Game-Theoretic Considerations for Optimizing Taxi System Efficiency

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Combining game-theoretic solution concepts with existing models of taxi markets helps model drivers' strategy-making process as a game, transforming the problem of optimizing taxi system efficiency to finding a market policy that leads to the desired equilibrium.

There are more than 60,000 licensed taxis in Beijing serving nearly 20 million citizens. However, despite rising customer demand during peak times, most taxi drivers act counterintuitively, intentionally avoiding working during those times. Consequently, passengers spend an unreasonably long amount of time trying to get a ride. Some even switch to unlicensed cabs, which typically charge a higher rate and create big threats to road safety. It turns out that the improper distance-based pricing scheme is the main cause of this situation, also called the *peak-time dilemma*. Low travel speed during peak time as a result of heavy traffic causes low or even negative revenue generation for taxi drivers, leaving them to pursue the only option that makes them money: not working during peak time.

We propose a solution: increasing fare price during the peak time to incentivize taxi drivers to work, specifically, with the use of a dynamic time-dependent fare structure that differentiates peak and non-peak times. The key is to calculate the best fare price that maximizes taxi system efficiency. We call this the TAxi System Efficiency Optimization (TASEO) problem.

Artificial Intelligence and Game Theory

Research about the economics of taxi markets dates back to 1969, when Daniel Orr pointed out the inadequacy of applying traditional cost-demand theory.[1] In 1972, George Douglas introduced an aggregate demand and supply model that assumes customer demand for taxi services depends on expected monetary and time costs, and expected customer waiting time depends on total vacant taxi hours.[2] Based on this model, Hai Yang and colleagues studied congestion externalities and time variance in service intensity.[3,4] Other studies investigated taxi fare pricing in various places, such as New York City.[5] Kim

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Young-Joo and Hark Hwang studied an incremental discount policy on taxi fare with the objective of maximizing average profit.[6] Hai Yang and colleagues examined a nonlinear fare structure and showed its advantages over existing options in Hong Kong.[7] However, none of these works have investigated the effect of taxi drivers' strategic behavior. To solve our problem, we need to know how taxi drivers will react to fare price changes.

Fortunately, the artificial intelligence field has provided many techniques for studying human behavior, and there's a growing trend of applying AI techniques to problems in the transportation sector, such as traffic control, intersection management, and transportation system simulation.[8–10] Most recently, we've seen the successful application of game theory in the AI community, such as in security resource allocation.[11] Game theory provides rich mathematic foundations and concepts for studying conflicts and cooperation between intelligent, rational decision makers. Existing applications have demonstrated the potential of game theory in addressing comprehensive real-world problems, motivating us to seek solutions along this direction.

Taxi System Efficiency Optimization

To improve system efficiency by adjusting fare price, the primary task is to know how system efficiency is affected by fare price. Results from existing research[3,12] suggest that the taxi market is determined by two key factors: fare price and taxi supply (that is, the number of working taxis). Due to the taxi system's decentralized management, taxi supply is determined by the drivers' operation strategy and isn't directly controllable by the market regulator. Thus, we have an indirect dependency: for a given fare price, drivers react with a best strategy to keep their profits maximized. As Figure 1 shows, we can combine a game-theoretic behavior model and an existing taxi market model.

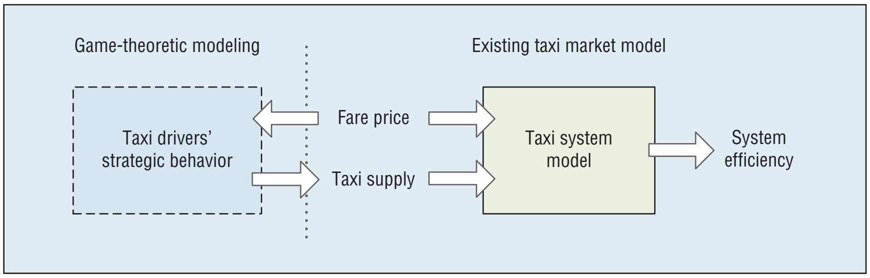

Figure 1. Game-theoretic modeling based on an existing taxi market model.

