Optimal Spot-Checking for Improving the Evaluation Quality of Crowdsourcing: Application to Peer Grading Systems

Wanyuan Wang, Bo An, and Yichuan Jiang, *Senior Member, IEEE*

***Abstract*— Peer grading is a natural crowdsourcing application, where dispersed students/peers resources are collected to evaluate others' assignments. Peer grading also offers a promising solution for scaling evaluation and learning to large-scale educational systems. A key challenge in peer grading is motivating peers to grade diligently and provide a high-quality evaluation. Spot-checking (SC) mechanisms, allowing instructors to check evaluations, can prevent peer collusion where peers grade arbitrarily and coordinate to report the uninformative grade. However, existing SC mechanisms unrealistically assume that peers have the same grading reliability and cost. This is limiting in practice, where we would expect peers to differ in reliability and cost. This article proposes the general Optimal SC (OptSC) model of determining the probability that each assignment needs to be checked to maximize assignments' evaluation accuracy aggregated from peers and takes into consideration: 1) peers' heterogeneous characteristics and 2) peers' strategic grading behaviors to maximize their own utility. We prove that the bilevel OptSC is NP-hard to solve. By exploiting peers' grading behaviors, we first formulate a single-level relaxation to approximate OptSC. By further exploiting structural properties of the relaxed problem, we propose an efficient algorithm to that relaxation, which also gives a good approximation of the original OptSC. Extensive experiments on both synthetic and real data sets show significant advantages of the proposed algorithm over existing approaches.**

***Index Terms*— Crowdsourcing, game theory, optimization, peer grading systems (PGSs).**

I. Introduction

CROWDSOURCING has become an effective computing paradigm of evaluating objectives of interest by collecting evaluations from the ubiquitous human resources [2]–[4]. Peer grading can be viewed as a natural crowdsourcing application, where the assignments of students/peers are dispersed to and evaluated by other students. Peer grading not only helps the instructor bring qualified feedbacks to classrooms but also helps students self-study using other peers' solutions [5]–[7]. Besides its direct application to educational systems (e.g., Coursera and EdX), peer grading is also useful in other crowdsourcing systems where it is difficult to evaluate peers' contributions [8]–[10]. One of the key challenges in peer grading is how to motivate students to grade assignments diligently, report the truthful grades, and provide high-quality evaluations [11], [12]. Well-known peer prediction (PP) [13], [14] and the Bayesian truth serum (BTS) [15], [16] mechanisms work by paying a reward to a peer if his reported grade is predicted to be correct based on other peers' reports (referring to the related work for the detail discussion of these two kinds of mechanisms). However, most of these incentive mechanisms are vulnerable to peer collusion, where peers could have a motivation to report the uninformative grade by a preagreed grading rule [12], [17], [18].

Spot-checking (SC) mechanisms can prevent the peer collusion issue [11], [19], [20]. In SC, the instructor checks some assignments by himself and offers a reward to a peer who grades diligently. Existing SC studies have shown that under the special setting where peers are homogeneous with the same grading reliability and cost, a simple SC mechanism, such as uniform [20] or random [21], is efficient to motivate peers to be diligent. Different from existing SC studies, our interest is on the optimization issue of maximizing assignments' evaluation accuracy under a more practical and general setting, in which peers have heterogeneous grading reliability and cost [16], [22], [23]. For example, in an empirical online peer grading test [24], peers with suitable backgrounds have 25% disagreements in average, and varied by peers, about 75% grading is completed in 9.5 to 17.3 min. Under such a general setting, randomized SC mechanisms might perform poorly (as we show in this article) on maximizing assignments' evaluation accuracy. The focus of this article is to find the optimal SC mechanism to maximize assignments' evaluation accuracy in such a practical setting with heterogeneous peers.

The peer grading literature has a great deal of work on optimizing evaluation accuracy [3], [25], and the PP literature has a lot of work on truthful reporting [26]–[29]; this is one of the few pieces of work that makes progress on the combination of these two problems [30], [31]. Our contributions are summarized as follows.

1) We first propose a general SC model for peer grading systems (PGSs) with strategic and heterogeneous peers. We assume that the instructor has an SC budget,

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Wanyuan Wang and Yichuan Jiang are with the School of Computer Science and Engineering, Southeast University, Nanjing 211189, China (e-mail: wywang@seu.edu.cn; yjiang@seu.edu.cn).

Bo An is with the School of Computer Science and Engineering, Nanyang Technological University, Singapore 639798 (e-mail: boan@ntu.edu.sg).

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denoting the maximum number of assignments that he is capable of checking. Given such a budget, the instructor's objective is maximizing assignments' evaluation accuracy aggregated from peers, which can be formulated as a bilevel Optimal SC (OptSC) problem. In the upper level, the instructor determines the probability each assignment needs to be checked, and in the lower level, peers are strategic that choose the optimal grading strategies to maximize their own utility.

2) To address the NP-hardness of OptSC, our second contribution is formulating a single-level relaxation to approximate the bilevel OptSC. Compared with the original OptSC, the relaxation not only achieves nearly the same solution but also has an elegant structure that can be well exploited. By further exploiting the structural properties of the relaxed problem, we propose an efficient algorithm that achieves accuracy within nearly a constant factor with respect to the original OptSC.
3) We extend and propose an exploration algorithm for the incomplete information OptSC variant where peers have uncertain reliability and cost information.
4) For the large-scale PGSs with millions of peers, we extend and propose a scalable algorithm by partitioning these peers into subgroups in a fair manner.
5) Finally, we conduct extensive experiments on both synthetic and real data sets to validate the advantage and robustness of the proposed algorithm over other existing approaches.

The remainder of this article is organized as follows. In Section II, we provide a brief review of related studies on PGSs. In Section III, we model the OptSC problem. We analyze the problem in Section IV and propose an efficient approximation algorithm in Section V. In Section VI, we consider the uncertain OptSC variant where peers have uncertain reliability and cost. In Section VII, we conduct a set of experiments to evaluate our proposed algorithm's performance on evaluation accuracy. Finally, we conclude this article and discuss future work in Section VIII.

II. Related Work

Many PGSs, such as peerScholar [6], Crowdgrader [32], Mechanical TA [21], and Peer Assessment [33], have been developed. Existing PGSs mainly include three phases, calibration, grading, and aggregation. In the following, we review and discuss the related studies in the three phases, respectively.

A. Calibration

The calibration studies can be grouped into two categories, i.e., gold-standard-based calibration and estimation-based calibration.

1) "Gold-Standard"-Based Calibration: In this calibration category, peers' grading reliability and cost are calibrated and learned on sample assignments with "gold-standard" grades [21], [34]. During calibration, peers can request to review any sampled assignment, and immediately after the review is submitted, peers' time/cost used for reviewing can be learned. Meantime, the peer's reliability can be calibrated by the configurable distance between his review and the gold-standard review [35]–[39]. Moreover, peers can also improve their reliability by highlighting the contexts in which the student's review differed from the gold standard [24].

2) Estimation-Based Calibration: No sample assignments that have the "gold standards" are available in this calibration category. The peers' reliability is modeled as the function of peers' and assignment characteristics, e.g., the Gaussian function with respect to the true score of the peer [22], [40]. Immediately after reviews are submitted, peers' reliability can be estimated by the inference approach, such as the simple Gibbs sampling approach [22], the expectation-maximization (EM) approach [41], and probabilistic graphical [40].

Those studies of learning peers' reliability and cost can be orthogonal to our work of optimizing SC mechanisms and providing parameters input for peer modeling. Given these inputs, this article mainly focuses on designing SC mechanisms to motivate peers to review articles diligently, which closes the loop of the PGSs.

B. Grading

In the grading phase, students grade peer assignments according to the predesigned incentive mechanisms. The key challenge in grading is to ensure that the strategic peers grade diligently and have the right incentives to report honest grade [26].

1) Peer Prediction Mechanisms: Miller *et al.* [13] first introduced the PP mechanisms by scoring a peer if his report is predicted to be correct based on other peers' reports. They theoretically prove that with appropriate payment rule, truthful reporting is a Nash equilibrium. Its extension of designing practical scoring rules, e.g., the reward should be scaled to be positive and within a range, is proposed for real-world applications [33]. Besides eliciting truthful reporting, effort exertion is necessary as well, and without effort contribution, even the truthful reporting is uninformative [2], [29]. Effort elicitation can be achieved by designing reward schemes to cover the cost of effort [27], [28]. Although PP mechanisms have theoretical advantages, there are two weaknesses preventing it from being practical. First, PP mechanisms hold the impractical assumption of common priors about an assignment's inherent quality and about the way in which grade is generated by peers given assignment quality type [42]. Second, there might be multiple equilibria of PP mechanisms, and the truthful equilibrium might be dominated by other uninformative equilibria, e.g., the peer collusion equilibrium where all peers coordinate to report the same uninformative grade [17], [43].

2) Bayesian Truth Serum Mechanisms: To relax the common prior assumption, BTS mechanisms elicit additional information to compensate for the lack of knowledge about peers' beliefs. The additional information provided by peers is a prediction of the empirical distribution of grades of other peers. BTS assigns high scores to grades that are more common than collectively predicted [15]. By taking advantage of a quadratic scoring rule, BTS can be extended to small population scenarios [44]. By relating the information score with peer reports, divergence-based BTS is proposed to handle nonbinary signal scenarios [45]. On the other hand, without the assumption of a common prior, Witkowski and Parkes [42] directly extended PP mechanisms by combining a peer's belief report and grade report to calculate a "shadow" posterior belief. Recently, Radanovic and Faltings [18] systemically analyzed how information structures, such as without knowing agents' beliefs (i.e., PP mechanisms) or eliciting additional information (i.e., BTS mechanisms), affect the efficiency of equilibrium solutions.

3) Spot-Checking Mechanisms: Jurca and Faltings [11] first realized that SC can prevent such an uninformative equilibrium issue by allowing the instructor to check some assignments and rewarding a peer if he is verified to grade diligently. Extensions of SC theoretically prove that with homogeneous peers, the simple uniform [20], or random [21] policies, peers can focal on truthful equilibrium. However, existing SC mechanisms do not address the optimization problem of maximizing assignments' evaluation accuracy with the general setting where the instructor has a limited SC budget and peers are heterogeneous on grading reliability and cost. There are also a number of other methods attempting to elicit truthful reporting to be focal. For example, in the binary signals' setting, by enumerating all Nash equilibria and carefully designing the payment rule, peers will prefer to the truthful equilibrium with the highest payment [46]. To guarantee the truthful reporting to be the unique equilibrium, a multitask-based PP (MTPP) mechanism is proposed [2]. In MTPP, given a peer i and her reference peer $r_j(i)$, if i and $r_j(i)$ agree with each other on their overlapping tasks but disagree on the nonoverlapping tasks, peer i will achieve a higher reward, and vice versa. Liu and Chen [31] proposed a machine learning-aided PP mechanism that compares a peer's label with the benchmark label that is generated by a classification algorithm. Shnayder *et al.* [12] extended MTPP to nonbinary signal scenarios by proposing correlated agreement rule. However, in MTPP, without the assumption about task quality, peer collusion is unavoidable.

In summary, compared with PP mechanisms, our mechanisms can prevent uninformative grading behavior. Compared with BTS mechanisms, our mechanism can elicit truthful reporting without the additional information of a prediction of other peers' report. Compared with the most related SC mechanisms that use simple random policy for homogeneous PGSs, we consider designing the optimal SC mechanism and the aggregation rule to maximize PGSs' reliability in a more practical and general setting where the instructor has the budget constraint and peers are strategic and heterogeneous.

C. Aggregation

In the aggregation phase, peers' grades are aggregated to estimate assignments' true grades [7]. The followings are four typical aggregation methods: 1) majority voting, where estimated type is determined by the most common report [47]; 2) EM that models the label given by a peer is conditioned on the task difficulty and peer expertise, and the estimated true label can be estimated by the EM inference rule [41]; 3) belief Propagation (BP)—to tackle the computational intractability issue of EM, the dynamic iterative BP algorithm can approximate the probability marginalization [48], [49]; and 4) ordinal ranking, where peers make ordinal ranks of assignments, e.g., "assignment A is better than assignment B" [50], [51]. However, traditional aggregation methods mainly focus on finding aggregation rules to maximize the probability that the aggregated grade correctly predicts the underlying true value. Most of these works assume honest peers who always grade diligently. One exception is a recent article that studies both elicitation and aggregation [10]; however, prior information about the true value of each assignment is required.

III. Model

In a typical PGS, there are n (≥ 2) peers/students I and n assignments J of these students. The true quality q_j of each assignment j is drawn from a set of possible categories Q. For ease of analysis, we use the binary grade criterion $Q = \{-1, 1\}$, which can be interpreted as categories bad (-1) and good (1). Let $G = (I, J, E)$ denote the bipartite grading graph between peers and assignments [2]. That is, $(i, j) \in E$ if peer i grades assignment j, and $(i, i) \notin E$ guarantees that each peer does not grade his own assignment. Let $I(j)$ denote peers who grade assignment j, and $J(i)$ denote assignments graded by peer i. $|I(j)| = |J(i)| = l$, where l is the load of peers. Table I shows the notations used throughout this article.

Peers: Peers' grades are denoted by $Z = (z_{ij})_{i \in I, j \in J}$, where $z_{ij} \in \{-1, 1\}$ if $(i, j) \in E$ and $z_{ij} = 0$ if $(i, j) \notin E$. Moreover, as required in many practical PGSs [32], peers should also provide detailed comments on assignments. The observed grade mainly depends on a peer's reliability, which denotes the probability of grading an assignment correctly. A peer's reliability is an increasing function of the effort level that he puts in grading. Let e_{ij} denote peer i's effort level on assignment j. For simplicity, we consider binary effort level, i.e., $e_{ij} \in \{0, 1\}$. Putting in full effort $e_{ij} = 1$ incurs cost $c_{ij}(1) \in [0, 1]$, while putting in zero effort $e_{ij} = 0$ incurs zero cost $c_{ij}(0) = 0$. To simplify notations, in the following, $c_{ij}(1)$ is substituted by c_{ij}. A peer who puts in zero effort grades arbitrarily with reliability $p_i^0 = \mathbb{P}(z_{ij} = q_j | e_{ij} = 0) = 0.5$ ($\mathbb{P}$ means the probability), denoting a random estimate [27]. A peer who grades with full effort or diligently produces his maximum reliability $p_i^1 = \mathbb{P}(z_{ij} = q_j | e_{ij} = 1) > 0.5$.

