Optimal Interdiction of Urban Criminals with the Aid of Real-Time Information

Appendix

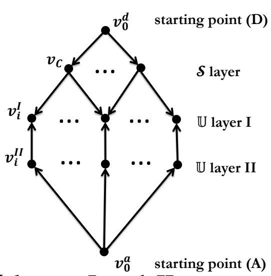
A Proofs

A.1 Proof of Proposition 1

Proof. We extend the example in Figure 1 to a more general form. That is, there are $|O| = n$ different paths from the adversary's initial location to the external world through n different exit nodes with two time steps. Specifically, any two paths do not intersect. On the other hand, there are n different paths starting from the defender resource's initial location v_0 to different exit nodes by one time step. Similar to the toy example in Figure 1, the probability of catching the adversary is $U_d = \frac{1}{n}$ by the optimal non-real-time strategy under the NE, while the probability of catching the adversary is $U_d^\star = 1$ by the optimal real-time strategy under the NE. Then, $\frac{U_d^\star}{U_d} = n$, can be made arbitrarily large by increasing n. □

A.2 Proof of Theorem 1

Proof. We reduce the Set Cover problem to computing the NE of NEST, which is described as follows: given a set $\mathbb{U}$ of elements, a collection $\mathcal{S} \subseteq 2^{\mathbb{U}}$ of subsets of $\mathbb{U}$, and an integer m, determine whether there exists a set $\mathcal{C} \subseteq \mathcal{S}$ of size m or less, such that $\cup_{C \in \mathcal{C}} = \mathbb{U}$.



Reduction: The network structure in NEST is demonstrated by the right figure, where v_0^a and v_0^d are starting points for the adversary and the defender respectively. Between them, there are three layers of nodes. The $\mathcal{S}$ layer is fully connected to v_0^d where each node v_C corresponds to the set C in $\mathcal{S}$. $\mathbb{U}$ layers I and II are two identical layers representing all the elements in the ground set $\mathbb{U}$. These two layers are connected with each other in an element-wise manner. Each node v_i^I in layer I is linked with v_C in the $\mathcal{S}$ layer if $i \in C$. On the other hand, the adversary can move from v_0^a to any node in the layer II. Moreover, the defender has m resources, all located at v_0^d initially. The time horizon $t_{\max}$ is set to 2, and the nodes in $\mathbb{U}$ layer I are characterized as exit nodes. It is easy to verify that this is a polynomial-time reduction.

With a horizon of two time steps, suppose that the adversary reaches v_i^{II} at the first time step, and the defender is aware of exit node v_i^I chosen by the adversary. If any one of the defender's resources reaches v_C in the $\mathcal{S}$ layer with $i \in C$ at the first time step, the adversary will be captured for sure. On the other hand, if there is no set cover, then it is easy to show that there is a positive probability that the adversary will move to a v_i^{II} such that the corresponding v_i^I is not protected by any defender resources and thus, the adversary can escape. Thus, the NE of NEST answers the Set Cover problem: there exists a set $\mathcal{C} \subseteq \mathcal{S}$ of size m or less covering the ground set $\mathbb{U}$ ⇔ in NE, the defender captures the adversary with probability 1, which requires the defender to move the m resources to nodes at $\mathcal{S}$ layer which fully protect $\mathbb{U}$ layer I. □

A.3 Proof of Theorem 2

Proof. Given $\mathbf{x}$, we first show that conditions (3a)-(3c) are satisfied. By Eq.(4), we have:

$$\textstyle\sum_{l \in A_{s_0}} f_{s_0,l} = \sum_{l \in A_{s_0}} P_{\mathbf{x}}(s_0) x_{s_0,l} = \sum_{l \in A_{s_0}} x_{s_0,l} = 1,$$

which is Eq.(3a). By Eq.(2c), $P_{\mathbf{x}}(s) = \sum_{l \in A_s} x_{s,l} P_{\mathbf{x}}(s)$ ($\forall s \in S \backslash S_t$). Moreover, by Eq.(1), $\forall s \in S \backslash (\{s_0\} \cup S_t)$,

$$\textstyle\sum_{s' \in S \backslash S_t : s \in S_{s',l_s}} P_{\mathbf{x}}(s') x_{s',l_s} = \sum_{l \in A_s} P_{\mathbf{x}}(s) x_{s,l}.$$