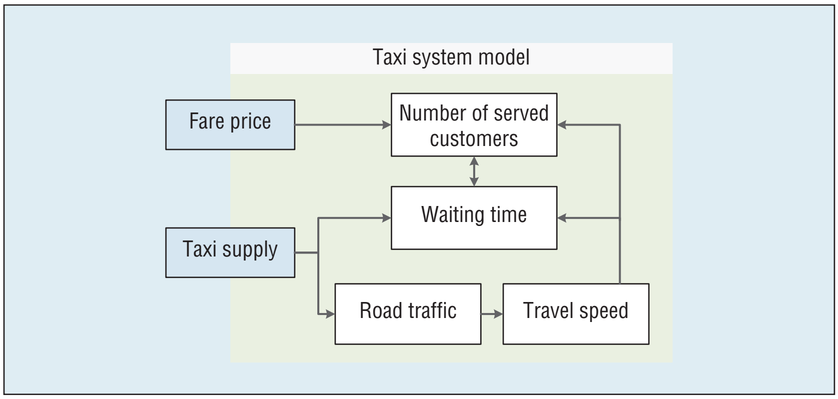

Figure 2. Interdependencies of factors in a taxi market.

The Existing Taxi Market Model

A taxi market is a dynamic time-varying system. To model its variances, we discretize the optimization horizon (such as a whole day) into a set of n equal-length periods, such that when each period's duration is sufficiently short, the market can be treated as uniform in each period. In a single period i, the number of passengers served by the whole taxi system is determined by the average monetary and time cost of a trip, that is,

$$D^i\left(F^i, L^i, W^i\right) = D^i e^{-\beta\left(\frac{F^i}{\gamma} + \varphi_1 L^i + \varphi_2 W^i\right)}, \tag{1}$$

where e is the base of the natural logarithm; F^i is the average fare price, L^i is the average travel time, and W^i is the average customer waiting time; $\beta > 0$ is a sensitivity parameter; φ_1 and φ_2 are parameters used for converting time costs into monetary costs; γ is the average number of passengers per ride; and D^i is the number of potential customers, an ideal number achieved when the total cost is zero. The waiting time W^i in turn depends on D^i as

$$W^i\left(D^i, L^i, p^i\right) = \frac{\omega}{p^i \cdot N_T - D^i L^i / (\gamma \cdot \tau)}, \tag{2}$$

where $\omega > 0$ is a parameter depending on the density of taxi stands; p^i is the proportion of working taxis (PoW); N_T is the total number of taxis; and $p^i \times N_T - D^i L^i/(\gamma \times \tau)$ represents vacant taxis in period i. It's proven that when F^i, L^i, and p^i are fixed, D^i and W^i are uniquely determined by Equations 1 and 2.[13] Therefore, D^i and W^i are in fact implicit functions of F^i, L^i, and p^i. We denote them as $D^i = D^i(F^i, L^i, p^i)$ and $W^i = W^i(F^i, L^i, p^i)$.

Given the average trip distance d^i, travel time can be represented by travel speed V^i as $L^i = d^i/V^i$. Travel speed in a road network can be approximated by a linear function of the number of on-road vehicles,[14] which is linear to PoW p^i as we assume that the number N_v^i of non-taxi vehicles in the network is a period-specified constant. Thus, V^i is a linear function of p^i, that is,

$$V^i\left(p^i\right) = \mu\left(p^i \cdot N_T + N_v^i\right) + \lambda,$$

where μ and λ are parameters depending on road conditions. We write L^i, D^i, and W^i as $L^i(p^i)$, $D^i(F^i, p^i)$, and $W^i(F^i, p^i)$, respectively.

Next, we adopt a distance-based fare structure

$$F^i = f_0 + f^i \cdot (d^i - d_0),$$

where f_0 is the initial charge and d_0 is the distance covered by f_0; f^i is the charge rate for period i, that is, the per-unit distance charge. We optimize the fare structure by adjusting the charge rate f^i and thus treat F^i as a function $F^i(f^i)$. Accordingly, all market factors, particularly the number D^i of served customers, now depend on f^i and p^i, that is, $D^i = D^i(f^i, p^i)$. For ease of description, we denote a market factor over all periods as a column vector, where each component corresponds to a period. For example, we denote charge rate over all periods as $\mathbf{f} = \langle f^i \rangle$. Figure 2 summarizes the interdependencies of the factors.