To motivate peers to grade diligently, the SC mechanism is introduced. In SC, the instructor himself can check and grade some assignments. Given a peer-assignment pair $(i, j) \in E$, if peer i is checked with grading assignment j diligently, i will gain a reward $r_{ij} \in [0, 1]$; otherwise, if i is checked with putting zero effort on j, he will not receive any reward. On the other hand, if j is not checked by the instructor, i will not receive any reward [52]. Assume that assignment j will be spot-checked with probability $x_j \in [0, 1]$, peer i's expected utility $u_{ij}(e_{ij}, x_j)$ gained by putting in effort $e_{ij} \in \{0, 1\}$ on j is[1]

$$u_{ij}(e_{ij}, x_j) = e_{ij}(x_j r_{ij} - c_{ij}). \tag{1}$$

[1]Our results can be extended to involving peer i's reliability $p_i^{e_{ij}}$ in his utility function, i.e., $u_{ij}(e_{ij}, x_j) = e_{ij}(p_i^{e_{ij}} x_j r_{ij} - c_{ij})$.

TABLE I
NOTATION OVERVIEW

Notation	Description
$I = \{1, \ldots, i, \ldots, n\}$	the set of peers
$J = \{1, \ldots, j, \ldots, n\}$	the set of assignments
$I(j)$	the peers who grade assignment j
$J(i)$	the assignments graded by peer i
$l = \|I(j)\| = \|J(i)\|$	the load (number of assignments) of peers
$e_{ij} \in \{0,1\}$	peer i's effort level on assignment j
$c_{ij} \in [0,1]$	peer i's cost of putting full effort on j
$p_i^0 = 0.5$	the reliability of peer i with zero effort
p_i^1	the reliability of peer i with full effort
$r_{ij} \in [0,1]$	the reward paid to i for diligent grading j
$x_j \in [0,1]$	the probability of j to be checked
$q_j \in \{-1,1\}$	the true quality of j
$\tilde{q}_j \in \{-1,1\}$	the estimated quality of j
$p_{ij} \in \{p_i^0, p_i^1\}$	the reliability of i on grading j
$w_{ij} \in [0,1]$	the weight of i's grade on j
$\mathbf{p}_j = (p_{ij})_{i \in I(j)}$	peers reliability profile on j
$\mathbb{P}_e(j, \mathbf{p}_j)$	j's error rate with $\mathbf{p}_j$
$\mathbb{P}_e^u(j, \mathbf{p}_j)$	peers upper bound error rate with $\mathbf{p}_j$
θ_{ij}	the critical checking probability of j on i
η_j	the critical checking probability of j on $I(j)$

Considering the external reward and intrinsic grading cost, peer i's best strategy on grading assignment j is

$$e_{ij}^* = \arg\max_{e_{ij} \in \{0,1\}} u_{ij}(e_{ij}, x_j). \quad (2)$$

Instructor: The instructor estimates unchecked assignments' quality by aggregating peers' grades. We adopt the widely used weighted majority voting (WMV) aggregation method [47], which can guarantee accuracy performance. For an assignment j, its estimated value $\tilde{q}_j$ aggregated by WMV can be computed by

$$\tilde{q}_j = \begin{cases} 1, & \sum_{i \in I(j)} w_{ij} z_{ij} \geq 0 \\ -1, & \sum_{i \in I(j)} w_{ij} z_{ij} < 0 \end{cases} \quad (3)$$

where $w_{ij} = 2p_{ij} - 1$ is the weight of peer i's grade on assignment j and $p_{ij} \in \{p_i^0, p_i^1\}$ is the reliability of peer i on grading assignment j. This design of WMV has two desirable properties: 1) the weight is proportional to peers' reliability and 2) if peer i grades arbitrarily with reliability $p_i^0 = 0.5$, his weight becomes zero, indicating that the arbitrary uninformative grade will be discarded in the final aggregation.

Given an assignment j and its peers' reliability profile $\mathbf{p}_j = (p_{ij})_{i \in I(j)}$, let $\mathbb{P}_e(j, \mathbf{p}_j) = \mathbb{P}(q_j \neq \tilde{q}_j, \mathbf{p}_j)$ denote j's exact error rate (i.e., the probability of returning the incorrect grade) under WMV, which can be computed by

$$\sum_{\mathcal{S} \subseteq I(j)} \Big(\prod_{i \in \mathcal{S}} (1 - p_{ij}) \prod_{i \in I(j) \setminus \mathcal{S}} p_{ij} \Big) \mathbb{1}_{\mathcal{X}(\mathcal{S}, j) \geq \mathcal{X}(I(j) \setminus \mathcal{S}, j)}.$$

The instructor considers all possible peer subsets $\mathcal{S} \subseteq I(j)$ who grade incorrectly and remaining peers $I(j) \setminus \mathcal{S}$ who grade correctly such that the aggregated grade is incorrect. The function $\mathbb{1}_{x \geq y}$ equals 1 if $x > y$, 0.5 if $x = y$, and 0 if $x < y$. The function $\mathcal{X}(\mathcal{S}, j) = \sum_{i \in \mathcal{S}} (2p_{ij} - 1)$, denoting the total weight of peers $\mathcal{S}$. We further define j's exact accuracy rate $\mathbb{P}_a(j, \mathbf{p}_j) = \mathbb{P}(q_j = \tilde{q}_j, \mathbf{p}_j) = 1 - \mathbb{P}_e(j, \mathbf{p}_j)$.

Computing the exact error rate $\mathbb{P}_e(j, \mathbf{p}_j)$ requires considering 2^l $(l = |I(j)|)$ peer combinations, which is intractable for large-scale PGSs peer where each assignment is graded by dozens of peers [22]. Moreover, the structure of $\mathbb{P}_e(j, \mathbf{p}_j)$ is complex and hard to analyze. Alternatively, inspired by error rate analysis of crowd labeling [53], we apply a simple but meaningful upper bound error rate $\mathbb{P}_e^u(j, \boldsymbol{p}_j)$ of WMV to approximate $\mathbb{P}_e(j, \mathbf{p}_j)$

$$\mathbb{P}_e^u(j, \boldsymbol{p}_j) = e^{-0.5 \sum_{i \in I(j)} (2p_{ij} - 1)^2}. \quad (4)$$

Proposition 1: Given an assignment j and reliability profile $\mathbf{p}_j = \{p_{ij}\}_{i \in I(j)}$, we have $\mathbb{P}_e(j, \mathbf{p}_j) \leq \mathbb{P}_e^u(j, \mathbf{p}_j)$

Refer to Appendix A for the proof. Next, we show that the upper bound error rate decreases with peer reliability, which is consistent with the exact error rate measure. We first define relationship '$\succ$' between two reliability profiles $\mathbf{p}_j$ and $\mathbf{p}'_j$: $\mathbf{p}_j \succ \mathbf{p}'_j$, iff $\exists i \in I(j) : p_{ij} > p'_{ij}$ and $\forall k \in I(j) \backslash i$, $p_{kj} \geq p'_{kj}$.

Proposition 2: For an assignment j and two peer reliability profiles $\mathbf{p}_j$ and $\mathbf{p}'_j$, where $\mathbf{p}_j \succ \mathbf{p}'_j$, $\mathbb{P}_e^u(j, \mathbf{p}_j) < \mathbb{P}_e^u(j, \mathbf{p}'_j)$.

Proof: To prove $\mathbb{P}_e^u(j, \mathbf{p}_j) = e^{-0.5 \sum_{i \in I(j)} (2p_{ij} - 1)^2}$ decreases with peer i's reliability p_{ij}, we only need to prove that the function $f(j, \mathbf{p}_j) = \sum_{i \in I(j)} (2p_{ij} - 1)^2$ increases with p_{ij}. Deviating $f(j, \mathbf{p}_j)$ by p_{ij}, we have $\partial f(j, \mathbf{p}_j) / \partial p_{ij} = 4(2p_{ij} - 1) \geq 0$ as $p_{ij} \geq 0.5$. □

Instructor's Objective: In practice, the instructor can only check a limited number of assignments, denoted as the SC budget K. Given such budget K, the instructor's objective is to optimize the SC policy $\mathbf{x} = (x_j)_{j \in J}$ of determining each assignment j's checking probability x_j, with the aim of maximizing assignments' average evaluation accuracy. We formulate a bilevel optimization program for the OptSC problem as follows:

$$\max_{\mathbf{x}} \ \Phi(\mathbf{x}) = \frac{\sum_{j \in J} \Big(1 - (1 - x_j) e^{-0.5 \sum_{i \in I(j)} (2p_i^{e_{ij}} - 1)^2} \Big)}{n}, \quad (5)$$

$$\textbf{s.t.} \ u_{ij}(e_{ij}, x_j) \geq u_{ij}(e'_{ij}, x_j) \quad \forall \ i \in I(j), \ e'_{ij} \in \{0,1\} \quad (6)$$

$$\sum_{j \in J} x_j \leq K \quad (7)$$

$$\forall j \in J, x_j \in [0,1]. \quad (8)$$

In the upper level (5), for each assignment $j \in J$.

1) $1 - x_j$ is the probability of not checking j.
2) $e^{-0.5 \sum_{i \in I(j)} (2p_i^{e_{ij}} - 1)^2}$ is j's upper bound error rate $\mathbb{P}_e^u(j, \mathbf{p}_j)$ if it is not checked, where $p_i^{e_{ij}} \in \{p_i^0, p_i^1\}$ is peer i's reliability on j.
3) $(1 - x_j)\mathbb{P}_e^u(j, \mathbf{p}_j)$ and $1 - (1 - x_j)\mathbb{P}_e^u(j, \mathbf{p}_j)$ are j's upper bound error rate and lower bound accuracy rate under $\mathbf{x}$, respectively.

In the lower level (6), each peer i maximizes his utility u_{ij} by choosing the optimal strategy e_{ij} on grading j. Given an SC policy $\mathbf{x}$, we define assignments' total lower bound accuracy rate, $\Phi_n(\mathbf{x}) = n \cdot \Phi(\mathbf{x})$.

Given an SC policy $\mathbf{x}$, each peer's best strategy can be uniquely determined for his monotone utility function. Thus, we can substitute $\mathbb{P}_e^u(j, \mathbf{p}_j)$ by $\mathbb{P}_e^u(j, \mathbf{x})$. In the following, for convenience, we substitute upper bound error rate and lower bound accuracy rate by error rate and accuracy rate.

IV. Problem Analysis

In Section IV-A, we show the NP-hardness of OptSC. In Section IV-B, we present some useful notations and rules. We provide a motivation example to show the weaknesses of traditional random SC mechanisms [20] in Section IV-C.

A. Complexity Analysis

We show that OptSC is NP-hard by reducing an arbitrary 0-1 knapsack decision problem (KDP) to an OptSC.

Theorem 1: The OptSC is NP-hard.

Proof: Given a set of items $\mathcal{I} = \{1, \ldots, n\}$, each with a cost $c_i \in \mathbb{Z}^+$ and a value $v_i \in \mathbb{Z}^+$, and the knapsack's capacity $C \in \mathbb{Z}^+$, here, without loss of generality, we assume that $\max_{i\in\mathcal{I}} c_i < C$. A KDP asks that given $K \in \mathbb{Z}^+$, whether there exists a subset $\mathcal{S} \subseteq \mathcal{I}$ so that $\sum_{i\in\mathcal{S}} c_i \leq C$ and $\sum_{i\in\mathcal{S}} v_i \geq K$. For any KDP $= \langle\mathcal{I}, C, K\rangle$, we construct the corresponding OptSC as follows: for each item $i \in I$, we create an assignment $j(i)$ and a peer i who grades $j(i)$. Each peer i's grading reliability is set as $p_i^0 = 0.5$ and

$$p_i^1 = 0.5\left(\left(-2\ln\left(1 - \frac{v_i/v_{\max}}{1 - c_i/C}\right)\right)^{0.5} + 1\right) \tag{9}$$

where $v_{\max} = \max_{i\in\mathcal{I}} v_i/(1 - c_i/C)/1 - e^{-0.5}$. Let SC budget be 1 and the cost and reward of peer-assignment pair $(i, j(i))$ be c_i and C, respectively. The construction can be done in polynomial time. We can show that the constructed OptSC has a SC policy with average accuracy rate $K/v_{\max} + 1/n$ "iff" KDP $= \langle\mathcal{I}, C, K\rangle$ has a solution.

1) 'If' Direction: Assume that KDP has a solution $\mathcal{S} \subseteq \mathcal{I}$ so that $\sum_{i\in\mathcal{S}} c_i \leq C$ and $\sum_{i\in\mathcal{S}} v_i \geq K$. Then, in OptSC, the instructor can check each assignment $j(i) \in J(\mathcal{S})$ ($J(\mathcal{S}) = \{j(i)|i \in \mathcal{S}\}$) with probability c_i/C and allocates the remaining budget to others $J(I \setminus \mathcal{S})$ uniformly. Let $\mathbf{x}$ denote this policy, where $\forall j(i) \in J(\mathcal{S})$, $x_{j(i)} = c_i/C$ and $\forall j(i') \in J(I\setminus\mathcal{S})$, $x_{j(i')} = 1 - \sum_{j(i)\in J(\mathcal{S})} c_i/C/|J(I \setminus \mathcal{S})|$, and we have $\Phi_n(\mathbf{x}) \geq \sum_{j(i)\in J(\mathcal{S})}(1 - (1 - (c_i/C))e^{-0.5(2p_i^1-1)^2}) + \sum_{j(i')\in J(I\setminus\mathcal{S})} x_{j(i')}$. Substituting p_i^1 by (9), we have $\Phi_n(\mathbf{x}) \geq \sum_{j(i)\in J(\mathcal{S})}(c_i/C+(v_i/v_{\max}))+1-\sum_{j(i)\in J(\mathcal{S})} c_i/C \geq K/v_{\max}+1$. Thus, the average accuracy rate is $\Phi(\mathbf{x}) = 1/n\Phi_n(\mathbf{x}) \geq K/v_{\max} + 1/n$.