Further, by Eq.(4), $\forall s \in S \backslash (\{s_0\} \cup S_t)$,

$$\textstyle\sum_{s' \in S \backslash S_t : s \in S_{s',l_s}} f_{s',l_s} = \sum_{l \in A_s} f_{s,l},$$

which is Eq.(3b). Obviously, Eq.(3c) is obtained from Eqs.(1), (2d) and (4). For each $o \in O$,

$$\begin{aligned} U_d(\mathbf{x}, o) &= \textstyle\sum_{s \in S_c : h_s \sqsubseteq o} P_{\mathbf{x}}(s) \\ &= \textstyle\sum_{s \in S_c : h_s \sqsubseteq o} \sum_{s' \in S \backslash S_t : s \in S_{s',l_s}} P_{\mathbf{x}}(s') x_{s',l_s} \\ &= \textstyle\sum_{s \in S_c : h_s \sqsubseteq o} \sum_{s' \in S \backslash S_t : s \in S_{s',l_s}} f_{s',l_s}, \\ &= U_d(\mathbf{f}, o). \end{aligned}$$

Therefore, $\forall \mathbf{x}$, $\exists \mathbf{f}$ defined by Eq.(4) such that $U_d(\mathbf{x}, o) = U_d(\mathbf{f}, o)$ ($\forall o \in O$).

Given $\mathbf{f}$, we define $\mathbf{x}$ in Eq. (5). Obviously, $P_{\mathbf{x}}(s) x_{s,l} = f_{s,l}$, and then $U_d(\mathbf{f}, o) = U_d(\mathbf{x}, o)$. Therefore, $\forall \mathbf{f}$, $\exists \mathbf{x}$ defined by Eq. (5) such that $U_d(\mathbf{f}, o) = U_d(\mathbf{x}, o)$ ($\forall o \in O$). □

A.4 Proof of Theorem 3

Proof. After calling our *BR* algorithm, we can obtain the best response policy starting from s_0 against $\mathbf{y}$ in $G(S, A)$. Note that $V(s_0)$ is the defender utility by the best response against $\mathbf{y}$ in $G(S, A)$ by our *BR* algorithm. Then $V(s_0) \geqslant U_d(\mathbf{x}, \mathbf{y}) \geqslant 0$. If $V(s_0) > U_d(\mathbf{x}, \mathbf{y})$, π, i.e., the output at Line 4, must contain some states s with $V(s) > 0$ or their actions that are not in $G(S', A')$. Consequently, new states and actions will be added to $G(S', A')$, and then $G(S', A')$ is expanded. In the worst case, $G(S', A') = G(S, A)$, where *IGRS* will stop and $V(s_0) = U_d(\mathbf{x}, \mathbf{y})$. Therefore, *IGRS* will converge with $V(s_0) = U_d(\mathbf{x}, \mathbf{y})$ with a finite number of iterations because the number of states and actions in $G(S, A)$ is finite. Therefore, $U_d(\mathbf{x}, \mathbf{y}) \geqslant U_d(\mathbf{x}', \mathbf{y})$ ($\forall \mathbf{x}'$). Note that $U_a(\mathbf{x}, \mathbf{y}) \geqslant U_a(\mathbf{x}, \mathbf{y}')$ ($\forall \mathbf{y}'$). Then, $(\mathbf{x}, \mathbf{y})$ is an NE. □

A.5 Proof of Theorem 4

Proof. Because $O_{h_{s_1}}$ in G_{s_1} is similar to $O_{h_{s_2}}$ in G_{s_2}, we have

$$\begin{aligned} V^{\pi_{s_1 \to s_2}}(s_2) &= \sum_{P_{\pi_{s_1 \to s_2}}(s_c) = 1, s_c \in S_c \cap S_{G_{s_2}} : h_{s_c} \sqsubseteq o \in O_{h_{s_2}}} y'_o \\ &= \sum_{P_{\pi_{s_1}}(s_c) = 1, s_c \in S_c \cap S_{G_{s_1}} : h_{s_c} \sqsubseteq o \in O_{h_{s_1}}} y_o \\ &= V^{\pi_{s_1}}(s_1). \end{aligned}$$