The Taxi Driver's Strategy

Taxi drivers decide which time periods to work by considering the potential profits. In other words, a taxi driver's strategy is a schedule specifying several time periods (in a day) in which to work. We allow randomization into the strategy, letting the driver play a *mixed strategy*, which is an assignment of probabilities to each schedule. In this sense, each schedule is called a *pure strategy*. We denote a pure strategy as a vector $\mathbf{s} \in \{0,1\}^n$, where $s^i = 1$ (respectively, $s^i = 0$) indicates working (respectively, not working) in period i. Let the pure strategy set, that is, the set of pure strategies feasible for the taxi driver to choose, be S. We then denote a mixed strategy as $\mathbf{x} \in \mathbb{R}^{|S|}$ such that $\mathbf{x} \geq \mathbf{0}$ and $\mathbf{1}^\top\mathbf{x} = 1$. Considering the capability of taxi drivers in real situations, we impose the following constraints on each schedule in the pure strategy set:

- **Constraint 1 (C1):** The taxi driver doesn't work for more than n_w periods.
- **Constraint 2 (C2):** The taxi driver doesn't work *continuously* for more than n_c periods.

Namely, $S = \{\mathbf{s} \in \{0,1\}^n \mid \mathbf{s}$ satisfies C1 and C2$\}$.

Our framework applies to different models of taxi driver behavior. We illustrate with two models: *symmetric strategy*, in which every taxi driver assumes that all other drivers adopt the same strategy as he or she does, given that all taxis are identical (same car type, operation cost, and charging scheme), and *egoistic strategy*, in which the classic solution concept Nash equilibrium (NE) is adopted and we assume a driver deviates from his or her strategy (and the others don't) unless he or she can't benefit from doing so.

Symmetric strategy. The assumption is in accordance with the focal point theory,[15] which states that people tend to use solutions depending on simple social beliefs (others drivers play the same strategy), especially in the absence of communication. Given the symmetric strategy $\mathbf{x}$, PoW is then given by

$$\mathbf{p}(\mathbf{x}) = \sum\nolimits_{\mathbf{s}\in\mathcal{S}} x_{\mathbf{s}} \cdot \mathbf{s}, \tag{3}$$

which is the same as the probability of taxi drivers working in each period and can be viewed as a compact representation of taxi driver strategy. Taxi drivers are profit-driven, and they always choose the best strategy to maximize their utility, that is,

$$\mathbf{x}^* \in \operatorname{argmax}_{\mathbf{x}:\mathbf{x}\geq 0,\ \mathbf{1}^\top\mathbf{x}=1} U\left(\mathbf{f}, \mathbf{p}(\mathbf{x})\right). \tag{4}$$

Before we define the utility function $U(\mathbf{f},\mathbf{p})$, note that the above optimization, although defined on a single strategy, captures the behavior of all players under the assumption that all drivers are the same (as we can see, the profit for working in a period depends also on how many other taxis are working in that same period). Under this condition, each player solves the same optimization problem in which the player's utility depends on the strategies of other players using the same mixed strategy.

$U(\mathbf{f}, \mathbf{p})$ is defined as the sum of utilities in all periods, that is,

$$U(\mathbf{f},\mathbf{p}) = \sum\nolimits_{i=1}^{n} p^i \cdot G^i\left(f^i, p^i\right),$$

where $G^i(f^i, p^i)$ is the profit of working in period i, defined as

$$G^i\left(f^i, p^i\right) = \frac{D^i\left(p^i\right)}{\gamma \cdot N_T \cdot p^i} \cdot F^i\left(f^i\right) - c^i \cdot \tau, \tag{5}$$

where $D^i/\gamma \cdot N_T$ represents the average number of trips each taxi serves, and c_g is the cost in gasoline consumption per unit time.

It follows that the fare price determines taxi driver strategy via the optimization in Equation 4, and strategy in turn determines PoW via Equation 3. $U(\mathbf{f}, \mathbf{p})$ is strictly concave with respect to $\mathbf{p}$,[16] so there's only one $\mathbf{p}$ maximizing U, given that the feasible set of $\mathbf{p}$ is convex.[17] This means that even if there's more than one solution to Equation 4, the solutions must all yield the same PoW, and a one-to-one correspondence from $\mathbf{f}$ to $\mathbf{p}$ is guaranteed.