2) 'Only If' Direction: Assume that there is a policy $\mathbf{x}$ for OptSC with average accuracy rate $\Phi(\mathbf{x}) \geq K/v_{\max} + 1/n$. Let $J(\mathcal{S}) = \{j(i)|x_{j(i)} \geq c_i/C\}$ be the assignments that are graded diligently. We show that, for KDP, the item set $\mathcal{S}$ is a feasible solution that satisfies $\sum_{i\in\mathcal{S}} c_i \leq C$ and $\sum_{i\in\mathcal{S}} v_i \geq K$. First, note that in OptSC, the total budget spent on assignments $J(\mathcal{S})$ should be less than 1, so we have $\sum_{j(i)\in J(\mathcal{S})} c_i/C \leq \sum_{j(i)\in J(\mathcal{S})} x_{j(i)} \leq 1$. Thus, in KDP, we have $\sum_{i\in\mathcal{S}} c_i \leq C$. Second, in OptSC, we define an alternative policy $\mathbf{x}^*$, where $\forall j(i) \in J(\mathcal{S})$, $x^*_{j(i)} = c_i/C$, and $\forall j(i') \in J(I \setminus \mathcal{S})$, $x^*_{j(i')} = 1 - \sum_{j(i)\in J(\mathcal{S})} c_i/C/|J(I \setminus \mathcal{S})|$. According to error rate first (ERF) rule, we have $\Phi(\mathbf{x}^*) \geq \Phi(\mathbf{x}) \geq K/v_{\max} + 1/n$. Unfolding $\Phi(\mathbf{x}^*) \geq 1/n\left(\sum_{j(i)\in J(\mathcal{S})}(c_i/C + v_i/v_{\max}) + 1 - \sum_{j(i)\in J(\mathcal{S})} c_i/C\right) = 1/n\left(\sum_{j(i)\in J(\mathcal{S})} v_i/v_{\max} + 1\right) \geq K/v_{\max} + 1/n$, Thus, for KDP, we have $\sum_{i\in\mathcal{S}} v_i \geq K$. □

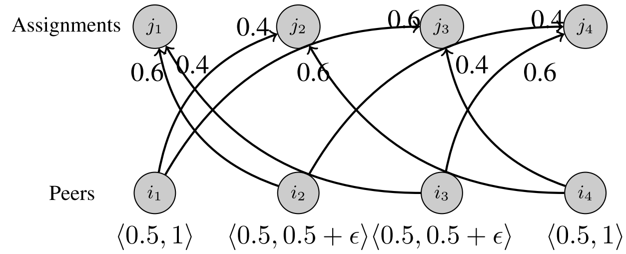


Fig. 1. Toy peer grading example.

B. Useful Notations and Rules

1) Critical Checking Probability, θ_{ij} and η_j: For an assignment j with checking probability x_j, peer i putting in full effort $e_{ij} = 1$ will gain utility $u_{ij}(1, x_j) = x_j r_{ij} - c_{ij}$ and putting in zero effort $e_{ij} = 0$ will gain $u_{ij}(0, x_j) = 0$. To elicit i to grade j diligently, the checking probability x_j should satisfy $x_j \geq \theta_{ij} = c_{ij}/r_{ij}$ such that $u_{ij}(1, x_j) \geq 0$. Thus, we first define $\theta_{ij} = c_{ij}/r_{ij}$, the critical checking probability of the assignment j with respect to peer i, above which i grades j diligently and under which i grades j arbitrarily. We next define $\eta_j = \max_{i\in I(j)} \theta_{ij}$, the critical checking probability of j with respect to peers $I(j)$. A diligent peer-assignment pair (i, j) denotes i grades j diligently.

2) Error Rate First Rule: Given a PGS $G = (I, J, E)$ and an SC policy $\mathbf{x}$, let $\mathbb{P}_e^u(j, \mathbf{x})$ denote the error rate of assignment j under $\mathbf{x}$. Now assume that there is extra tiny budget ϵ that cannot elicit any nondiligent peer-assignment pair to be diligent. It is optimal to maximize PGS's evaluation accuracy by allocating ϵ to the assignment j^* that has the largest error rate, i.e., $j^* = \arg\max_{j\in J} \mathbb{P}_e^u(j, \mathbf{x})$. We verify this rule by analyzing the structure of (5). Let $\mathbf{x}^* = (x_{-j^*}, x_{j^*} = x_{j^*} + \epsilon)$ and $\mathbf{x}' = (x_{-j'}, x_{j'} = x_{j'} + \epsilon)$ ($x_{j^*} + \epsilon < 1$, $x_{j'} + \epsilon < 1$) denote policies of allocating ϵ to j^* and $j'(\neq j^*)$, where x_{-j} is the checking probabilities of all assignments except j under $\mathbf{x}$. The total accuracy difference between $\mathbf{x}^*$ and $\mathbf{x}'$ is $\Phi_n(\mathbf{x}^*) - \Phi_n(\mathbf{x}') = \partial\Phi_n(\mathbf{x})/\partial x_{j^*}\epsilon - \partial\Phi_n(\mathbf{x})/\partial x_{j'}\epsilon = \epsilon(\mathbb{P}_e^u(j^*, \mathbf{x}) - \mathbb{P}_e^u(j', \mathbf{x})) \geq 0$.

C. Inefficiency of Random SC Mechanism

We use an example to show the disadvantage of traditional random SC mechanism. In Fig. 1, there are four peers $I = \{i_1, i_2, i_3, i_4\}$ and four assignments $J = \{j_1, j_2, j_3, j_4\}$. Each assignment is allocated to two peers. Each peer i is associated with a tuple $\langle p_i^0, p_i^1\rangle$, indicating his zero and full effort reliability, e.g., for peer i_1, his zero effort reliability is 0.5, and full effort reliability is 1. Each edge (i, j) is associated with a value $\theta_{ij} = c_{ij}/r_{ij}$, indicating the critical checking probability of the peer-assignment pair (i, j), e.g., for peer-assignment pair (i_1, j_2), to elicit peer i_1 to be diligent on assignment j_2, the critical checking probability of j_2 is 0.4. Now, assume that the instructor has SC budget $K = 1$, and the exact and lower bound accuracy rates of the random and optimal SC policies are computed as follows.

1) Random SC Policy: Under this random policy, the probability of checking each assignment is randomly generated. For instance, the random policy is $\mathbf{x}^R = (0.25, 0.25, 0.25, 0.25)$, i.e., checking all assignments with uniform probability 0.25. Under $\mathbf{x}^R$, peers i_1, i_2, i_3, and i_4 all grade arbitrarily on assignments j_1, j_2, j_3, and j_4. The assignment j_1's exact accuracy rate is $\mathbb{P}_a(j_1, \mathbf{x}^R) = 1 - (1 - x_1^R)\mathbb{P}_e(j_1, \mathbf{x}_R) = 1 - (1 - 0.25) \cdot (1 - 0.5) = 0.625$. Similarly, the exact accuracy rates of assignments j_2, j_3, and j_4 are $\mathbb{P}_a(j_2, \mathbf{x}^R) = 0.625$, $\mathbb{P}_a(j_3, \mathbf{x}^R) = 0.625$, and $\mathbb{P}_a(j_4, \mathbf{x}^R) = 0.625$. The average exact accuracy rate of all assignments is $\Phi_e(\mathbf{x}^R) = \sum_{1\le k\le 4} \mathbb{P}_a(j_k, \mathbf{x}^R)/4$ =0.625. Similarly, the average lower bound accuracy rate is

$$\begin{aligned}\Phi_l(\mathbf{x}^R) &= \big[(1-(1-0.25)\cdot e^{-0.5\cdot 0}) + (1-(1-0.25)\cdot e^{-0.5\cdot 0}) \\ &\quad +(1-(1-0.25)\cdot e^{-0.5\cdot 0}) + (1-(1-0.25)\cdot e^{-0.5\cdot 0})\big]/4 \\ &= 0.25.\end{aligned}$$

2) Optimal SC Policy: The optimal checking policy is $\mathbf{x}^O = (0.2, 0.4, 0.4, 0)$. Then, peer i_1 grades diligently on both j_2 but arbitrarily on j_3; peer i_2 grades arbitrarily on both j_1 and j_4; peer i_3 grades arbitrarily on both j_1 and j_4; and peer i_4 grades diligently on j_3 but arbitrarily on j_2. The average exact accuracy rate under the optimal solution $\mathbf{x}_O$ is $\Phi_e(\mathbf{x}^O) = [(1-(1-0.2)\cdot 0.5) + (1-0.6\cdot(1-1)) + (1-0.6\cdot(1-1)) + (1-(1-0)\cdot 0.5)]/4 = 0.775$, and the average lower bound accuracy rate is $\Phi_l(\mathbf{x}^O) = [(1-0.8\cdot 1) + (1-0.6\cdot e^{-0.5\cdot 2}) + (1-0.6\cdot e^{-0.5\cdot 2}) + (1-1\cdot 1)]/4 \approx 0.44$.

From the abovementioned example, we can see that: 1) the random SC policy performs poorly on maximizing accuracy rate and 2) although the lower bound accuracy rate might be far from the exact accuracy rate, it is consistent with the accuracy rate on SC policy, indicating that lower bound accuracy rate is a satisfiable indicator to measure the exact accuracy rate.

V. Efficient Approximation Algorithm

The key idea behind our algorithm is that we first formulate a single-level relaxation to approximate the bilevel OptSC (see Section V-A). The relaxed formulation: 1) guarantees the limited solution loss of the original OptSC (see Theorem 2) and 2) has an elegant structure that can be well exploited (see Theorem 3). In Section V-B, we design an efficient approximation algorithm for the relaxed problem, which also offers performance guarantee for the original OptSC (see Section V-C).

A. Relaxing OptSC

Given an SC policy $\mathbf{x}$, let $\mathcal{S}(\mathbf{x})$ be the set of diligent peer-assignment pairs (i, j) where peer i grades assignment j diligently, i.e., $\mathcal{S}(\mathbf{x}) = \{(i, j)|x_j \ge \theta_{ij}, (i, j) \in E\}$. Let $J(i, \mathcal{S}(\mathbf{x})) = \{j|\exists(i, j) \in \mathcal{S}(\mathbf{x})\}$ be assignments in $\mathcal{S}(\mathbf{x})$ that are graded diligently by peer i and $I(j, \mathcal{S}(\mathbf{x})) = \{i|\exists(i, j) \in \mathcal{S}(\mathbf{x})\}$ be peers in $\mathcal{S}(\mathbf{x})$ who grade assignment j diligently. We formulate a single-level peer-assignment-oriented relaxation OptSC_PA. This relaxation is a combinatorial optimization problem of finding the optimal diligent peer-assignment pair set $\mathcal{S} \subseteq E$ to maximize assignments' accuracy rate $\Phi^\tau(\mathcal{S})$, shown as follows:

$$\max_{\mathcal{S}\subseteq E} \Phi^\tau(\mathcal{S}) = \frac{\sum_{j\in J}\Big(1-(1-x_j)e^{-0.5\sum_{i\in I(j)}(2p_{ij}-1)^2}\Big)}{n} \tag{10}$$
$$\textbf{s.t. } x_j = \max_{i\in I(j,\mathcal{S})} \theta_{ij} \tag{11}$$
$$p_{ij} = p_i^1 \quad \forall(i,j)\in\mathcal{S};\quad p_{ij} = p_i^0 \quad \forall(i,j)\notin\mathcal{S} \tag{12}$$
$$\sum_{j\in J} x_j \le K. \tag{13}$$

In (10), $\mathcal{S} \subseteq E$ is the set of selected peer-assignment pairs. To elicit all peer-assignment pairs in $\mathcal{S}$ to be diligent, (11) proposes a critical checking policy x_j of checking each assignment j with the maximal critical checking probability with respect to peers $I(j, \mathcal{S})$. This critical checking policy guarantees that the original OptSC and the relaxation OptSC_PA achieve nearly the same accuracy rate.

Theorem 2: Given $K \le \sum_{j\in J} \eta_j$, let $\mathbf{y} = (y_j)_{j\in J}$ and Φ_{opt} be OptSC's optimal SC policy and accuracy rate. Let $\mathcal{S}$ and Φ^τ_{opt} be OptSC_PA's optimal peer-assignment pair set and accuracy rate. We have $\Phi_{\text{opt}} - \Phi^\tau_{\text{opt}} \le 1 + n^{\mathbb{1}}/n$, where n is the number of peers and $n^{\mathbb{1}}$ is the number of assignments that are checked by probability 1 in $\mathbf{y}$.

Proof: Under $\mathbf{y}$, we split all assignments J into three disjoint groups, $\mathcal{L}$, $\mathcal{H}$, and $\mathcal{F}$, where $\mathcal{L} = \{j|y_j = \max_{i\in I(j,\mathcal{S}(\mathbf{y}))} \theta_{ij}\}$ denotes assignments checked by the critical checking probability, $\mathcal{H} = \{j|y_j > \max_{i\in I(j,\mathcal{S}(\mathbf{y}))} \theta_{ij}, y_j \ne 1\}$ denotes assignments neither checked by the critical checking probability nor probability 1, and $\mathcal{F} = \{j|y_j = 1, y_j \notin \mathcal{L}\}$ denotes assignments checked by probability 1, but not in $\mathcal{L}$, where $|\mathcal{F}| = n^{\mathbb{1}}$.