Suppose $\pi_{s_1 \to s_2}$ is not the best response in G_{s_2} against $\mathbf{y}'$. Then, there is a best response strategy π'_{s_2} against $\mathbf{y}'$ such that $V^{\pi'_{s_2}}(s_2) > V^{\pi_{s_1 \to s_2}}(s_2)$. Let $O_{y'}$ be the support set of $\mathbf{y}'$. If $O_y \cap O_{h_{s_2}} = \varnothing$, then $V^{\pi'_{s_2}}(s_2) = V^{\pi_{s_1 \to s_2}}(s_2) = 0$, which leads to a contradiction. If $O_{y'} \cap O_{h_{s_2}} = O^* \neq \varnothing$, then $V^{\pi'_{s_2 \to s_1}}(s_1) = V^{\pi'_{s_2}}(s_2) > V^{\pi_{s_1 \to s_2}}(s_2) = V^{\pi_{s_1}}(s_1)$, i.e., π_{s_1} is not the best response in G_{s_1} against $\mathbf{y}$, which causes a contradiction. Therefore, $\pi_{s_1 \to s_2}$ is the best response in G_{s_2} against $\mathbf{y}'$. □

A.6 Proof of Lemma 1

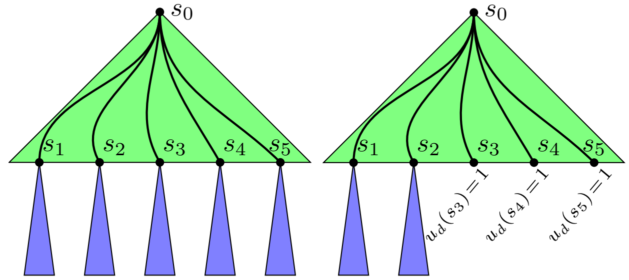
Proof. If there is a strategy $\mathbf{y}$ such that $V^{\pi^\star_s}(s) = \sum_{o \in O_{h_s}} y_o \sum_{s_c \in S_c \cap S_{G_s}: h_{s_c} \sqsubseteq o} P_{\pi^\star_s}(s_c) < \sum_{o \in O_{h_s}} y_o$ under $\pi^\star_s$, then there is a path $o^* \in O_{h_s}$ such that $\sum_{s_c \in S_c \cap S_{G_s}: h_{s_c} \sqsubseteq o^*} P_{\pi^\star_s}(s_c) = 0$. Therefore, if $\pi^\star_s$ is played against $\mathbf{y}$ with $y_o > 0$ $(\forall o \in O_{h_s})$, $V^{\pi^\star_s}(s) = \sum_{o \in O_{h_s}} y_o \sum_{s_c \in S_c \cap S_{G_s}: h_{s_c} \sqsubseteq o} P_{\pi^\star_s}(s_c) = \sum_{o \in O_{h_s} \setminus \{o^*\}} y_o \sum_{s_c \in S_c \cap S_{G_s}: h_{s_c} \sqsubseteq o} P_{\pi^\star_s}(s_c) < \sum_{o \in O_{h_s}} y_o$, which contradicts the definition of $\pi^\star_s$ in Eq.(9). □

A.7 Proof of Theorem 5

Proof. Suppose $\pi^\star_s$ is not optimal in NEST. Then, there is a strategy $\mathbf{y}$, and π_s is the best response in G_s such that with $V^{\pi_s}(s) > V^{\pi^\star_s}(s)$. Let O_y be $\mathbf{y}$'s support set. If $O_y \cap O_{h_s} = \varnothing$, then $V^{\pi_s}(s) = V^{\pi^\star_s}(s) = 0$, which leads to a contradiction. If $O_y \cap O_{h_s} = O^* \neq \varnothing$, then, by Lemma 1, $V^{\pi^\star_s}(s) = \sum_{o \in O_{h_s}} y_o = \sum_{o \in O^*} y_o \geqslant V^{\pi_s}(s)$, which also causes a contradiction. □