Egoistic strategy. To analyze taxi driver behavior under NE, we let the strategy profile be $\langle \mathbf{x}_1,\ldots,\mathbf{x}_{N_T} \rangle$. PoW is then given by

$$p^i\left(\mathbf{x}_1,\ldots,\mathbf{x}_{N_T}\right)=\frac{1}{N_T}\sum\nolimits_{j=1}^{N_T}\sum\nolimits_{\mathbf{s}\in\mathcal{S}}x_{j,\mathbf{s}}\cdot s^i, \tag{6}$$

and the utility of each taxi is

$$U_j\left(\mathbf{f},\mathbf{x}_1,\ldots,\mathbf{x}_{N_T}\right)=\sum\nolimits_{i=1}^{n}p^i\left(\mathbf{x}_j\right)\cdot G^i\left(f^i,p^i\left(\mathbf{x}_1,\ldots,\mathbf{x}_{N_T}\right)\right). \tag{7}$$

Under NE, no player can benefit from changing his or her strategy, assuming that the other players stick to theirs. Namely, $\langle \mathbf{x}_1,\ldots,\mathbf{x}_{N_T} \rangle$ is said to be in NE if, for every taxi j,

$$U_j\left(\mathbf{f},\mathbf{x}_1,\ldots,\mathbf{x}_{N_T}\right)\geq U_j\left(\mathbf{f},\mathbf{x}_1,\ldots,\mathbf{x}_{j-1},\mathbf{x}',\mathbf{x}_{j+1},\ldots,\mathbf{x}_{N_T}\right),\quad \forall\mathbf{x}'\geq 0\wedge\mathbf{1}^{\top}\mathbf{x}'=1. \tag{8}$$

Because there are a large number of taxis, the game is *non-atomic*, meaning that the effect of a single taxi is negligibly small. When a single taxi deviates, it doesn't change the overall PoW, that is,

$$\mathbf{p}\left(\mathbf{f},\mathbf{x}_1,\ldots,\mathbf{x}_{N_T}\right)=\mathbf{p}\left(\mathbf{f},\mathbf{x}_1,\ldots,\mathbf{x}_{j-1},\mathbf{x}',\mathbf{x}_{j+1},\ldots,\mathbf{x}_{N_T}\right),\quad \forall\mathbf{x}'\geq 0\wedge\mathbf{1}^{\top}\mathbf{x}'=1.$$

The criterion in Equation 8 is then rewritten as

$$\sum\nolimits_{i=1}^{n}p^i\left(\mathbf{x}_j\right)\cdot G^i\left(f^i,p^i\left(\mathbf{x}_1,\ldots,\mathbf{x}_{N_T}\right)\right)\geq \sum\nolimits_{i=1}^{n}p^i\left(\mathbf{x}'\right)\cdot G^i\left(f^i,p^i\left(\mathbf{x}_1,\ldots,\mathbf{x}_{N_T}\right)\right),\ \forall\mathbf{x}'. \tag{9}$$

In addition, as the utility function (Equation 7) is identical to all taxis, the game is symmetric. For symmetric games, there always exists a symmetric equilibrium under which all players play the same strategy.[18] In this case, the strategy profile can be concisely represented with a universal strategy, say, $\mathbf{x}$. We again write PoW as $\mathbf{p}(\mathbf{x})=\Sigma_{\mathbf{s}\in\mathcal{S}}x_{\mathbf{s}}\cdot\mathbf{s}$ as in Equation 3, and the equilibrium criterion in Equation 9 becomes

$$\sum\nolimits_{i=1}^{n}p^i\left(\mathbf{x}_j\right)\cdot G^i\left(f^i,p^i\left(\mathbf{x}\right)\right)\geq \sum\nolimits_{i=1}^{n}p^i\left(\mathbf{x}'\right)\cdot G^i\left(f^i,p^i\left(\mathbf{x}\right)\right),\ \forall\mathbf{x}'. \tag{10}$$