Case 1 ($n^{\mathbb{1}} = 0$): Let j^* be the assignment in $\mathcal{H}$ that has the largest error rate under $\mathbf{y}$, i.e., $\mathbb{P}^u_e(j^*, \mathbf{y}) = \max_{j\in\mathcal{H}} \mathbb{P}^u_e(j, \mathbf{y})$. According to the ERF rule, we can improve $\mathbf{y}$ by transferring some budget from other assignments $j \in \mathcal{H}$ to j^* until the transferred budget can elicit certain peer in $I(j^*)\backslash I(j^*, \mathcal{S}(\mathbf{y}))$ to be diligent on j^*. This budget transfer will not decrease any assignment's error rate. We proceed this budget transfer process until one of the two scenarios happens: 1) all assignments in $\mathcal{H}$ are checked by the critical checking probability and 2) there is only one assignment in $\mathcal{H}$ that is not checked by the critical checking probability. Let $\mathbf{y}'$ be the final policy after this budget transfer, and we have $\Phi(\mathbf{y}') \ge \Phi(\mathbf{y})$ in OptSC.

For scenario 1, we have $\Phi(\mathbf{y}') \le \Phi^\tau(\mathcal{S})$ because $\mathcal{S}$ is the optimal critical checking policy. Thus, $\Phi_{\text{opt}} - \Phi^\tau_{\text{opt}} = \Phi(\mathbf{y}) - \Phi^\tau(\mathcal{S}) \le \Phi(\mathbf{y}') - \Phi^\tau(\mathcal{S})) \le 0 \le 1/n$.

For scenario 2, let j^* be the assignment that is not checked by the critical checking probability under $\mathbf{y}'$ and $z_{j^*} = y'_{j^*} - \max_{i\in I(j^*,\mathcal{S}(\mathbf{y}'))} \theta_{ij^*} \le 1$ be the redundant noncritical checking budget on j^*. Removing z_{j^*} from j^* and defining the corresponding critical checking policy $\mathbf{y}'' \ge (y'_{-j^*}, y_{j^*}'' = y'_{j^*} - z_{j^*})$. Because z_{j^*} is the redundant noncritical checking budget, we have $\Phi(\mathbf{y}'') = \Phi(\mathbf{y}') - z_{j^*}/n$ in OptSC. Let $\Phi^\tau(\mathcal{S}) = \Phi^\tau(\mathcal{S}(K))$ denote the optimal accuracy rate of OptSC_PA with budget K. Then, we have $\Phi^\tau(\mathcal{S}(K)) \ge \Phi^\tau(\mathcal{S}(K - z_{j^*}))$. With budget $K - z_{j^*}$, the policy y'' is a critical checking policy, and $\mathcal{S}(K - z_{j^*})$ is the optimal critical checking policy. Thus, we have $\Phi^\tau(\mathcal{S}(K - z_{j^*})) \ge \Phi(\mathbf{y}'') \ge \Phi(\mathbf{y}') - z_{j^*}/n \ge \Phi(\mathbf{y}) - z_{j^*}/n$. Finally, we have $\Phi_{\text{opt}} - \Phi^\tau_{\text{opt}} = \Phi(\mathbf{y}) - \Phi^\tau(\mathcal{S}) \le \Phi(\mathbf{y}) - \Phi^\tau(\mathcal{S}(K - z_{j^*})) \le z_{j^*}/n \le 1/n$.

Case 2 ($n^{\mathbb{1}} > 0$): Besides $j^* = \arg\max_{j\in\mathcal{H}} \mathbb{P}_e^u(j, \mathbf{y})$, there are other $n^{\mathbb{1}}$ assignments in $\mathcal{F}$ that have higher accuracy rates in $\mathbf{y}$ for OptSC than those in $\mathcal{S}$ for OptSC_PA. Similar to Case 1, we can further derive that $\Phi_{\text{opt}} - \Phi_{\text{opt}}^{\tau} \leq 1 + n^{\mathbb{1}}/n$. The condition that the assignment $j \in \mathcal{F}$ checked by probability 1 in OptSC is harsh, where the error rate $\mathbb{P}_e^u(j, \mathbf{y})$ of j under full critical checking probability η_j must be larger than all assignments $j' \in \mathcal{L}$ that are checked by the partial critical checking probability $y_{j'} \leq \eta_j$. With budget $K \leq \sum_j \eta_j$, the scenario that an assignment is checked by probability 1 happens infrequently, and compared with n, $n^{\mathbb{1}}$ is very small. □

Properties of OptSC_PA: We observe that in OptSC_PA, the objective function Φ^{τ} satisfies monotone and submodular properties with respect to $\mathcal{S}$. Let U be a nonempty finite set and f be a function $f : 2^U \to \mathbb{R}$, where 2^U denotes the power set of U. The function f is ***monotone*** if $f(\mathcal{A}) \leq f(\mathcal{B})$ for all $\mathcal{A} \subseteq \mathcal{B} \subseteq U$ and ***submodular*** if $f(\mathcal{A} \cup s) - f(\mathcal{A}) \geq f(\mathcal{B} \cup s) - f(\mathcal{B})$ for all $\mathcal{A} \subseteq \mathcal{B} \subseteq U$ and $s \in U \backslash \mathcal{B}$. We first define a couple of quantities that will be useful in Theorem 3. Let $\mathbb{P}_e^u(j, I(j, \mathcal{S})) = e^{-0.5\sum_{i\in I(j,\mathcal{S})}(2p_{ij}^{1}-1)^2}$ denote the error rate of assignment j when peers $I(j, \mathcal{S})$ grade diligently on j. For two disjoint sets $I(j, \mathcal{S}_1)$ and $I(j, \mathcal{S}_2)$, where $I(j, \mathcal{S}_1) \cap I(j, \mathcal{S}_2) = \emptyset$, we have $\mathbb{P}_e^u(j, I(j, \mathcal{S}_1) \cup I(j, \mathcal{S}_2)) = \mathbb{P}_e^u(j, I(j, \mathcal{S}_1)) \cdot \mathbb{P}_e^u(j, I(j, \mathcal{S}_2))$.

Theorem 3: The objective function Φ^{τ} defined in (10) is monotone and submodular with respect to $\mathcal{S}$.

Proof: Monotone Property: Given a set of diligent peer-assignment pairs $\mathcal{S}$ and another peer-assignment pair $(i^*, j^*) \notin \mathcal{S}$, let $\mathcal{S}^* = \mathcal{S} \cup (i^*, j^*)$ and $\mathbf{x}^{\mathcal{S}} = (x_j^{\mathcal{S}})_{j\in J}$, $\mathbf{x}^{\mathcal{S}^*} = (x_j^{\mathcal{S}^*})_{j\in J}$ denote critical checking polices for $\mathcal{S}$ and $\mathcal{S}^*$, respectively. Then, we have $I(j^*, \mathcal{S}^*) \setminus I(j^*, \mathcal{S}) = i^*$; $\forall j \in J \setminus j^*$, $x_j^{\mathcal{S}} = x_j^{\mathcal{S}^*}$ and $\mathbb{P}_e^u(j, I(j, \mathcal{S})) = \mathbb{P}_e^u(j, I(j, \mathcal{S}^*))$; and $x_{j^*}^{\mathcal{S}^*} \geq x_{j^*}^{\mathcal{S}}$. Finally, the difference between $\Phi_n(\mathcal{S}^*)$ and $\Phi_n(\mathcal{S})$ is $\Phi_n(\mathcal{S}^*) - \Phi_n(\mathcal{S}) = (1 - x_{j^*}^{\mathcal{S}})\mathbb{P}_e^u(j^*, I(j^*, \mathcal{S})) - (1 - x_{j^*}^{\mathcal{S}^*})\mathbb{P}_e^u(j^*, I(j^*, \mathcal{S}^*)) = \mathbb{P}_e^u(j^*, I(j^*, \mathcal{S}))\Big((1 - x_{j^*}^{\mathcal{S}}) - (1 - x_{j^*}^{\mathcal{S}^*})\mathbb{P}_e^u(j^*, i^*)\Big)$. Since $\mathbb{P}_e^u(j^*, I(j^*, \mathcal{S})) \geq 0$, we have $\Phi_n(\mathcal{S}^*) - \Phi_n(\mathcal{S}) \propto 1 - x_{j^*}^{\mathcal{S}} - (1 - x_{j^*}^{\mathcal{S}^*})\mathbb{P}_e^u(j^*, i^*) \geq x_{j^*}^{\mathcal{S}^*} - x_{j^*}^{\mathcal{S}} \geq 0$. The operator $\propto$ means the positive relation.

Submodular Property: Let $\mathcal{S}_1$ and $\mathcal{S}_2$ denote two diligent peer-assignment pair sets, where $\mathcal{S}_1 \subseteq \mathcal{S}_2$. For any diligent peer-assignment grading $(i^*, j^*) \notin \mathcal{S}_2$, $\mathcal{S}_1^* = \mathcal{S}_1 \cup (i^*, j^*)$ and $\mathcal{S}_2^* = \mathcal{S}_2 \cup (i^*, j^*)$; then, we have

$$
\begin{aligned}
&\Phi_n(\mathcal{S}_1^*) - \Phi_n(\mathcal{S}_1) - (\Phi_n(\mathcal{S}_2^*) - \Phi_n(\mathcal{S}_2)) \\
&= \mathbb{P}_e^u(j^*, I(j^*, \mathcal{S}_1))\Big((1 - x_{j^*}^{\mathcal{S}_1}) - (1 - x_{j^*}^{\mathcal{S}_1^*})\mathbb{P}_e^u(j^*, i^*)\Big) \\
&\quad -\mathbb{P}_e^u(j^*, I(j^*, \mathcal{S}_2))\Big((1 - x_{j^*}^{\mathcal{S}_2}) - (1 - x_{j^*}^{\mathcal{S}_2^*})\mathbb{P}_e^u(j^*, i^*)\Big) \\
&= \mathbb{P}_e^u(j^*, I(j^*, \mathcal{S}_1))\Big[\big(1 - x_{j^*}^{\mathcal{S}_1}\big) - \big(1 - x_{j^*}^{\mathcal{S}_1^*}\big)\mathbb{P}_e^u(j^*, i^*) \\
&\qquad -\mathbb{P}_e^u\big(j^*, I(j^*, \mathcal{S}_2 \setminus \mathcal{S}_1)\big) \\
&\qquad \Big(\big(1 - x_{j^*}^{\mathcal{S}_2}\big) - \big(1 - x_{j^*}^{\mathcal{S}_2^*}\big)\mathbb{P}_e^u(j^*, i^*)\Big)\Big] \\
&\propto \big(1 - x_{j^*}^{\mathcal{S}_1}\big) - \big(1 - x_{j^*}^{\mathcal{S}_1^*}\big)\mathbb{P}_e^u(j^*, i^*) \\
&\quad -\mathbb{P}_e^u(j^*, I(j^*, \mathcal{S}_2 \setminus \mathcal{S}_1))\Big(\big(1 - x_{j^*}^{\mathcal{S}_2}\big) - \big(1 - x_{j^*}^{\mathcal{S}_2^*}\big)\mathbb{P}_e^u(j^*, i^*)\Big)
\end{aligned}
$$

Algorithm 1 Peer-Assignment Pair-Based SC Algorithm PASC (G, K)

Input : The PGS $G = (I, J, E)$ and the SC budget K.
Output: The SC policy $\mathbf{x} \subseteq [0, 1]^n$.
1 Initialize $\mathbf{x} = (0)_{j\in J}$, $\Omega = E$, $K^r = K$, $K^* = K$;
2 **while** $\Omega \neq \emptyset$ **do**
3 $\quad (i^*, j^*) = \arg\max_{(i,j)\in\Omega} \frac{\Phi^{\tau}(\mathbf{x}_{-j}, x_j'=\theta_{ij}) - \Phi^{\tau}(\mathbf{x})}{x_j' - x_j}$;
4 $\quad$ **if** $\theta_{i^*j^*} - x_{j^*} \leq K^r$ **then**
5 $\quad\quad x_{j^*} = \theta_{i^*j^*}$, $K^r = K^r - (\theta_{i^*j^*} - x_{j^*})$;
6 $\quad\quad \Omega = \Omega \setminus \{(i, j^*) \in \Omega | \theta_{ij^*} \leq x_{j^*}\}$;
7 $(i^*, j^*) = \arg\max_{(i,j)\in E} \Phi(0, \ldots, x_j = \theta_{ij}, \ldots, 0)$;
8 $\mathbf{x}^* = (0, \ldots, x_{j^*} = \theta_{i*j*}, \ldots, 0)$, $K^* = K^* - x_{j^*}$;
9 **If** $\Phi(\mathbf{x}^*) > \Phi(\mathbf{x})$, **then** $K^r = K^*$ and $\mathbf{x} = \mathbf{x}^*$;
10 **while** $K^r > 0$ **do**
11 $\quad j^* = \arg\max_{j\in J} \mathbb{P}_e^u(j, \mathbf{x})$, $\delta_b = \max\{1 - x_{j*}, K^r\}$;
$\quad K^r = K^r - \delta_b$, $x_{j^*} = x_{j*} + \delta_b$;
12 **Return** the SC policy $\mathbf{x} = (x_1, x_2, \ldots, x_n)$.

$$
\begin{aligned}
&\geq \Big(1 - x_{j^*}^{\mathcal{S}_1}\Big) - \Big(1 - x_{j^*}^{\mathcal{S}_1^*}\Big)\mathbb{P}_e^u(j^*, i^*) \\
&\quad -\Big(\Big(1 - x_{j^*}^{\mathcal{S}_2}\Big) - \Big(1 - x_{j^*}^{\mathcal{S}_2^*}\Big)\mathbb{P}_e^u(j^*, i^*)\Big) \\
&= \Big(x_{j^*}^{\mathcal{S}_2} - x_{j^*}^{\mathcal{S}_1}\Big) - \mathbb{P}_e^u(j^*, i^*)\Big(x_{j^*}^{\mathcal{S}_2^*} - x_{j^*}^{\mathcal{S}_1^*}\Big) \\
&\geq \Big(x_{j^*}^{\mathcal{S}_2} - x_{j^*}^{\mathcal{S}_1}\Big) - \Big(x_{j^*}^{\mathcal{S}_2^*} - x_{j^*}^{\mathcal{S}_1^*}\Big) = \Big(x_{j^*}^{\mathcal{S}_1^*} - x_{j^*}^{\mathcal{S}_1}\Big) - \Big(x_{j^*}^{\mathcal{S}_2^*} - x_{j^*}^{\mathcal{S}_2}\Big) \\
&\geq 0.
\end{aligned}
$$

□

B. Approximation Algorithm for OptSC_PA

Based on monotone and submodular properties of Φ^{τ}, we propose PASC, a peer-assignment pair-based *SC* algorithm (i.e., Algorithm 1). Algorithm 1 mainly consists of two stages.