A.8 Proof of Theorem 6

Proof. Suppose $\pi^\star_{s_1 \to s_2}$ is not optimal in G_{s_2}. Then, there is a strategy $\mathbf{y}$, and π_{s_2} is the best response in G_{s_2} such that with $V^{\pi_{s_2}}(s_2) > V^{\pi^\star_{s_1 \to s_2}}(s_2)$. Let O_y be $\mathbf{y}$'s support set. If $O_y \cap O_{h_{s_2}} = \varnothing$, then $V^{\pi_{s_2}}(s_2) = V^{\pi^\star_{s_1 \to s_2}}(s_2) = 0$, which leads to a contradiction. If $O_y \cap O_{h_{s_2}} = O^* \neq \varnothing$, by the assumption $V^{\pi_{s_2}}(s_2) > V^{\pi^\star_{s_1 \to s_2}}(s_2)$, then there is a path $o' \in O_{h_{s_2}}$ which is not interdicted by $\pi^\star_{s_1 \to s_2}$, i.e., $P_{\pi^\star_{s_1 \to s_2}}(s_c) = 0$ $(\forall s_c \in S_c \cap S_{G_{s_2}}$ with $h_{s_c} \sqsubseteq o')$. Note that, given a deterministic policy π_s in G_s, for each path $o \in O_{h_s}$, there is at most one capture state s_c such that $P_{\pi^\star_s}(s_c) = 1$ and $h_{s_c} \sqsubseteq o$. For each $o_1 \in O_{h_{s_1}}$, if there is a capture state s_c with $P_{\pi^\star_{s_1}}(s_c) = 1$ and $h_{s_c} \sqsubseteq o_1$, then for its similar o_2 in $O_{h_{s_2}}$ such that $q(h_{s_1}, o_1) = q(h_{s_2}, o_2)$, there is a capture state s'_c with $P_{\pi^\star_{s_1 \to s_2}}(s'_c) = 1$ and $h_{s'_c} \sqsubseteq o_2$ because resources at the key locations take the same actions in semi-similar states in both subgames by Eq.(10). Therefore, $o^* \in O_{h_{s_1}}$ with $q(h_{s_1}, o^*) = q(h_{s_2}, o')$ does not generate the history in any capture state reached from s_1 by $\pi^\star_{s_1}$, i.e., $P_{\pi^\star_{s_1}}(s_c) = 0$ $(\forall s_c \in S_c \cap S_{G_{s_1}}$ with $h_{s_c} \sqsubseteq o^*)$. It means that, for $\mathbf{y}'$ with $y'_o > 0 (\forall o \in O_{h_{s_1}})$, $V^{\pi^\star_{s_1}}(s_1) = \sum_{o \in O_{h_{s_1}}} y'_o \sum_{s_c \in S_c \cap S_{G_{s_1}}: h_{s_c} \sqsubseteq o} P_{\pi^\star_{s_1}}(s_c) = \sum_{o \in O_{h_{s_1}} \setminus \{o^*\}} y'_o \sum_{s_c \in S_c \cap S_{G_{s_1}}: h_{s_c} \sqsubseteq o} P_{\pi^\star_{s_1}}(s_c) < \sum_{o \in O_{h_{s_1}}} y'_o$,



(a) Before

(b) After using optimal strategies in G_{s_3}, G_{s_4}, and G_{s_5}

Figure 3: The full game before and after using the optimal strategies in subgames.

which contradicts Lemma 1 for $\pi^\star_{s_1}$. Therefore, $V^{\pi^\star_{s_1 \to s_2}}(s_2) = \sum_{o \in O_{h_{s_2}}} y_o$ for $\mathbf{y}$ with $y_o > 0$ $(\forall o \in O_{h_{s_2}})$, and $\pi^\star_{s_1 \to s_2}$ is optimal in NEST by Theorem 5. □

A.9 Proof of Theorem 7

Proof. This theorem is implied by Theorems 3–6. □

B Illustration for Two Techniques

B.1 Mapping Between Subgames

As shown in Figure 3(a), in the full game, the defender's strategy space (S, A) includes all states and actions, while the adversary's strategy space O includes all escaping paths. Subgames of G_{s_1}, G_{s_2}, G_{s_3}, G_{s_4}, and G_{s_5} are part of the full game starting from states s_1, s_2, s_3, s_4, and s_5, respectively.

