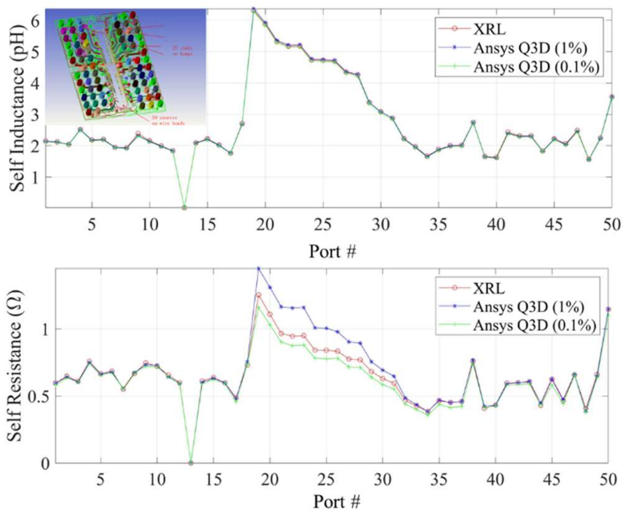

Figure 1. Self-inductances (top) and self-resistances (bottom) of each port at 1.6 GHz obtained by XRL and Q3D.

TABLE II
Memory and CPU Time Requirements of XRL and Q3D

	XRL	Q3D (1%)	Q3D (0.1%)
Memory requirement (GB)	3.84	16.3	54.7
CPU time requirement (h)	0.155	3.05	14.45

IV. Conclusion

An SIE simulator for extracting the RL parameters under MQS assumption is proposed. XRL leverages novel CM basis functions and an accurate ESI model. While the former gives rise to simple implementation and a reduced computational cost, the latter ensures accurate RL parameter extraction at low frequency regime.

References

[1] M. Kamon, M. J. Tsuk, and J. K. White, "FASTHENRY: A multipole-accelerated 3-D inductance extraction program," *IEEE Trans. Microw. Theory Techn.*, vol. 42, no. 9, pp. 1750-1758, Sept. 1994.

[2] A. C. Yucel, I. P. Georgakis, A. G. Polimeridis, H. Bagci, and J. K. White, "VoxHenry: FFT-accelerated inductance extraction for voxelized geometries," *IEEE Trans. Microw. Theory Techn.*, vol. 66, no. 4, pp. 1723–1735, Apr. 2018.

[3] K. Jackman and C. J. Fourie, "Tetrahedral modeling method for inductance extraction of complex 3-D superconducting structures," *IEEE Trans. Appl. Supercond.*, vol. 26, no. 3, pp. 1-5, April 2016.

[4] Z. Zhu, B. Song, and J. K. White, "Algorithms in FastImp: a fast and wide-band impedance extraction program for complicated 3-D geometries," *IEEE Trans. Comput.-Aided Design Integr. Circuits Syst.*, vol. 24, no. 7, pp. 981-998, July 2005.

[5] Z. Zhu, Z. Chen, and S. Yang, "An SIE formulation with triangular discretization and loop analysis for parameter extraction of arbitrarily shaped interconnects," *IEEE Trans. Microw. Theory Techn.*, vol. 71, no. 10, pp. 4282-4294, Oct. 2023.

[6] M. Al-Qedra, J. Aronsson, and V. Okhmatovski, "A novel skin-effect based surface impedance formulation for broadband modeling of 3-D interconnects with electric field integral equation," *IEEE Trans. Microw.Theory Techn.*, vol. 58, no. 12, pp. 3872-3881, Dec. 2010.

[7] J. C. Rautio, "An investigation of microstrip conductor loss," *IEEE Microw. Mag.*, pp. 60–67, Dec. 2000.