1) *Stage 1–Diligent Grading Elicitation:* In Lines 2–6, Algorithm 1 first finds one candidate SC policy by greedily eliciting the peer-assignment pair (i^*, j^*) that has the largest margin accuracy gain-cost ratio to be diligent, that is

$$(i^*, j^*) = \arg\max_{(i,j)\in E} \frac{\Phi^{\tau}(\mathbf{x}_{-j}, x_j'=\theta_{ij}) - \Phi^{\tau}(\mathbf{x})}{x_j' - x_j} \tag{14}$$

where $x_j' = \theta_{ij}$ is the critical probability of eliciting diligent grading of (i, j) and $x_j' - x_j$ is the budget necessary for this elicitation under the policy $\mathbf{x}$. In Lines 7 and 8, Algorithm 1 also finds another candidate policy $\mathbf{x}^*$ that only actives the optimal peer-assignment pair (i^*, j^*) that has the largest accuracy rate gain. To elicit this peer-assignment pair (i^*, j^*) to be diligent, a budget of θ_{ij} should be allocated to the assignment j. In Line 9, Algorithm 1 selects the policy from the two candidates $\mathbf{x}$ and $\mathbf{x}^*$ with larger accuracy rate.

2) *Stage 2–Remaining Budget Allocation:* In lines 10 and 11, if there is remaining budget $K^r = K - \sum_j x_j > 0$, Algorithm 1 iteratively allocates K^r to assignments that have the largest error rates.

C. Algorithm Property Analysis

Approximation Ratio Analysis: In this section, we provide the approximation ratio $\Phi_{\text{pasc}}/\Phi_{\text{opt}}$ of PASC, where Φ_{pasc} and Φ_{opt} are accuracy rates returned by PASC and optimum (OPT) of OptSC. Let $n^{\mathbb{1}}$ be the number of assignments that are checked by probability 1 in OPT, which has been discussed in Theorem 2.

Theorem 4: If $K < \sum_j \eta_j$, $\Phi_{\text{pasc}}/\Phi_{\text{opt}} \geq K/2(K + n^{\mathbb{1}} + 1)(1 - 1/e)$.

Proof: Let Φ^{τ}_{opt} be the optimal solution of OptSC_PA. For such a budgeted maximization problem with submodular objective function and linear cost function, we have $\Phi_{\text{pasc}}/\Phi^{\tau}_{\text{opt}} \geq 1/2(1 - (1/e))$ [54]. Derived from Theorem 2 that $\Phi_{\text{opt}} - \Phi^{\tau}_{\text{opt}} \leq n^{\mathbb{1}} + 1/n$, we have $\Phi_{\text{opt}}/\Phi^{\tau}_{\text{opt}} \leq 1 + (n^{\mathbb{1}} + 1/n \cdot \Phi^{\tau}_{\text{opt}})$. We also have $\Phi^{\tau}_{\text{opt}} \geq K/n$ by checking K assignments. Thus, $\Phi_{\text{pasc}}/\Phi_{\text{opt}} \geq K/2(K + n^{\mathbb{1}} + 1)$ $(1 - (1/e))$. □

For $K \geq \sum_j \eta_j$, we first present a useful property of OPT.

Proposition 3: Given the budget $K = K_1 + K_2$, we have $\boldsymbol{x}_{\text{opt}}(K) = \boldsymbol{x}_{\text{opt}}(K_1) + \boldsymbol{x}_{\text{opt}}(K_1 \oplus K_2)$, where $\boldsymbol{x}_{\text{opt}}(K)$ is the optimal SC policy with budget K and $\boldsymbol{x}_{\text{opt}}(K_1 \oplus K_2)$ is the optimal SC policy with budget K_2 under existing optimal SC policy $\mathbf{x}_{\text{opt}}(K_1)$.

Proof: Let $\boldsymbol{x}'_{\text{opt}} = \boldsymbol{x}_{\text{opt}}(K_1) + \boldsymbol{x}_{\text{opt}}(K_1 \oplus K_2)$ and its solution be OPT'. Let $\boldsymbol{x}^*_{\text{opt}}(K)$ be the optimal policy with budget K and its solution be OPT*. We split assignments J into two disjoint groups, $\mathcal{H}$ and $\mathcal{L}$, where $\mathcal{H} = \{j | x'_{\text{opt}}(j) \geq x^*_{\text{opt}}(j)\}$ and $\mathcal{L} = \{j | x'_{\text{opt}}(j) < x^*_{\text{opt}}(j)\}$. Then, for OPT', in the second stage with budget K_2, we can transfer the excess budget $x'_j - x^*_j$ of "high" assignments $j \in \mathcal{H}$ to these "low" assignments $j \in \mathcal{L}$ to obtain $\mathbf{x}^*_{\text{opt}}$. The only reason that this transfer cannot finish is that for certain "high" assignment $j \in \mathcal{H}$, its checking probability $x'_{\text{opt}}(j, K_1)$ has already been higher than $x^*_{\text{opt}}(j, K)$. However, this scenario is impossible. If it is optimal to allocate larger checking probability $x'_{\text{opt}}(j, K_1)$ to j within budget K_1, OPT* should also allocate at least such checking probability $x'_{\text{opt}}(j, K_1)$ to j within budget K. This is due to the monotone property of error rate on SC checking probability proved in Proposition 2. □

Theorem 5: If $K \geq \sum_j \eta_j$, $\Phi_{\text{pasc}}/\Phi_{\text{opt}} \geq K/K + n^{\mathbb{1}} + 1$.

Proof: We divide this setting into two subsettings: 1) $K = \sum_j \eta_j$ and 2) $K > \sum_j \eta_j$.

Setting 1 ($K = \sum_j \eta_j$): PASC can elicit all peers to be diligent, and we have $\Phi^{\tau}_{\text{opt}} = \Phi_{\text{pasc}}$. According to Theorem 4, we have $\Phi^{\tau}_{\text{opt}}/\Phi_{\text{opt}} \geq K/K + n^{\mathbb{1}} + 1$, which derives that $\Phi_{\text{pasc}}/\Phi_{\text{opt}} \geq K/K + n^{\mathbb{1}} + 1$.

Setting 2 ($K > \sum_j \eta_j$): Splitting K into two parts $K_1 = \sum_j \eta_j$ and $K_2 = K - K_1$. Based on Proposition 3, we have $\Phi_{\text{opt}} = \Phi_{\text{opt}}(K_1) + \Phi_{\text{opt}}(K_1 \oplus K_2)$. Given the budget K_1, by Theorem 2, OPT can be divided into two cases.

Case 1 ($n^{\mathbb{1}} = 0$): In the case that each assignment $j \in J$ is checked by the critical checking probability η_j, with budget K_1, we have $\Phi_{\text{pasc}}(K_1) = \Phi_{\text{opt}}(K_1)$ by eliciting all peer-assignment pairs to be diligent. In the second stage with budget K_2, we have $\Phi_{\text{pasc}}(K_1 \oplus K_2) = \Phi_{\text{opt}}(K_1 \oplus K_2)$ by the ERP rule. Thus, we have $\Phi_{\text{opt}} = \Phi_{\text{opt}}(K_1) + \Phi_{\text{opt}}(K_1 \oplus K_2) = \Phi_{\text{pasc}}(K_1) + \Phi_{\text{pasc}}(K_1 \oplus K_2) = \Phi_{\text{pasc}}$.

On the other hand, if one assignment j^* that has the largest error rate is not checked by the critical checking probability, i.e., $x_{\text{opt}}(j^*) > \eta_{j^*}$, any other assignment $j \in J \backslash \{j^*\}$ is checked by the critical checking probability $x_j = \max_{i \in I(j, \mathcal{S}(\mathbf{x}_{\text{opt}}))} \theta_{ij}$. We further consider three cases according to the scale of the remaining budget K_2 in the second stage.

1) K_2 is tiny such that the assignment j^* cannot be checked by probability 1 in OPT. Then, we have that in OPT, only the assignment j^* has a larger checking probability than that in PASC, i.e., $x_{\text{opt}}(j^*) > x_{\text{pasc}}(j^*), \forall j \neq j^*, x_{\text{opt}}(j) \leq x_{\text{pasc}}(j)$. For this case, we have $\Phi_{\text{opt}} - \Phi_{\text{pasc}} \leq \frac{1}{n}$.
2) K_2 is moderate that can make j^* be checked by probability 1 in OPT but by less than probability 1 in PASC. Here, we have that $\Phi_{\text{pasc}}(K_1 \oplus K_2) \geq \Phi_{\text{opt}}(K_1 \oplus K_2)$ because PASC allocates the whole K_2 to the assignment j^* that has the largest error. Thus, for this case, we have $\Phi_{\text{opt}} - \Phi_{\text{pasc}} = \Phi_{\text{opt}}(K_1) + \Phi_{\text{opt}}(K_1 \oplus K_2) - (\Phi_{\text{pasc}}(K_1) + \Phi_{\text{pasc}}(K_1 \oplus K_2)) \leq 1/n$.
3) K_2 is large that can make j^* be checked by probability 1 in PASC, and OPT is checking another assignment j', where $x_{\text{opt}}(j') \geq \eta_{j'}$. For this case, we have that in OPT, only j' has a larger checking probability than that in PASC, i.e., $x_{\text{opt}}(x'_j) \geq x_{\text{pasc}}(x'_j) \forall j \neq j', x_{\text{opt}}(x_j) \leq x_{\text{pasc}}(x_j)$, which derives $\Phi_{\text{opt}} - \Phi_{\text{pasc}} \leq 1/n$.

Other cases with larger budget can be reduced to abovementioned three cases.

Case 2 ($n^{\mathbb{1}} > 0$): Similar to the Case 1 analysis, we can also derive that $\Phi_{\text{opt}} - \Phi_{\text{pasc}} \leq (1 + n^{\mathbb{1}})/n$.

Combining the abovementioned conclusion that $\Phi_{\text{opt}} - \Phi_{\text{pasc}} \leq 1 + n^{\mathbb{1}}/n$, $\Phi_{\text{pasc}}/\Phi_{\text{opt}} \geq K/K + n^{\mathbb{1}} + 1$ follows readily from Theorem 4. □

Complexity Analysis of Algorithm 1: In this section, we provide the time complexity of Algorithm 1.

Proposition 4: The complexity of Algorithm 1 is $O(|E|^2 + K \cdot n)$, where $|E| = l \cdot n$, indicating the number of edges in PGS $G = (I, J, E)$, $|I| = |J| = n$, l is the assignment load of peers, and K is the SC budget.

Proof: In the first diligent grading elicitation stage (Steps 2–6), in each iteration (Steps 3–6), the most cost-effective peer-assignment pair (i^*, j^*) with the most gain-cost ratio can be selected in $O(|E|)$. There are at most $|E|$ iterations, so the complexity of the first stage is $O(|E|^2)$. Next, in the remaining budget allocation stage (Steps 10 and 11), in each iteration, the assignment with the highest error rate can be selected in $O(n)$. There are at most K iterations, so the complexity of the second stage is $O(K \cdot n)$. This leads to a total of $O(|E|^2 + K \cdot n)$ operations for Algorithm 1. □

VI. Uncertainty About Reliability and Cost

So far, we have addressed the optimal SC problem with complete information where each peer i's reliability p^1_i and cost c_{ij} are known. We extend it to the incomplete information setting where p^1_i and c_{ij} are unknown. We mainly consider two kinds of uncertain settings.

1) The interval uncertainty, where $p^1_i \in [p^{1,\min}_i, p^{1,\max}_i]$ and $c_{ij} \in [c^{\min}_{ij}, c^{\max}_{ij}]$.

2) *Compete Uncertainty:* The instructor does not know p_i^1 and c_{ij}, neither their intervals, but we assume that the instructor knows the mechanism structural information, such as peers are reward sensitive, and there are two effort levels.

We extend the proposed PASC mechanism to address these two kinds of uncertainties, respectively.

A. Interval Uncertainty

The instructor's objective is to determine an SC policy, $\mathbf{x}$, that maximizes the accuracy rate $\Phi^*(\mathbf{x})$ over all of the possibilities that each p_i^1 and c_{ij} could be chosen from the defined intervals, formulated as follows:

$$\begin{aligned} &\max_{\mathbf{x}}\ \Phi^*(\mathbf{x}) \qquad (15)\\ &\text{s.t. } \Phi^*(\mathbf{x}) = \min_{\mathbf{p}^1,\mathbf{c}} \frac{\sum_j \left(1-(1-x_j)e^{-0.5\sum_{i\in I(j)}(2p_i^{e_{ij}}-1)^2}\right)}{n}\\ &p_i^{1,\min} \le p_i^1 \le p_i^{1,max}, c_{ij}^{\min} \le c_{ij} \le c_{ij}^{\max} \quad \forall i \in I, j \in J\\ &(6)--(8). \end{aligned}$$

We can convert this incomplete information problem (**IP**) to the equivalent complete information problem (**CP**) defined in (5) with $p_i^1 = p_i^{1,\min}$ and $c_{ij} = c_{ij}^{\max}$.