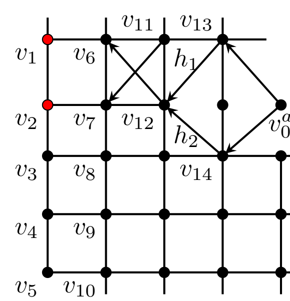


Figure 4: Scenario

To illustrate the characteristic of subgames, we analyze the scenario shown in the right figure. The adversary arrives at v_{12} through two different directions generating history h_1 $(= \langle v_0^a, v_{13}, v_{12} \rangle)$ and history h_2 $(= \langle v_0^a, v_{14}, v_{12} \rangle)$, respectively. Exit nodes are v_1 and v_2.

We consider three initial locations for two defender resources: $l_1 = (v_{11}, v_5)$, $l_2 = (v_3, v_4)$, and $l_3 = (v_3, v_5)$. By combining the observed adversary history with the locations of two defender resources, we consider five states $s_1 = ((v_{11}, v_5), h_1)$, $s_2 = ((v_{11}, v_5), h_2)$, $s_3 = ((v_3, v_4), h_1)$, $s_4 = ((v_3, v_4), h_2)$, and $s_5 = ((v_3, v_5), h_2)$, which are the initial states of five subgames: G_{s_1}, G_{s_2}, G_{s_3}, G_{s_4}, and G_{s_5}.

We consider time horizon $t_{\max} = 4$. Then, the set of paths generating h_1 is $O_{h_1} = \{o_1, o_2\}$ with $o_1 = h_1 \cup \langle v_6, v_1 \rangle = h_{13}$ and $o_2 = h_1 \cup \langle v_7, v_2 \rangle = h_{14}$, while $CH_{h_1} = \{h_{11}, h_{12}\}$ with $h_{11} = h_1 \cdot v_6$ and $h_{12} = h_1 \cdot v_7$, $CH_{h_{11}} = \{h_{13}\}$, and $CH_{h_{12}} = \{h_{14}\}$. The set of paths generating h_2 is $O_{h_2} = \{o_3, o_4\}$ with $o_3 = h_2 \cup \langle v_6, v_1 \rangle = h_{23}$ and $o_4 = h_2 \cup \langle v_7, v_2 \rangle = h_{24}$, while $CH_{h_2} = \{h_{21}, h_{22}\}$ with $h_{21} = h_2 \cdot v_6$ and $h_{22} =$

$h_2 \cdot v_7$, $CH_{h_{21}} = \{h_{23}\}$, and $CH_{h_{22}} = \{h_{24}\}$. Then, O_{h_1} and O_{h_2} are similar after v_{12} because $\eta(h_1) = \eta(h_2) = v_{12}$, $q(h_1, o_1) = \langle v_6, v_1 \rangle = q(h_2, o_3)$, and $q(h_1, o_2) = \langle v_7, v_2 \rangle = q(h_2, o_4)$.

O_{h_1} is the adversary's strategy space of G_{s_1} and G_{s_3}, and O_{h_2} is the adversary's strategy space of G_{s_2}, G_{s_4}, and G_{s_5}. Thus, G_{s_1} and G_{s_2} are similar because $(11,5) = l_{s_1} = l_{s_2}$, and O_{h_1} and O_{h_2} are similar after node $\eta(h_{s_1}) = v_{12}$ ($h_{s_1} = h_1$). After we compute the best response in G_{s_1} (i.e., π_{s_1} with $\pi_{s_1}(s_1) = (v_6, v_5)$ transiting to a capture state $((6,5), h_{11})$ such that $V^{\pi_{s_1}}(s_1) = 0.2$) against the adversary strategy $\mathbf{y}$ with $y_{o_1} = 0.2$ and $y_{o_2} = 0.1$, we can map it to G_{s_2} against the adversary strategy with $y_{o_3} = 0.2$ and $y_{o_4} = 0.1$ (Theorem 4). That is, the best response strategy for state s_1 is taking $l = (v_6, v_5)$, which is also the best response strategy for its similar state s_2 after the mapping.

