Optimal Sample Size for Adword Auctions

(Extended Abstract)

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ABSTRACT

Generalized Second Price (GSP) mechanism is widely used in ad auctions and reserve price is an effective tool for revenue maximization. The optimal reserve price depends on bidders' value distribution, which, however, is generally unknown to auctioneers. A common practice for auctioneers is to first collect information about the value distribution by a *sampling procedure* and then apply the reserve price estimated with the sampled bids to the following auctions. In order to maximize his/her total revenue over finite GSP ad auctions, it is important for the auctioneer to find a proper sample size to trade off between the cost of the sampling procedure and the optimality of the estimated reserve price. We first propose the revenue bounds during and after sampling. Then we formulate the problem of finding the optimal sample size that maximizes the auctioneer's worse-case total revenue as an constrained optimization problem, the solution of which is independent of the value distribution.

Keywords

Revenue Maximization, Ad Auction, Optimal Reserve Price, Generalized Second Price

1. INTRODUCTION

Ad auctions have become a major monetization channel for Internet economy, including sponsored search auctions [9] and realtime bidding (RTB) [2]. In sponsored search, when a user issues a query[1] to a search engine, in addition to a list of relevant webpages, a selective set of ads related to the query will also be shown to the user. A position auction is used to determine which ads to show and how much to charge the corresponding advertisers. In RTB for display advertising, when a user visits a publisher's website, an ad impression with related information will be sent to the advertisers (or ad networks) through an ad exchange. Then the bids from the advertisers are collected and an auction is used to determine which ad to show and how much to charge the advertiser. In these applications, GSP is the most popularly used auction mechanism and has attracted a lot of research attention in recent years [4, 11].

When bidders' value distribution is known, the auctioneer can compute the optimal reserve price based on the Myerson's theory [7]. However, in the real world, bidders' valuations are private information and are invisible to the auctioneer. In practice, auctioneers usually need to collect information about the value distribution using the auction mechanism with a heuristically set reserve price. With the sampled data, the auctioneer can infer the optimal reserve price and use it in the future auctions. Existing works have neglected the revenue loss during the sampling period. Obviously, more rounds of sampling will lead to a more accurate estimation of the optimal reserve price. However, the sampling process usually cannot achieve the optimal revenue by itself, i.e., there is a "cost" of sampling. On the other hand, fewer samples will make lower revenue loss during the sampling period but lead to worse estimation of the optimal reserve price. In the real world, the auctioneer usually cares for his/her revenue in finite auctions, e.g., one month, one quarter, or one year. Hence, the auctioneer need to determine how many rounds should be used for sampling in order to maximize the overall revenue.

In this paper, we study the trade-off between the cost of sampling and the optimality of the estimated reserve price for revenue maximization, in the context of finite-horizon GSP auctions. Specifically, we first consider the revenue loss for two phases: the sampling period and the period after sampling. Then we formulate the trade-off problem as a constrained optimization problem, which aims to maximize the auctioneer's worse-case total revenue. We show that the optimal sample size is independent of the value distribution.

2. GSP MECHANISM AND OPTIMAL RESERVE PRICE

There are N bidders competing for K ad slots ($K < N$). Each ad slot has a corresponding click-through-rate (CTR). Let v_i denote the value of bidder i's ad and $v = (v_1, v_2, \ldots, v_N)$ represent the value profile. Bidders' values are usually assumed to be independent and identically distributed (i.i.d.) with cumulative distribution function F (probability density function f), which is common knowledge to bidders but unknown to the auctioneer [3, 8].

In the theory of revenue maximization, it is usually assumed that bidders were playing the lowest-revenue envy-free (LREF) equilibrium for GSP [4, 5, 8, 11], which is an efficient Nash Equilibrium. Then according to [6, 7], the auctioneer's expected revenue with respect to the reserve

[1]For simplicity, we only consider the exact match between queries and keywords.