Theorem 6: Let $\mathbf{x}^*$ be the optimal SC policy of **CP**. Then, $\mathbf{x}^*$ is also the optimal SC policy of **IP**.

Proof: Assume that there is an alternative policy $\mathbf{x}'$, where $\Phi^*(\mathbf{x}') > \Phi^*(\mathbf{x}^*)$. For $\mathbf{x}'$, we have the following.

1) $\Phi^*(\mathbf{x}') = \sum_j \left(1-(1-x_j')e^{-0.5\sum_{i\in I(j)}(2p_i^{e_{ij}}-1)^2}\right)/n$, where $p_i^{e_{ij}} = p_i^{1,\min}$ if $x_j' \ge c_{ij}/r_{ij}$; $p_i^{e_{ij}} = p_i^0$ if $x_j' < \frac{c_{ij}}{r_{ij}}$. That means the minimal accuracy rate Φ^* under $\mathbf{x}'$ is that each peer i has the minimum reliability $p_i^1 = p_i^{1,min}$. This conclusion can be derived in Proposition 2 that each assignment j's error rate $\mathbb{P}_e^u(j, \mathbf{x})$ decreases with peer reliability.
2) $c_{ij} = c_{ij}^{\max}$, denoting i, has the maximum cost. Given an SC policy $\mathbf{x}$, each peers' reliability decreases with his cost, and assignments' error rate decreases with peer reliability. Thus, assignments' error rate decreases with peers' cost.

Derived from Proposition 2 that Φ increases with peers' reliability, we have that for two cost profiles $\mathbf{c} = (c_{ij})_{i\in I, j\in J}$ and $\mathbf{c}' = (c_{ij}')_{i\in I, j\in J}$, if $\mathbf{c} \prec \mathbf{c}'$, $\Phi^*(\mathbf{x}, \mathbf{c}) \ge \Phi^*(\mathbf{x}, \mathbf{c}')$. Since $\mathbf{x}^*$ is the optimal SC policy in **CP**, we have $\Phi^*(\mathbf{x}') \le \Phi^*(\mathbf{x}^*)$, which contradicts $\Phi^*(\mathbf{x}') > \Phi^*(\mathbf{x}^*)$. Thus, $\mathbf{x}^*$ is the optimal SC policy in both **CP** and **IP**. □

B. Complete Uncertainty

We extend the proposed PASC mechanism for the complete uncertain scenarios without knowing peers' reliability and cost, neither their intervals. The extended method can be called the exploration-exploitation SC mechanism. In terms of the exploration, we use the first ϵB SC budget to explore and elicit peers' reliability and cost, where B is the total budget. In terms of the exploitation, with the explored and estimated reliability and cost, PASC is exploited to optimize the evaluation accuracy using the remaining $(1-\epsilon)B$.

Algorithm 2 MAB-Based Reliability and Cost Exploration (ϵPASC)

Input : The PGS $G = (I, J, E)$ and arms $\mathcal{K} = \{1, 2, \ldots, T\}$, and the exploration budget $\tilde{B} = \epsilon B$.

Output: Each peer i's estimated diligent reliability p_i^1 and threshold checking probability θ_i.

1 **while** $\tilde{B} > 0$ **do**
2   Select m assignments $J_m = \{j_1, j_2, \ldots, j_m\}$ randomly (with the truth grading) and allocate J_m to each peer i;
3   **for** $\forall j \in J_m$ **do**
4   Select the arm k randomly for j and collect the grading z_{ij} of i on j;
5   Set $L(k) = L(k) + 1$ and $l_i(k) = l_i(k) + \mathbf{1}_{\{z_{ij}=q_j\}}$;
6   $\tilde{B} = \tilde{B} - m$;
7 **for** $\forall i \in I, k \in \mathcal{K}$ **do**
8   $p_i^1(k) = \frac{l_i(k)}{L(k)}$;
9 **for** $\forall i \in I$ **do**
10   Set max_r=0;
11   **for** $\forall k \in \mathcal{K}$ **do**
12   **If** $p_i^1(k) - p_i^1(k-1) > max_r$ **then**
      $max_r = p_i^1(k) - p_i^1(k-1)$, $p_i^1 = p_i^1(k)$ and $\theta_i = k$.

Multiarmed Bandit (MAB)-Based Reliability and Cost Exploration: We model the threshold SC probability $\theta_i = c_i/r_i$ of each peer i for exerting high effort as drawn from a distribution with cumulative distribution function $F(\theta_i)$ within $[0, 1]$, where c_i is the cost for diligent grading, and r_i is the known reward given for diligent grading behavior. Given the SC probability x, let $p_i(x) = P[z_{ij} = q_j]$ denote the probability of grading an assignment j correctly (where q_j is the truth grade of j and z_{ij} is the grade returned by i), which can be computed as $p_i(x) = F(x)p_i^1 + (1-F(x))p_i^0$ when $F(\theta_i)$ and p_i^1 are known. The main motivation behind the MAB-based exploration phase is that we would like to try different levels of SC probability x and search for the optimal level x^* that elicits the most evaluation accuracy with the least SC probability. This SC level x^* can be regarded as the peer i's threshold SC probability θ_i, his cost can be estimated by $c_i = r_i\theta_i$, and his reliability can be estimated by $p_i(x^*)$.

We would like to treat each checking probability level as an "arm" (as in standard MAB context) to explore with. Since we have a continuous space of checking level $(0, 1]$, we separate the support of checking probability $(0, 1]$ into $T = \{(0, 1/T], (1/T, 2/T], \ldots, (T-1/T, 1]\}$ finite intervals. Then, we treat each checking probability interval as an arm. For each interval, we take its right endpoint as the checking probability level to offer: $x_k = k/T$. Our goal is to select the best one of them with the maximized accuracy-checking probability ratio. The detail of the MAB-based reliability and cost exploration is shown in Algorithm 2.

In Algorithm 2, each round, we randomly select m gold-standard assignments J_m (which have been graded by the instructor) and allocate them to each peer i (step 2). In step 4,

Algorithm 3 Fair Partition for Large-Scale PGSs (FP-PGSs)

```
  Input  : The PGS G = (I, J, E) and reliability ranges R.
  Output: Subgroups G = {g_1, g_2, ..., g_m}.
1 Initialize p(g_j, r_k) = 0, ∀g_j ∈ G, r_k ∈ R;
2 for ∀i ∈ I do
3     Set k = arg_l{p_i^1 ∈ r_l};
4     Set min = ∞ and q = −1;
5     for ∀g_j ∈ G do
6         If p(g_j, r_k) < min, then min = p(g_j, r_k) and
          g_q = g_j;
7     Dispatch peer i to subgroup g_q, and
      p(g_q, r_k) = p(g_q, r_k) + 1.
```

we randomly choose an arm k for each assignment $j \in J_m$, i.e., j is checked with the probability x_k. In step 5, let $L(k)$ denote the number of arms k chosen and $l_i(k)$ denote the correct grades of peer i under the arm k (where the function $\mathbf{1}_{\{z_{ij}=q_j\}}$ returns 1 if $z_{ij} = q_j$; otherwise, return 0). After using the exploration budget ϵB (step 6), the estimated reliability under each arm k is estimated by $p_i^1(k) = l_i(k)/L(k)$ in step 8. Finally, in step 12, we determine the threshold checking probability θ_i as the arm that has the maximal marginal evaluation accuracy improvement, i.e., $\theta_i = k^*$, where $k^* = \arg\max_k p_i^1(k) - p_i^1(k-1)$, where $p_i^1(0) = p_i^0$. The corresponding estimated diligent reliability is $p_i^1 = p_i^1(k^*)$.

VII. Scalability on Large-Scale PGSs With Millions of Peers

To tackle the large-scale PGSs with millions of peers, we propose a scalable method by partitioning these peers into subgroups in a fair manner. Each subgroup can implement the proposed PASC effectively in parallel. In terms of fairness, we mean that peers' reliability distribution in each subgroup is similar. The detail of this fair partition is shown in Algorithm 3.

In Algorithm 3, for the input, we divide the reliability $[0.5, 1]$ into T discrete distributions $R = \{[0.50, 0.50 + 0.50 \times (1/T)), [0.50 + 0.50 \times (1/T), 0.50 + 0.50 \times (2/T)), \ldots, [0.50 + 0.50 \times (T-1/T), 1]\}$. The output is the partitioned m subgroups $G = \{g_1, g_2, \ldots, g_m\}$. For each peer i, let $r_k \in R$ denote the range that his diligent reliability belongs to, i.e., $p_i^1 \in r_k$ (step 3). Let $p(g_j, r_k)$ denote the number of peers in subgroup g_j whose reliability belongs to r_k (step 1). In steps 5 and 6, the subgroup g_q with the minimal number of peers whose reliability distributed in the range r_k is selected. To guarantee the fairness that each subgroup has the similar reliability distribution, we always dispatch the peer i to such a minimal subgroup g_q (step 7). After the partition of peers into m subgroups, each subgroup implements the proposed PASC in parallel with the budget m/B. For Algorithm 3, we can conclude the following.

1) For any range $r_k \in R$, and any two subgroups g_p and g_q, the difference between $p(g_p, r)$ and $p(g_q, r)$ is at most 1, i.e., $\forall g_p, g_q \in G, r \in R, \mid p(g_p, r) - p(g_q, r) \mid \leq 1$.
2) For each peer i, it needs to take T computation time to find his reliability range and m computation time to find the minimal subgroup. Since there are n peers, the time complexity of Algorithm 3 is $O(n(T+m)) = O(nm)$, where $T \geq m$. Thus, the time complexity of returning the SC solution of the extended PASC is $O(nm + (n/m)^2)$ ($n/m$is the number of peers in each subgroup), which reduces significantly compared with original PASC's $O(n^2)$ time complexity.

VIII. Experimental Evaluation

We experimentally verify the evaluation accuracy and scalability of the proposed algorithm on synthetic and real data sets. All computations are performed on a 64-bit PC with a dual-core 3.2-GHz CPU and 16-GB memory. All results are averaged over 500 instances.

A. Experiment on Synthetic Data Set

There are 10 000 peers and 10 000 assignments. For each peer i, his diligent reliability follows the Gaussian distribution $N(\mu, \delta^2)$, where $\mu = 0.75$ and $\delta = 0.125$. We allocate each assignment j to l peers randomly. The cost c_{ij} and reward r_{ij} follow the uniform distributions $U(0, 1)$ and $U(c_{ij}, 1)$.

We compare our **PASC** algorithm with three algorithms.

1) **Random**, where budget K is allocated to n assignments randomly, i.e., choose one assignment j and allocate a random probability $x_j \leq 1$ to j.
2) Assignment-oriented SC algorithm (ASC), where the most cost-effective assignment (with the critical SC probability η_j) is selected and a budget of η_j is allocated to elicit diligent peer grading.
3) Assignment allocation first (AAF), where we first partition these 10 000 students into 2500 groups uniformly, each group has the equal-sized four members. Each assignment is randomly allocated to $\frac{l}{4}$ groups of these 2500 groups. After this assignment allocation, the uniform SC policy is exploited, where the budget is allocated to the assignments uniformly.

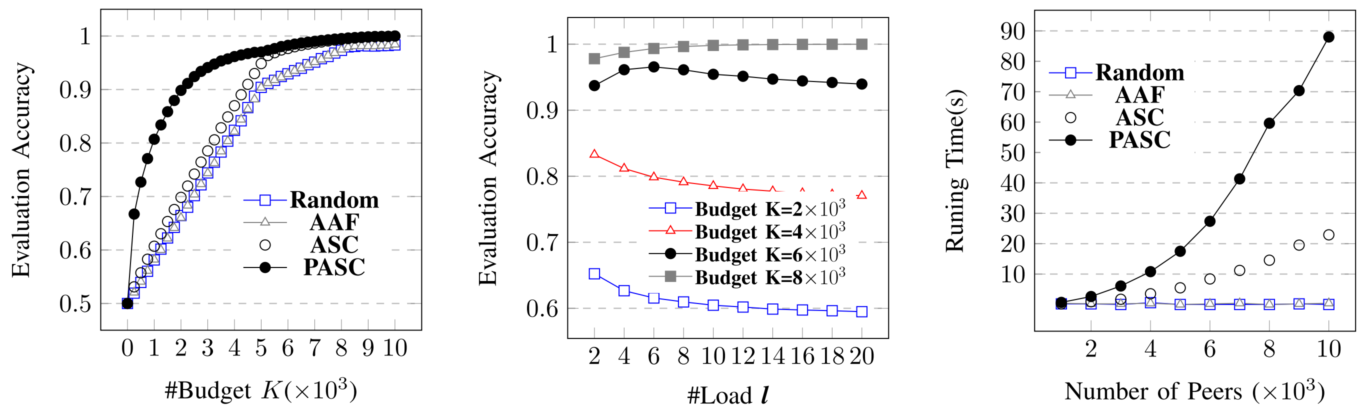
Evaluation Accuracy: Table II shows the evaluation accuracy rate under various budgets K and loads l, from which we observe that: 1) given any load and budget, PASC always produces the largest accuracy rate; 2) given the light load (i.e., $l = 4$) and any budget, the accuracy rate of PASC is followed by ASC, AAF, and Random, while given the heavy load (i.e., $l > 4$) and any budget, the accuracy rate of PASC is followed by AAF, ASC, and Random; 3) given any budget, accuracy rates of PASC, AAF, and Random increase with load, while the accuracy rate of ASC decreases with load; and 3) given a load, these algorithms' accuracy rates increase with the budget, and the increment becomes smaller with the increase of budget.

Fig. 2(a) shows the convergence of evaluation accuracy rate on a budget, from which we observe that when the budget K is smaller than the critical budget $K_c = \sum_j \eta_j$, PASC performs much better than ASC, AAF, and Random. When $K > K_c$, PASC and ASC produce the same accuracy rate and converge to the optimal accuracy.