Some computed strategies in subgames are the best response against any adversary strategy. The defender strategy in G_{s_3} against the adversary strategy with $y_{o_1} = 0.2$ and $y_{o_2} = 0.1$ is π_{s_3} such that $V^{\pi_{s_3}}(s_3) = 0.3$: $\pi_{s_3}(s_3) = (v_2, v_3)$ transiting to states $s_{31} = ((v_2, v_3), h_{11})$ and $s_{32} = ((v_2, v_3), h_{12})$, $\pi_{s_3}(s_{31}) = (v_1, v_2)$ transiting to capture state $s_{33} = ((v_1, v_2), h_{13})$, and $\pi_{s_3}(s_{32}) = (v_2, v_3)$ transiting to capture state $s_{34} = ((v_2, v_3), h_{14})$. Indeed, by π_{s_3} in G_{s_3}, given any adversary strategy, we have $V^{\pi_{s_3}}(s_3) = y_{o_1} + y_{o_2}$ (Lemma 1). That is, starting from s_3, the adversary will be captured certainly. This strategy is denoted as $\pi^\star_{s_3}$, which is optimal in NEST (Theorem 5). Then, we can define the payoff 1 for the defender in state s_3 as shown in Figure 3(b), i.e., the defender's expected utility in subgame G_{s_3} is $y_{o_1} + y_{o_2}$. Then, if this subgame is reached again, the defender's expected utility $y_{o_1} + y_{o_2}$ is returned immediately.

$\pi^\star_{s_3}$ in G_{s_3} can also be mapped to its semi-similar subgame G_{s_5}. In G_{s_3} and G_{s_5}, the adversary has the same move space and the first defender resource in G_{s_3} and the first defender resource in G_{s_5} has the same initial location (v_3) in their initial states s_3 and s_5, respectively. For strategy $\pi^\star_{s_3}$, the first resource will finally catch the adversary in state s_{33} or state s_{34} by moving to v_2 first, then moving to v_1 if she observes the adversary history h_{11} (i.e., in state s_{31}), or staying at v_2 if she observes the adversary history h_{12} (i.e., in state s_{32}). That is, v_3 is the key location in G_{s_3}. Therefore, given the mapping of $\pi^\star_{s_3}$ guaranteeing that the first defender resource in G_{s_3} and the first defender resource in G_{s_5} take the same action after observing the adversary's same move in both subgames, this mapping can guarantee optimality (Theorem 6). That is, the first defender resource in G_{s_5} will finally catch the adversary in state $s_{53} = ((v_1, v_5), h_{23})$ or state $s_{54} = ((v_2, v_5), h_{24})$ if she moves to v_2 first, then moves to v_1 if she observes the adversary history h_{21} (i.e., in state $s_{51} = ((v_2, v_5), h_{21})$), or stays at v_2 if she observes the adversary history h_{22} (i.e., in state $s_{52} = ((v_2, v_5), h_{22})$) when the first resource does not move. More specifically, the optimal strategy for state $s_{31} = ((v_2, v_3), h_{11})$ (note that $P_{\pi^\star_{s_{31}}} = 1$) is taking $l = (v_1, v_2)$, i.e., the first resource in s_{31}, locating at the key location v_2 that is reached from v_3 in s_1, moves to v_1; and then the mapping strategy for its semi-similar state $s_{51} = ((v_2, v_5), h_{21})$ is taking $l' = (v_1, v_5)$, i.e., the first resource in s_{51}, locating at the key location v_2 that is reached from v_3 in s_5, moves to v_1 while the second resource in s_{51} stays at the current node by Eq.(10). This $\pi^\star_{s_3 \to s_5}$ defined by Eq.(10) in G_{s_5} is optimal in the full game. By $\pi^\star_{s_3 \to s_5}$, as shown in Figure 3(b), we define the payoff 1 for the defender in state s_5, i.e., the defender's expected utility in subgame G_{s_5} is $y_{o_3} + y_{o_4}$. Similarly, G_{s_3} and G_{s_4} are semi-similar and $\pi^\star_{s_3 \to s_4}$ in G_{s_4} by Eq.(10) is optimal in NEST.