To analyze the equilibrium strategy, we construct the following function:

$$\mathcal{G}(\mathbf{f},\mathbf{p})=\sum\nolimits_{i=1}^{n}\int_0^{p^i}G^i\left(f^i,p^i\right)\mathrm{d}p. \tag{11}$$

Let $\mathbf{x}^*$ be an equilibrium strategy. We claim that the induced PoW $\mathbf{p}^*=\mathbf{p}(\mathbf{x}^*)$ maximizes $\mathcal{G}(\mathbf{f},\mathbf{p})$ for a given $\mathbf{f}$, which implies that, similar to Equation 4 $\mathbf{x}^*$ is captured by the following optimization:

$$x^*\in\operatorname{argmax}_{\mathbf{x}:\mathbf{x}\geq 0,\ 1^{\top}\mathbf{x}=1}\mathcal{G}\left(\mathbf{f},\mathbf{p}\left(\mathbf{x}\right)\right). \tag{12}$$

To see this, we observe that $\max_{\mathbf{p}\in\{\mathbf{p}(\mathbf{x})\mid \mathbf{x}\geq 0,\ \mathbf{1}^{\top}\mathbf{x}=1\}}\mathcal{G}(\mathbf{f},\mathbf{p})$ is a convex optimization: the feasible space of $\mathbf{p}$ is clearly convex, and $\mathcal{G}(\mathbf{f},\mathbf{p})$ is concave to $\mathbf{p}$ (which we will show later). According to the optimality criterion of convex optimization,[17] $\mathbf{p}^*$ is optimal if and only if $\nabla\mathcal{G}(\mathbf{p}^*)^{\top}(\mathbf{p}^*-\mathbf{p}')\geq 0$ for all feasible $\mathbf{p}'$, which is exactly the same as the equilibrium criterion in Equation 10.

To see the concaveness of $\mathcal{G}(\mathbf{f},\mathbf{p})$, we note that $D^i(f^i,p^i)$ is strictly concave to p^i, as pointed out elsewhere[16] (as suggested by the strict concaveness of U^i). We verify the concaveness through checking the Hessian matrix of $\mathcal{G}(\mathbf{f},\mathbf{p})$:

$$\frac{\partial^2\mathcal{G}(\mathbf{f},\mathbf{p})}{\partial p^i\,\partial p^j}=\begin{cases}0, & \text{if } i\neq j\\ \dfrac{F^i\left(f^i\right)}{\gamma\cdot N_T}\cdot\dfrac{1}{(p^i)^2}\cdot\left(p^i\cdot\dfrac{\partial D^i\left(f^i,p^i\right)}{\partial p^i}-D^i\left(f^i,p^i\right)\right), & \text{if } i=j\end{cases}.$$

We have $p^i\cdot\left(\partial D^i\left(f^i,p^i\right)/\partial p^i\right)-D^i\left(f^i,p^i\right)<0$ because

$$\lim_{p^i\to 0}\left(p^i\cdot\frac{\partial D^i\left(f^i,p^i\right)}{\partial p^i}-D^i\left(f^i,p^i\right)\right)=0$$

and

$$\frac{\partial\left(p^i\cdot\dfrac{\partial D^i\left(f^i,p^i\right)}{\partial p^i}-D^i\left(f^i,p^i\right)\right)}{\partial p^i}=p^i\cdot\frac{\partial^2D^i\left(f^i,p^i\right)}{\left(\partial p^i\right)^2},$$

where the last inequality holds because $\left(\partial^2D^i\left(f^i,p^i\right)/\left(\partial p^i\right)^2\right)<0$ due to the strict concaveness of D^i. Therefore, the Hessian matrix is positive definite. This also implies that there's only one $\mathbf{p}$ maximizing $\mathcal{G}(\mathbf{f},\mathbf{p})$, which, similar to that for $U(\mathbf{f},\mathbf{p})$, guarantees a one-to-one correspondence from $\mathbf{f}$ to $\mathbf{p}$.