TABLE II

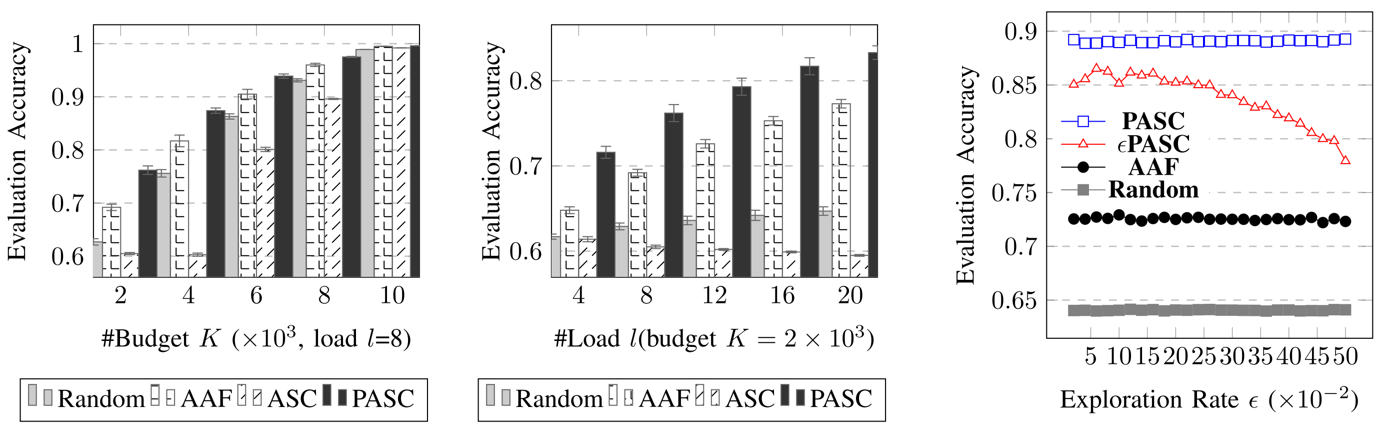
EVALUATION ACCURACY ON SYNTHETIC DATA SET. EACH CELL IS STATISTICALLY SIGNIFICANT AT 95% CONFIDENCE LEVEL

Budget K	$2{\times}10^3$				$4{\times}10^3$				$6{\times}10^3$				$8{\times}10^3$			
Load l	4	8	12	16	4	8	12	16	4	8	12	16	4	8	12	16
PASC	**0.804**	**0.875**	**0.920**	**0.952**	**0.894**	**0.956**	**0.978**	**0.989**	**0.954**	**0.986**	**0.994**	**0.997**	**0.991**	**0.998**	**0.999**	**0.999**
ASC	0.715	0.695	0.687	0.682	0.856	0.871	0.862	0.855	0.954	0.986	0.993	0.997	0.990	0.998	0.997	0.999
AAF	0.699	0.768	0.824	0.866	0.835	0.903	0.941	0.964	0.918	0.964	0.980	0.991	0.959	0.982	0.991	0.995
Random	0.634	0.650	0.663	0.670	0.767	0.802	0.822	0.841	0.870	0.908	0.928	0.942	0.943	0.963	0.974	0.981



(a) Convergence on budget, load l=8. (b) The evaluation accuracy of ASC on load. (c) Running time on number of peers, l=8.

Fig. 2. Properties of algorithms. (a) Convergence on budget. (b) Effect of load on ASC. (c) Running time. (a) Convergence on budget, load $l = 8$. (b) Evaluation accuracy of ASC on load. (c) Running time on number of peers, $l = 8$.



(a) (b) (c)

Fig. 3. Robustness on uncertain reliability and cost. (a) Interval uncertainty on budget. (b) Interval uncertainty on load. (c) Evaluation accuracy on exploitation rate ϵ.

TABLE III

EFFECT OF THE NUMBER OF SUBGROUPS ON LARGE-SCALE PGSS WITH RESPECT TO EVALUATION ACCURACY AND RUNNING TIME. EACH CELL IS AVERAGED OVER 20 INSTANCES AND STATISTICALLY SIGNIFICANT AT 95% CONFIDENCE LEVEL

No. of Subgroups m	10^3		10^4		10^5	
Comparison Metrics	Evaluation Accuracy	Running Time(s)	Evaluation Accuracy	Running Time(s)	Evaluation Accuracy	Running Time(s)
PASC	0.807	$1.261{\times}10^4$	0.806	$1.258{\times}10^4$	0.806	$1.263{\times}10^4$
Group_PASC	0.806	1.2	0.805	3.6	0.798	28.1
AAF	0.711	0.02	0.712	0.02	0.711	0.02
Random	0.641	0.05	0.641	0.06	0.642	0.06

Running Time: Fig. 2(c) shows the running time (in seconds) on the number of peers. We can find that: 1) the running time increases with the number of peers and 2) for the large-scale scenarios with 10 000 peers, our proposed algorithm PASC needs to take nearly 90 s to return the solution, and ASC takes about 20 s, while the random and AAF can return the solution within 1 s. Compared with the advantage of PASC on improving evaluation accuracy by 10% over other algorithms, 90 s are worth of waiting.

Robustness on Uncertainty of Reliability and Cost: The instructor might have uncertain information on peers' reliability p_i^1 and cost c_{ij}.

1) For the interval uncertainty where $p_i^1 \in [\tilde{p}_i^1 - \delta_p, \tilde{p}_i^1 + \delta_p]$ and $c_{ij} \in [\tilde{c}_{ij} - \delta_c, \tilde{c}_{ij}^1 + \delta_c]$, $\tilde{p}_i^1$ and $\tilde{c}_{ij}$ are observed values. δ_p and δ_c are noise parameters, where $\delta_p = \tilde{p}_i^1/10$ and $\delta_c = \tilde{c}_{ij}/10$. We compare algorithm's worst case accuracy rate in uncertain settings defined in (15). From Fig. 3(a) and (b), we observe that PASC always produces the largest accuracy rate under the interval uncertainty scenarios.
2) Fig. 3(c) shows the effect of exploration rate ϵ on evaluation accuracy, where ϵPASC is the extended PASC for reliability and cost exploration (i.e., Algorithm 2). In Fig. 3(c), there are 10 000 peers whose truth reliability follows the Gaussian distribution, the total budget used for SC is 2000, and the load $l = 8$. From Fig. 3(c), we can observe that compared with the Random and AAF without knowing peers' reliability and cost information, using the first ϵB budget to explore these information is beneficial to improve evaluation accuracy; evaluation accuracy of ϵPASC decreases with ϵ. This can be explained by the fact that the more the budget (i.e., larger ϵ) used for reliability and cost exploration, the less the budget remained for exploitation in PASC. Moreover, $\epsilon \leq 0.1$ might be a good option for the tradeoff between reliability exploration and exploitation.

Scalability on Large-Scale PGSs With Millions of Peers: Table III shows the evaluation accuracy and running time with 10^6 peers, where Group_PASC is the extended PASC in which these millions of peers are first partitioned into subgroups fairly. The total budget used for SC is $B = 2 \times 10^5$, the load $l = 8$, and the reliability range [0.5, 1] is divided into 50 discrete ranges $R = \{(0.50, 0.51], (0.51, 0.52], \ldots, (0.99, 1]\}$. For such m subgroups, each subgroup has B/m budget. From Table III, we can find that partitioning these millions of peers into subgroups and each subgroup implements PASC in parallel can significantly reduce running time but loses the limited evaluation accuracy. For example, by partitioning these peers into 10^3 subgroups, the SC solution can be returned by Group_PASC within several seconds, while there is only 0.1% percentage loss of evaluation accuracy compared with PASC. The more the subgroups partitioned, the lower the evaluation accuracy, e.g., the evaluation accuracy of 10^5 subgroups is less the evaluation accuracy of 10^3 subgroups. This can be explained by the fact that more fine-grained partitions (i.e., m is larger) will cause the budget B/m allocated to each subgroup more fragmented and inefficiently.

Discussion: We show that PASC, AAF, and Random perform better with the increase of load. This is because, for PASC, AAF, and Random, larger load indicates that assignments have a higher possibility to be graded by peers with higher reliability. However, for ASC, when each assignment j has a large load, more critical checking budget η_j is required, but such an incremental budget cannot proportionally improve accuracy due to the submodular property. Fig. 2(b) shows the evaluation accuracy of ASC on load. In Appendix B, we also test the evaluation accuracy when peers reliability follows the uniform distribution U(0.75,0.125), in which we observe similar results as that in the Gaussian distribution.

For the scenarios when $K > K_c$, PASC and ASC produce the same accuracy rate and converge to the optimal accuracy. This is because when $K > K_c$, all peers can be elicited to be diligent in both PASC and ASC. In terms of the large-scale scenarios with millions of peers, to make PASC practical with respect to running time, one potential method is to partition these millions of peers into smaller subgroups first (e.g., each group includes thousands of peers). The group should be fairly partitioned such that the reliability distribution of peers is similar.

B. Experiment on Real Data Set

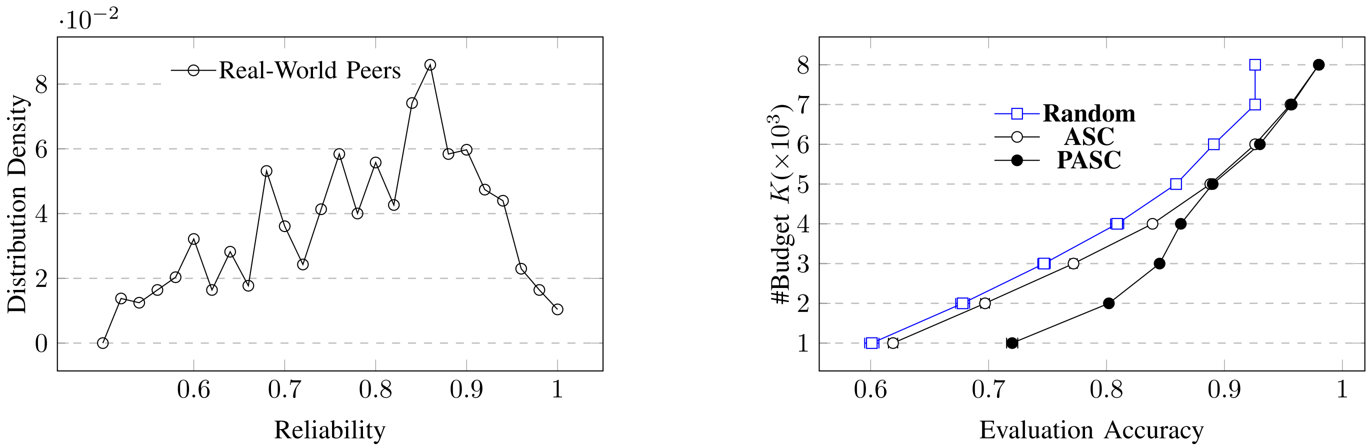
Data Set: TREC[2] is a collection of topic-document relevance judgments labeled by workers on AMT. This data sets data structure is similar to PGS's, where each worker (i.e., peer) is asked to judge whether a topic document (i.e., assignment) is relevant (i.e., good) or not (i.e., bad). Each assignment has a true answer. This data set contains 9161 judgments collected from 763 workers.

Results: We first use l_{tra} training tasks to calibrate worker reliability. For each worker i who judges l_i^{cor} correct labels among l_{tra} tasks, his reliability is estimated as $p_i^1 = l_i^{\text{cor}}/l_{\text{tra}}$. We model worker's cost and reward in a similar way with that in Section VIII-A. We compute the workers' average reliability of 0.82 and variance of 0.17. Under the SC mechanism, a worker–task pair (i, j) that is elicited to be diligent, we directly use i's label in the data set as i's judgment; otherwise, i reports a random judgment on j. Finally, we use the WMV to aggregate the estimated judgment. Fig. 4(a) depicts the reliability distribution of these 763 peers in the real-data set, which can be fit by a Gaussian distribution.

Fig. 4(b) shows the evaluation accuracy in real data set (in real data set, the task allocation has been determined; thus, AAF is unnecessary), from which we observe that: 1) PASC performs the best on improving evaluation accuracy and 2) for PASC and ASC, before the critical budget point ($K \leq 5 \times 10^3$), their increment rates drop (i.e., the gradient increases) with the budget, while exceeding the critical budget, their increment rates goes up again.

Discussion: In the real data set, peers have high average reliability accuracy (~0.82). Even with the limited budget, these highly reliable peers can be elicited to be diligent, leading to a high base accuracy. When the budget becomes moderate, such an incremental budget can only improve limited accuracy due to the submodular property and the high base accuracy. Finally, when the budget becomes so large that it exceeds the critical budget, all peers will be diligent. The remaining budget will be allocated to the assignment that has the largest error rate, thereby improving the increment rate again. Although the fit Gaussian distribution of peers reliability might not have the same average and variance with the statistical results (e.g., for this real data set, the fit Gaussian distribution might be $\mathcal{N}(0.86, 0.14)$), it is practical of modeling peers' reliability to follow a Gaussian distribution.

[2]https://sites.google.com/site/treccrowd/



(a) (b)

Fig. 4. Results on real data set. (a) Reliability distribution of peers in the real data set. (b) Evaluation accuracy in real data set with respect to the budget.

IX. Conclusion

This article studies the problem OptSC of optimal SC assignments to maximize assignments evaluation accuracy in general PGSs. The NP-hardness complexity of OptSC is analyzed. A combinational optimization problem OptSC_PA is proposed to approximate OptSC. The monotone and submodular properties of OptSC_PA are exploited, and an efficient SC approximation algorithm is proposed. Experimental results show that in both syntectic and real data sets, the proposed algorithm achieves higher evaluation accuracy than other benchmark algorithms.