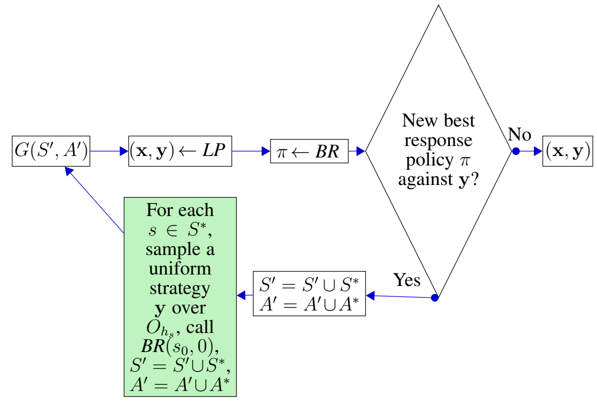


Figure 5: Adding multiple best response strategies at one iteration.

B.2 Adding Multiple Strategies at One Iteration

For using our technique to add multiple best response strategies at one iteration, the procedure of *IGRS* is shown in Figure 5. Specifically, before the convergence, we sample the uniform adversary strategies based on the states that are part of the best response and then call $BR(s_0, 0)$ at Line 4 in *IGRS* to compute the best response strategies against them. Here, S^* is the set of the states involved in the best response, A^* is the set of actions involved in the best response, and a uniform strategy over O_{h_s} means $y_o = 1/|O_{h_s}|$ $(\forall o \in O_{h_s})$. Especially, we sample a strategy with $y_o = 1/|O|$ $(\forall o \in O)$ at the initial step in *IGRS* (i.e., after s_0 is added to S') because we can compute the best response efficiently by using our effective pruning techniques including the mapping technique. Each strategy will be sampled at most once.

C The Reason to Consider Similar Subgames

We consider the scenario shown in Figure 4 again. For subgame G_{s_1} we know that π_{s_1} ($\pi_{s_1}(s_1) = (6,5)$ transiting to a capture state $s_{11} = ((6,5), h_{11})$ such that $V^{\pi_{s_1}}(s_1) = 0.2$) is the best response in G_{s_1} against the adversary strategy with $y_{o_1} = 0.2$ and $y_{o_2} = 0.1$. Here, only the first resource contributes to the interdiction, i.e., interdicting the adversary in s_{11} due to $\eta(h_{11}) = 6$. Now we consider subgame G_{s_6} with initial state $s_6 = ((11,8), h_2)$. Obviously, G_{s_1} and G_{s_6} are semi-similar on $\bar{l} = (11)$ that is also the key location of G_{s_1}. Let us define the mapping strategy of π_{s_1} as $\pi_{s_1 \to s_6}$ by Eq.(10). That is, $\pi_{s_1 \to s_6}(s_6) = (6,8)$. However, $\pi_{s_1 \to s_6}$ is not the best response in G_{s_6} against the adversary strategy with $y_{o_3} = 0.2$ and $y_{o_4} = 0.1$ because taking action $(6,7)$ in s_6 will result in $V(s_6) = 0.3$ (larger than

$V^{\pi_{s_1 \to s_6}}(s_6) = 0.2$). That is, given a best response (non-optimal) strategy in G_{s_1} with the key location set $l^*_{s_1}$, resources in a semi-similar subgame G_{s_6} of G_{s_1} who are not initially at nodes in $l^*_{s_1}$ may contribute to the interdiction, which results in a better strategy than its mapping strategy. Therefore, we cannot have the property similar to Theorem 4 between semi-similar subgames. However, Theorem 4 holds between similar subgames.