

Mathematical Statistics

MAS 713

Tutorial about Chapter 3

Exercise 1

Question:

Let X be a r.v. with cumulative distribution function $F(x)$ and density $f(x) = F'(x)$. Find the probability density function of

- (a) the maximum of n independent random variables all with cumulative distribution function $F(x)$.
- (b) the minimum of n independent random variables all with cumulative distribution function $F(x)$.

Exercise 2

Question:

An article in the review *Knee Surgery, Sports Traumatology and Arthroscopy* in 2005 cites the following results:

- ① a success rate of more than 90% for meniscal tears with a rim width of less than 3mm,
- ② but only a 67% success rate for tears of 3–6mm.

If you are unlucky enough to suffer from:

- a meniscal tear of less than 3mm on your **left** knee and
- one of width 3–6mm on your **right** knee,
- what is the **probability mass function** of the number of successful surgeries? (Assume the surgeries are independent)
- Find the **mean** and **variance** of the number of successful surgeries that you would undergo.

Exercise 3

Question:

The article “*Error Distribution in Navigation*” (J. Institute of Navigation, 1971) suggests that the distribution of the lateral position error, say X (in nautical miles), which can be either positive or negative, is well approximated by a **density** like

$$f(x) = c e^{-0.2|x|} \quad \text{for } -\infty < x < \infty,$$

for a constant c .

- a) Find the value of c which makes f a legitimate density function, and sketch the corresponding density curve.
- b) In the long-run, what proportion of errors is negative? At most 2? Between -1 and 2 ?

Exercise 4

Question:

The probability density function of the weight X (in kg) of packages delivered by a post office is

$$f(x) = \frac{70}{69x^2} \quad \text{for } 1 < x < 70$$

and 0 elsewhere.

- a) Determine the mean and the variance of the weight X .
- b) If the shipping cost is \$2.50 per kg, what is the average shipping cost of a package? What is the variance of the shipping cost?
- c) In the long-term, what is the proportion of packages whose weight exceeds 50 kg?

Exercise 5

Question:

Compute $\mathbb{E}(x)$ and $\text{Var}(x)$ for each of the following probability distributions:

- 1 $f(x) = ax^{a-1}, 0 < x < 1, a > 0$
- 2 $f(x) = \frac{1}{n}, x = 1, 2, \dots, n, n > 0$ an integer

Exercise 6

In each of the following situations state whether it is reasonable to use **Binomial** or **Poisson** distributions for X .

If so, tell which one, and (if it is possible) determine what are the values of the parameters:

- a) Toss a fair coin 6 times, X is the number of 'Heads'.
- b) Toss a fair coin until the first time a head appears, X is the count of the number of tosses you make.
- c) A factory makes carpets. Sometimes there are flaws in the carpet. On average a square metre of carpet has 3 flaws. X is the number of flaws in a random square metre of carpet.
- d) Most calls made at random by sample surveys don't succeed in talking to a person. Of calls in New York City, only $1/12$ succeed. A survey calls 500 randomly selected numbers in New York City, and X is the number that reach a live person.
- e) Calls to a telephone exchange come in at an average of 250 an hour, X is the number of calls in a given hour.
- f) A die (6 faces, numbered 1,2,3,4,5,6) is tossed twice and X is the number of 6s obtained.

Exercise 7

Question:

An individual claims to have extrasensory perception (ESP). As a test, a fair coin is tossed ten times, and he is asked to predict in advance the outcome. Our individual gets seven out of ten correct.

- What is the probability he would have done at least this well if he had no ESP?
- Would you believe in his powers?

Exercise 8

Question:

The analysis of results from a leaf transmutation experiment (turning a leaf into a petal) is summarised by type of transformation completed :

		Textural transformation	
		Yes	No
Colour transformation	Yes	243	26
	No	13	18

A naturalist randomly selects three different leaves from this set. Determine the following probabilities :

- a) Exactly one has undergone both types of transformations.
- b) At least one has undergone both types of transformations.
- c) Exactly one has undergone one but not both transformations.
- d) At least one has undergone at least one transformation.

Exercise 9

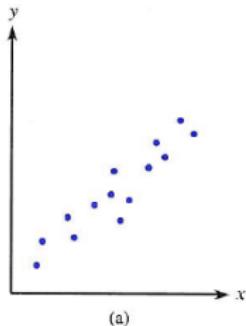
Question:

Suppose a value z is repeatedly randomly chosen from a standard normal distribution :

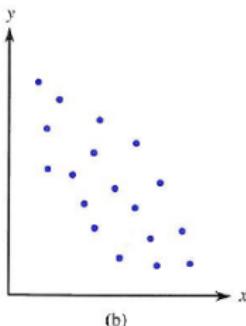
- (a) In the long run, what is the proportion of times that z will be at most 2.15? Less than 2.15?
- (b) What is the long run proportion of times that z will be between -1.23 and 2.85?
- (c) What is the long run proportion of times that z will exceed 5? Will exceed -5?
- (d) What is the long run proportion of times that z will satisfy $|z| < 2.50$?

Exercise 10

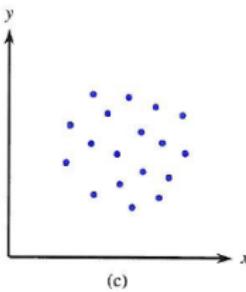
Given the four scatter plots for two random variables X , Y in the Figure



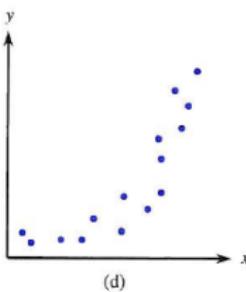
(a)



(b)



(c)



(d)

Figure:

Question:

- 1 which of the plots demonstrates a positive relationship ?
- 2 which of the plots demonstrates a positive linear relationship ?
- 3 which of the plots demonstrates a negative relationship ?
- 4 which of the plots demonstrates no relationship ?
- 5 for which of the plots would you expect positive correlation ?
Negative correlation ? No or little correlation ?

Exercise 11

Question:

For each of the following pairs of variables, indicate whether you would expect a positive correlation, a negative correlation, or little or no correlation. Explain your choices.

- ① Maximum daily temperature and cooling cost
- ② Interest rate and number of loan applications
- ③ Distance a student doing MAS713 lives from NTU campus and their marks at the Matlab online quizzes