There are mainly three limitations of the proposed approach: 1) for the scenarios where there are no prior knowledge peers' reliability and cost, the proposed SC mechanism would not have any desirable theoretical guarantee on estimation accuracy; 2) for the large-scale applications where there are millions of peers, it will take intolerable running time to return the solution; and 3) peers can be bounded rationality that has only partial knowledge about their utilities and, not always, are capable of effectively optimizing their utilities. These limitations can lead to several interesting topics for future work. First, the assumptions of knowing peers' reliability, cost, and peers' perfect rationality can be relaxed. Novel online learning-based SC mechanisms by learning peers' reliability, cost, and rationality level should be developed. Second, partitioning millions of peers into small-scale subgroups (e.g., each group includes thousands of peers) might a potential option for large-scale applications, and how to group these peers fairly should be cleverly designed. Third, the diligent peer can be modeled as the instructor and for the scenarios where is not enough budgets, utilizing the peer-assignment allocation network by comparing diligent peers' assignments with other unknown peers might a good option to improve evaluation accuracy.

Appendix A
Proof of Proposition 1

Proof: Define $\gamma_{ij} = w_{ij}z_{ij}$, where $w_{ij} = 2p_{ij} - 1$ is the weight of peer i's grade on assignment j and $p_{ij} \in \{p_i^0, p_i^1\}$. Let $\xi_j^+ = \mathbb{E}^+(\sum_{i\in I(j)} w_{ij}z_{ij}) = \sum_{i\in I(j)} w_{ij}\mathbb{E}(z_{ij}|q_j = 1) = \sum_{i\in I(j)} w_{ij}(1\cdot p_{ij} + (-1)\cdot(1 - p_{ij})) = \sum_{i\in I(j)} w_{ij}(2p_{ij} - 1)$ denote the expected value aggregated by WMV when the true value of j is 1. Similarly, let $\xi_j^- = \sum_{i\in I(j)} w_{ij}\mathbb{E}(z_{ij}|q_j = -1) = -\sum_{i\in I(j)} w_{ij}(2p_{ij}-1)$ denote the expected value if the true value of j is -1. Let $\beta_j^+ = \mathbb{P}(\sum_{i\in I(j)} w_{ij}z_{ij} \geq 0|q_j = 1)$ denote the expected accuracy rate of WMV if the true value of j is 1. This accuracy rate satisfies

$$\begin{aligned}\beta_j^+ &= \mathbb{P}\Big(\sum_{i\in I(j)} w_{ij}z_{ij} \geq 0|q_j = 1\Big)\\ &= \mathbb{P}\Big(\sum_{i\in I(j)} w_{ij}z_{ij} - \mathbb{E}^+\Big[\sum_{i\in I(j)} w_{ij}z_{ij}\Big] \geq -\xi_j^+\Big)\\ &\geq 1 - e^{\frac{-2(\xi_j^+)^2}{\sum_{i\in I(j)}(w_{ij} - (-w_{ij}))^2}}\\ &= 1 - e^{\frac{-(\xi_j^+)^2}{2\sum_{i\in I(j)} w_{ij}^2}}.\end{aligned} \tag{16}$$

The inequality is derived from the Hoeffding inequality

$$\begin{aligned}&\mathbb{P}\Big(\sum_{i\in I(j)} w_{ij}z_{ij} - \mathbb{E}^+\Big[\sum_{i\in I(j)} w_{ij}z_{ij}\Big] \geq -\xi_j^+\Big)\\ &\quad= 1 - \mathbb{P}\Big(\sum_{i\in I(j)} w_{ij}z_{ij} - \mathbb{E}^+\Big[\sum_{i\in I(j)} w_{ij}z_{ij}\Big] < -\xi_j^+\Big)\\ &\quad\geq 1 - e^{\frac{-2(\xi_j^+)^2}{\sum_{i\in I(j)}(b_i - a_i)^2}}\end{aligned}$$

where the value $w_{ij}z_{ij}$ distributes in the range $[a_i, b_i] = [-w_{ij}, w_{ij}]$ uniformly [55].[3] Similarly, we can derive that the expected accuracy rate of WMV if the true value of j is -1 satisfies

$$\begin{aligned}\beta_j^- &= \mathbb{P}\Big(\sum_{i\in I(j)} w_{ij}z_{ij} < 0|q_j = -1\Big)\\ &= \mathbb{P}^+\Big(\sum_{i\in I(j)} w_{ij}z_{ij} - \mathbb{E}^-\Big[\sum_{i\in I(j)} w_{ij}z_{ij}\Big] < -\xi_j^-\Big)\\ &\geq 1 - e^{\frac{-(\xi_j^-)^2}{2\sum_{i\in I(j)} w_{ij}^2}}.\end{aligned} \tag{17}$$

[3]To make the Hoeffding inequality hold, the value of $\xi_j^+ \geq 0$, and in the later, we will set the weight w_{ij} to ensure that this nonnegative condition holds.

TABLE IV

EVALUATION ACCURACY ON SYNTHETIC DATA SET WHERE PEERS' RELIABILITY FOLLOWS THE UNIFORM DISTRIBUTION U(0.75, 0.125). EACH CELL IS STATISTICALLY SIGNIFICANT AT 95% CONFIDENCE LEVEL

Budget K	2×10^3				4×10^3				6×10^3				8×10^3			
Load l	4	8	12	16	4	8	12	16	4	8	12	16	4	8	12	16
PASC	**0.873**	**0.934**	**0.961**	**0.976**	**0.948**	**0.81**	**0.992**	**0.996**	**0.977**	**0.992**	**0.997**	**0.997**	**0.993**	**0.998**	**0.999**	**0.999**
ASC	0.722	0.702	0.692	0.687	0.893	0.876	0.866	0.859	0.977	0.992	0.997	0.993	0.994	0.998	0.999	0.999
AAF	0.791	0.874	0.917	0.944	0.913	0.961	0.981	0.990	0.964	0.989	0.995	0.998	0.982	0.994	0.998	0.999
Random	0.653	0.670	0.675	0.680	0.808	0.843	0.859	0.865	0.913	0.944	0.959	0.967	0.966	0.981	0.989	0.991

Finally, we have the expected accuracy rate of WMV on j, and β_j satisfies

$$\begin{aligned}\beta_j &= \mathbb{P}(\tilde{q}_j = q_j)\\ &= \mathbb{P}(q_j = 1, \tilde{q}_j = 1) + \mathbb{P}(q_j = -1, \tilde{q}_j = -1)\\ &= \mathbb{P}(q_j = 1)\mathbb{P}\Big(\sum\nolimits_{i\in I(j)} w_{ij} z_{ij} \geq 0\Big)\\ &\quad +\mathbb{P}(q_j = -1)\mathbb{P}\Big(\sum\nolimits_{i\in I(j)} w_{ij} z_{ij} < 0\Big)\\ &= \mathbb{P}(q_j = 1)\beta_j^+ + \mathbb{P}(q_j = -1)\beta_j^-.\end{aligned} \tag{18}$$

Substituting β_j^+ by inequality (16) and β_j^- by inequality (17), we have

$$\begin{aligned}\beta_j &\geq \mathbb{P}(q_j = 1)\left(1 - e^{\frac{-(\xi_j^+)^2}{2\sum_{i\in I(j)} w_{ij}^2}}\right)\\ &\quad +\mathbb{P}(q_j = -1)\left(1 - e^{\frac{-(\xi_j^-)^2}{2\sum_{i\in I(j)} w_{ij}^2}}\right)\\ &= 1 - e^{\frac{-\left(\sum_{i\in I(j)} w_{ij}(2p_{ij}-1)\right)^2}{2\sum_{i\in I(j)} w_{ij}^2}}.\end{aligned} \tag{19}$$

The last equality holds because $(\xi_j^+)^2 = (\xi_j^-)^2 = (\sum_{i\in I(j)} w_{ij}(2p_{ij} - 1))^2$. Thus, the expected error rate $\mathbb{P}(q_j \neq \tilde{q}_j)$ of each assignment j by the WMV aggregation method satisfies

$$\mathbb{P}_e(j) = \mathbb{P}(q_j \neq \tilde{q}_j) \leq e^{\frac{-(\sum_{i\in I(j)} w_{ij}(2p_{ij}-1))^2}{2\sum_{i\in I(j)} w_{ij}^2}}. \tag{20}$$

Substituting w_{ij} by $2p_{ij} - 1$, we have

$$\mathbb{P}_e(j) = \mathbb{P}(q_j \neq \tilde{q}_j) \leq e^{\frac{-\sum_{i\in I(j)}(2p_{ij}-1)^2}{2}}. \tag{21}$$

Equation (21) concludes this proposition. □

APPENDIX B
TEST OF EVALUATION ACCURACY WITH UNIFORM PEERS RELIABILITY DISTRIBUTION

From Table IV, we can find the similar observation with the Gaussian distribution, where: 1) PASC has the largest accuracy rate, which is followed by AAF, ASC, and Random; 2) accuracy rates of PASC, AAF, and Random increase with load, while the accuracy rate of ASC decreases with load; and 3) algorithms' accuracy rates increase with the budget, and the increment becomes smaller with the increase of budget. The potential reason that AAF always performs better than the ASC in the uniform scenario is that the uniform reliability distribution of peers might be more suitable for the uniform budget allocation mechanism that is incorporated in the AAF.

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This article is an extension of work [1]. The authors have extended the article in the following ways: 1) the original article focused on the interval uncertain settings where each peer's reliability and cost are uncertain within an interval; They have extended it to the complete uncertain settings, where the instructor does not know peers' reliability and cost, neither their intervals, but we assume that the instructor knows the mechanism structural information, such as peers are reward sensitive, and there are two effort levels; 2) the original article mainly focused on PGSs with only 10 000 peers; they have extended it to the large-scale PGSs with millions of peers; 2) they have extended the experiment settings to large-scale scenarios with millions of peers; and 4) they have extended the experiment settings to complete uncertain scenarios where peers' reliability and cost are unknown, neither their intervals.

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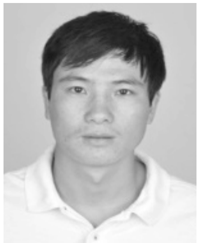

Wanyuan Wang received the Ph.D. degree in computer science from Southeast University, Nanjing, China, in 2016.

He is currently an Assistant Professor with the School of Computer Science and Engineering, Southeast University. He has published several articles in refereed journals and conference proceedings, such as the IEEE TRANSACTIONS, Association for the Advancement of Artificial Intelligence (AAAI), and International Conference on Autonomous Agents and MultiAgent Systems (AAMAS). His main research interests include artificial intelligence, multiagent systems, and game theory.

Dr. Wang received the Best Student Paper Award from ICTAI14.

Bo An received the Ph.D. degree in computer science from the University of Massachusetts at Amherst, Amherst, MA, USA.

He is currently a President's Council Chair Associate Professor of computer science and engineering with Nanyang Technological University, Singapore. He has published over 100 referred articles at the International Conference on Autonomous Agents and MultiAgent Systems (AAMAS), International Joint Conference on Artificial Intelligence (IJCAI), the Association for the Advancement of Artificial Intelligence (AAAI), International Conference on Automated Planning and Scheduling (ICAPS), ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD), Conference on Uncertainty in Artificial Intelligence (UAI), ACM Conference on Economics and Computation (EC), International Conference on World Wide Web Companion (WWW), Conference on Learning Representations (ICLR), Annual Conference on Neural Information Processing Systems (NeurIPs), *Journal of Autonomous Agents and Multi-Agent Systems* (JAAMAS), *Journal of Artificial Intelligence* (AIJ), and ACM/IEEE TRANSACTIONS. His current research interests include artificial intelligence, multiagent systems, computational game theory, reinforcement learning, and optimization. His research results have been successfully applied to many domains including infrastructure security and e-commerce.

Dr. An is also a member of the Editorial Board of the *Journal of Artificial Intelligence Research* (JAIR) and the Associate Editor of JAAMAS, the IEEE INTELLIGENT SYSTEMS, and the *ACM Transactions on Intelligent Systems and Technology* (TIST). He was elected to the board of directors of the International Foundation for Autonomous Agents and Multiagent Systems (IFAAMAS) and a Senior Member of AAAI. He was a recipient of the 2010 IFAAMAS Victor Lesser Distinguished Dissertation Award, the Operational Excellence Award from the Commander, the First Coast Guard District of the United States, the 2012 INFORMS Daniel H. Wagner Prize for Excellence in Operations Research Practice, and the 2018 Nanyang Research Award (Young Investigator). His publications won the Best Innovative Application Paper Award at AAMAS'12 and the Innovative Application Award at IAAI'16. He was invited to give Early Career Spotlight talk at IJCAI'17. He led the team HogRider that won the 2017 Microsoft Collaborative AI Challenge. He was named to the IEEE INTELLIGENT SYSTEMS' "AI's 10 to Watch" list for 2018. He is also the PC Co-Chair of AAMAS'20.

Yichuan Jiang (Senior Member, IEEE) received the Ph.D. degree in computer science from Fudan University, Shanghai, China, in 2005.

He is currently a Full Professor with the School of Computer Science and Engineering, Southeast University, Nanjing, China. He has published more than 90 scientific articles in refereed journals and conference proceedings, such as the IEEE TRANSACTIONS ON PARALLEL AND DISTRIBUTED SYSTEMS, the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, the IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS—PART A: SYSTEMS AND HUMANS, the IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS—PART C: APPLICATIONS AND REVIEWS, the IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS: SYSTEMS, the IEEE TRANSACTIONS ON CYBERNETICS, the *ACM Transactions on Autonomous and Adaptive Systems*, the *Journal of Autonomous Agents and Multi-Agent Systems*, the *Journal of Parallel and Distributed Computing*, IJCAI, AAAI, and AAMAS. His main research interests include multiagent systems, social computing, and social networks.

Dr. Jiang received the Best Paper Award from PRIMA06 and the Best Student Paper Awards twice from ICTAI13 and ICTAI14.