Solution Algorithm

We use the total number of served customers $D(\mathbf{f},\mathbf{p})=\Sigma_iD(f^i,p^i)$ to measure system efficiency and formulate a TASEO as the following bilevel optimization program:

$$\max_{\mathbf{f},\ \mathbf{x}^*}D\left(\mathbf{f},\mathbf{p}\left(\mathbf{x}^*\right)\right) \tag{13}$$

$$\text{s.t.}\quad \mathbf{x}^*\in\operatorname{argmax}_{\mathbf{x}:\mathbf{x}\geq 0,\ 1^{\top}\mathbf{x}=1}U\left(\mathbf{f},\mathbf{p}\left(\mathbf{x}\right)\right), \tag{14}$$

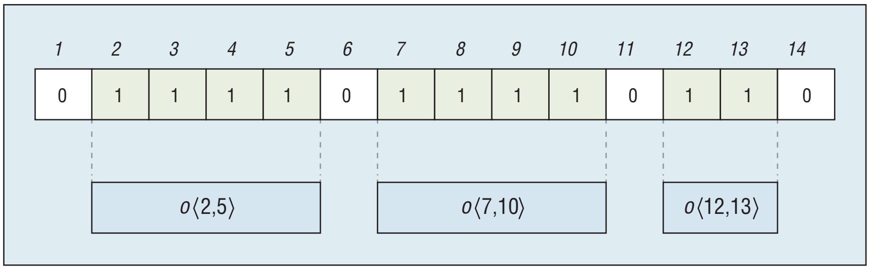

Figure 3. Atom Schedule Method (ASM): representing a schedule with a set of atom schedules.

where for the egoistic case, we replace $U(\mathbf{f},\mathbf{p})$ with $\mathcal{G}(\mathbf{f},\mathbf{p})$ in Equation 14. The model can also handle other measures of system efficiency with the same form of optimization program as long as the optimization objective is a function of **f** and **p**.

To solve this bilevel optimization problem, we can discretize the continuous fare price space into a small set of candidate prices, such as {¥1.00, ¥1.20, …, ¥5.00}, and solve the lower-level program (Equation 14) under each of the candidate prices to find the optimal fare price. Thus the problem reduces to the lower-level program. Unfortunately, the lower-level program suffers from a scalability issue as a result of the exponential growth of the drivers' pure strategy set. For example, when $n = 18$, the pure strategy set contains more than 1.7×10^5 strategies. How to design efficient algorithms to address the scalability issue is therefore the key to our approach.

One way to compactly represent strategy is to break drivers' working schedules into sections with fewer patterns. Each section is a set of consecutive periods during which taxi drivers work continuously (see Figure 3). We call these working sections *atom schedules* (or *atom* for short), and we refer to the approach as the Atom Schedule Method (ASM).[16] Obviously, given an n-period model, we only need $O(n^2)$ atom schedules to represent any original schedule (that is, by specifying a starting and an ending period). This is in contrast to the exponential size of the original strategy space.

We denote an atom as a tuple $o\langle j,k\rangle$, where j and k are the indices of the starting and ending periods, respectively. To reformulate the lower level on atom schedules, we assign a weight w_o to each atom o to denote the percentage of taxis using this atom. It follows that PoW can be computed as

$$p^i = \sum\nolimits_{o\in\mathcal{O}} w_o \cdot \delta(o,i), \quad \forall i = 1,\ldots,n,$$

where $\delta(o, i)$ encodes whether atom o covers period i or not, that is, $\delta(o, \langle j,k\rangle, i) = 1$ if $j \le i \le k$ and $\delta(o, \langle j,k\rangle, i) = 0$ otherwise. O is the set of all atoms we need. Clearly, **p** is now defined as a function $\mathbf{p} = \mathbf{p}(\mathbf{w})$, so that the lower-level program can be reformulated as a compact one that takes **w** (instead of **x**) as a variable. Specially, when **C2** is enforced on S, we only need atoms of at most n_c periods, so that

$$O \subseteq \{o\langle j,k\rangle \mid 1 \le j \le k \le n,\ 0 \le k - j < n_c\},$$

and there are less than $n_c \times n$ atoms in O and as many variables in the compact formulation. The new formulation is structured as

$$\max_{\mathbf{f},\,\mathbf{w}^*} D\left(\mathbf{f},\mathbf{p}\left(\mathbf{w}^*\right)\right)$$

s.t. $\mathbf{w}^* \in \operatorname{argmax}_{\mathbf{w}\in\mathcal{W}} U(\mathbf{f}, \mathbf{p}(\mathbf{w}))$,