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price r can be written as

$$R(r) = E_v\{\sum_{i=1}^{N} \psi(v_i) x_i^r(v)\},$$

where $\psi(v_i) = v_i - \frac{1-F(v_i)}{f(v_i)}$ is called the *virtual value function* and $x_i^r(v)$ is the CTR bidder i can receive given v and r. The optimal reserve price r^* satisfies that [7]

$$\psi(r^*) = 0.$$

3. PROBLEM FORMULATION

We realistically consider that bidders' value distribution is unknown to the auctioneer who aims to maximize his/her overall revenue for M rounds . The first τ rounds are used for sampling and the reserve price is set as zero in this period in order to observe complete (i.e., non-truncated) value distribution. Note that the auctioneer can only observe bids of the advertisers but not their values. We assume that bidders were playing the LREF equilibrium and compute the values from bids according to the method in [11]. Then we make an estimation of the optimal reserve price, represented as $\overline{r}$, with the sampled data and set it for the remaining $M - \tau$ rounds. The total revenue the auctioneer will get is

$$\tau \cdot R(0) + (M - \tau) \cdot R(\overline{r}).$$

The revenue loss is thus

$$M \cdot R(r^*) - (\tau \cdot R(0) + (M - \tau) \cdot R(\overline{r})),$$

which is equal to

$$M \cdot R(r^*)(1 - (\frac{\tau}{M} \cdot \frac{R(0)}{R(r^*)} + (1 - \frac{\tau}{M}) \cdot \frac{R(\overline{r})}{R(r^*)})).$$

Then minimizing the total loss means that

$$\max_{\tau} \ \frac{\tau}{M} \cdot \frac{R(0)}{R(r^*)} + (1 - \frac{\tau}{M}) \cdot \frac{R(\overline{r})}{R(r^*)}, \quad (1)$$

$$\text{s.t. } \tau \in \{1, 2, \ldots, M\}.$$

4. OPTIMAL SAMPLE SIZE

Further analysis shows that both $\frac{R(0)}{R(r^*)}$ and $\frac{R(\overline{r})}{R(r^*)}$ are related to the density function $f(\cdot)$. As a result, the solution of the problem is a function of $f(\cdot)$. Since the priori $f(\cdot)$ is unknown to the auctioneer, we need to find a solution that is independent of $f(\cdot)$ and has good performance on optimality at the same time. Specifically, inspired by [1] and [3] which proposed the distribution-independent lower bounds of $\frac{R(0)}{R(r^*)}$ and $\frac{R(\overline{r})}{R(r^*)}$ respectively for a simple mechanism where winners pay the same minimal price, we can derive the corresponding bounds for the GSP mechanism. We use l_1 to denote the lower bound of $\frac{R(0)}{R(r^*)}$. Since more samples will lead to a better estimation of the optimal reserve price, the lower bound $l_2(\tau)$ of $\frac{R(\overline{r})}{R(r^*)}$ is a function with respect to τ. Based on this result, we can re-formulate the optimization problem with l_1 and $l_2(\tau)$ as follows:

$$\max_{\tau} \ \widehat{\mathcal{R}}(\tau) = \frac{\tau}{M} l_1 + (1 - \frac{\tau}{M}) l_2(\tau), \quad (2)$$

$$\text{s.t. } \tau \in \{1, 2, \ldots, M\}. \quad (3)$$

Since both l_1 and $l_2(\tau)$ do not relay on $f(\cdot)$, the optimal sample size τ^* of the problem defined in Eqs. (2) and (3), which aims to maximize the auctioneer's worse-case total revenue, is robust against any distribution.

5. EXTENSIONS AND FUTURE WORK

This paper is based on the unweighted GSP, but the model can be extended to the weighted GSP [10]. First note that the LREF equilibrium also works in weighted GSP. Then we can follow [10] to assume that $s_i = v_i e_i$ is drawn from an i.i.d. regular distribution F, where e_i is bidder i's ad quality. The optimal reserve price for bidder i is modified as s^*/e_i, where s^* satisfies $\psi(s^*) = 0$. All the equations still hold if we replace v_i with $v_i e_i$. Thus the model can be extended to the weighted GSP.

The model can also be applied to VCG auctions. It is known that bidders' payments are the same for the (dominant) truth-telling equilibrium with VCG mechanism and the LREF equilibrium with GSP mechanism. Hence, if we assume bidders in VCG auctions to have i.i.d. values and bid truthfully, the model can be directly extended to VCG.

In future work, we will derive the tight bounds for $\frac{R(0)}{R(r^*)}$ and $\frac{R(\overline{r})}{R(r^*)}$ and propose efficient algorithms to solve the optimization problem.

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