where

$$\mathcal{W} = \left\{ \mathbf{w} \in \mathbb{R}^{|\mathcal{O}|} \;\middle|\; \begin{array}{c} 0 \le w_o \le 1, \quad \forall o \in \mathcal{O} \\ p^i(\mathbf{w}) + q^i(\mathbf{w}) \le 1, \quad \forall i = 1,\ldots,n \\ \sum\nolimits_{i=1}^{n} p^i(\mathbf{w}) \le n_w \end{array} \right\},$$

and, similar to PoW, $q^i(\mathbf{w})$ is the percentage of taxis switching from working to resting at period $i - 1$, that is, $q^i(\mathbf{w}) = \Sigma_{o\in\mathcal{O}} w_o \cdot \delta'(o,i)$, where $\delta'(o, \langle j,k\rangle, i) = 1$ if $k = i - 1$ (if o ends at period $i - 1$) and $\delta'(o, \langle j,k\rangle, i) = 0$ otherwise. It can be proven that $\mathcal{W}$ ensures equivalence of the compact formulation to the original formulation.[16] Without $\mathcal{W}$, the obtained solution might not find a feasible mixed strategy whose compact representation corresponds to it.

Experimental Evaluations

We conducted empirical experiments with real data from the Beijing Transportation Research Centre. We computed the optimal fare price for the real taxi market, examined the effects of scheduling constraints, and evaluated ASM performance. Taxi driver behavior is modeled with the assumption of symmetric strategies.

Optimal Fare Price

We examined prices from ¥1.00 to ¥5.00 with an interval of 0.20. For each price, we computed the drivers' optimal operation strategy and checked system efficiency with the obtained driver strategy against the existing taxi market model (the flow in Figure 1). Figure 4a shows system efficiency variance. As suggested by the blue curve, system efficiency peaks at ¥2.60 when constraints C1 and C2 are considered.

Effects of scheduling constraints. We also evaluated how C1 and C2 affect driver behavior (and consequently, system efficiency) by removing them from the model. As shown by the red curve in Figure 4a, system efficiency continues to increase when constraints are ignored, leading to an imprecise optimal fare of ¥5.00 (or even higher). The additional increase in system efficiency improvement is actually unreachable due to impractical overworking of the drivers. This can be seen in Figure 4b, where PoW variances show that taxi drivers are reluctant to

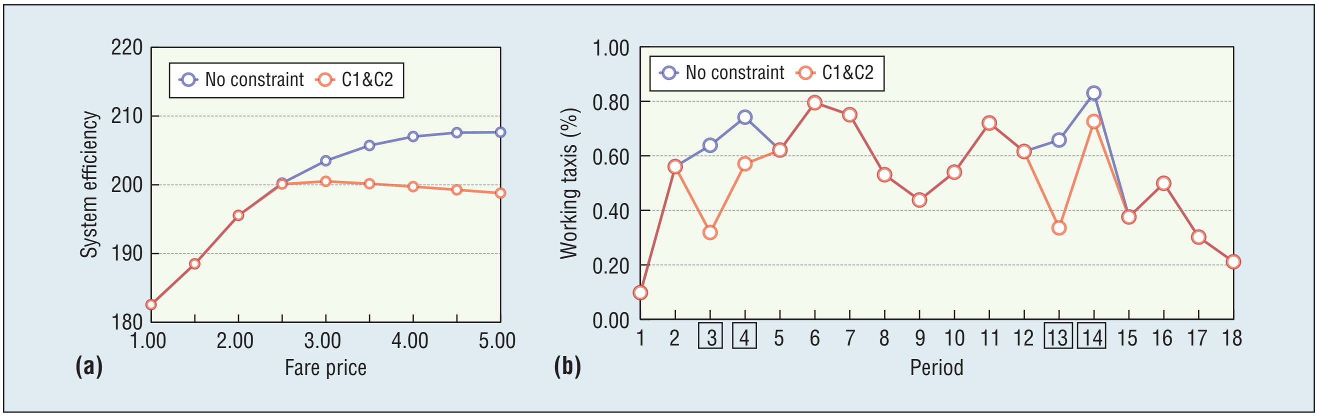

Figure 4. Effects of scheduling constraints: (a) system efficiency and (b) PoW variance (periods with indicies in boxes are peak periods).

work during peak times because of scheduling constraints. In this case, the system doesn't benefit from higher fare prices because when a higher price fails to improve service quality by incentivizing more taxis to work, it only leads to a decrease in the number of customers and undermines efficiency.

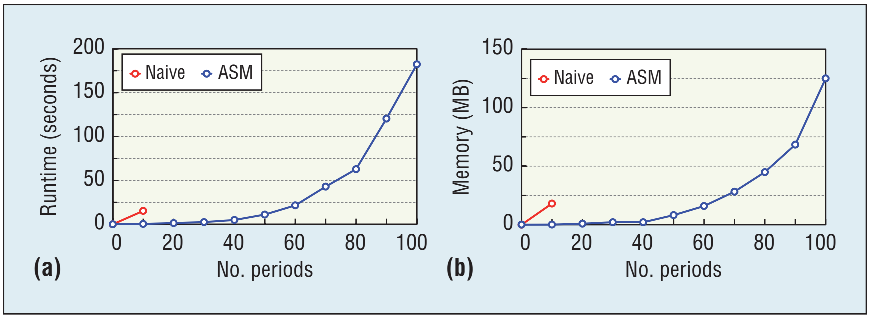

Figure 5. ASM performance: (a) runtime and (b) memory use.

ASM performance. Finally, we evaluated our solution algorithm ASM's scalability. First, we discretized the time horizon into different numbers of periods to scale up the problem. Figures 5a and 5b depict ASM's runtime and memory use for problems of different sizes compared to the naive formulation (Equations 13 and 14). Whereas the naive formulation runs out of memory at 15 periods, ASM can handle problems of up to 100 periods very easily. When data is available, the capability of scaling up to more periods allows us to use a more fine-grained model to achieve higher accuracy. It also lets us consider longer market cycles (for example, a week, considering the difference in customer demand on weekends and weekdays).

While the current model and algorithm are capable of handling TASEOs with specific settings, They're still inadequate for more extensive and complex real-world scenarios. We point out the following directions for future research:

- *Algorithms with better scalability.* In practice, customer demand and road condition might not be the same on different days. The taxi market's cycle is more likely to be a week rather than a day, so to cover a whole week with the same granularity, more periods are needed. Similarly, when the model needs to be more fine-grained to achieve higher accuracy, shorter periods, such as half an hour or even 10 minutes, are required, and the number of periods increases accordingly. Although a more scalable algorithm based on conversion of representations of polytopes is presented elsewhere,[19] the algorithm might not scale well when we consider other realistic constraints and uncertainties.
- *Heterogeneous taxis and taxi drivers.* Our current model is established on the assumption that all taxis and drivers are homogenous. Although this is generally true in taxi systems in many cities, some exceptions require special consideration. For example, a taxi cab can be operated by more than one driver in some cities to maximize the usage. In this case, taxis can run for a longer time, and constraints C1 and C2 might actually be violated. Differences in car types and taxi companies are also worthy of our consideration.
- *Uncertainties in human behaviors.* Uncertainties have always been issues in modeling intelligent behaviors. In a taxi system, drivers face uncertainties when implementing their strategies. They can't decide when the next customer will show up or the time needed to serve the next customer. In practice, taxi drivers would choose their actions according to the market conditions they face. How to model their behavior under uncertainties is another question to focus on.
- *Impact of app-based services.* The fast development of smartphones in

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recent years has made available vast new apps and services. Ride-sharing apps and customer-to-driver taxi-booking apps, such as Uber and Didi Dache, which connect taxi drivers with customers looking for a ride, are reshaping the traditional taxi market. Notably, these services are more than a simple dispatching system—they also make available negotiation between customers and drivers and provide wider choices to both parties. Growing uses of these new services suggest the necessity of considering them in taxi system research.

- *Spatial variances.* Although our model only considered time variance in a taxi system, spatial variance is a common feature in taxi systems, especially in megacities. Density of customer demand and levels of congestion might vary over different locations, which pose significant impact on the taxi system's performance. We'll consider them in our future work. ■

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