Toward Efficient City-Scale Patrol Planning Using Decomposition and Grafting

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***Abstract*— Motivated by the increasing need of the real-world patrolling, this paper studies a practical city-scale patrolling (CSP) variant. In CSP, the police are scheduled to patrol city regions, and the objective is not only to protect public security but also to respond to incidents timely. We use an integer program (IP) to formulate the CSP problem, with the objective of maximizing the police visibility rate (PVR) to improve public safety and the additional constraint of response time guarantee to handle incidents timely. For such an NP-hard problem, existing studies either cannot scale-up or do not provide a bound from optimum. To fill the research gap, we propose a decomposition and grafting approach. We first decompose the original CSP into two weakly-coupled subproblems, minimizing police problem (MinP) and maximizing PVR (MaxP) problem. By exploiting the subproblem structures, a polynomial time approximation algorithm is proposed for MinP, and a polynomial time optimal algorithm is proposed for MaxP. We prove that such a decomposition can provide the $1-\alpha$ approximation ratio, where α is the percentage of the police used in MinP. To further improve patrolling efficiency, a grafting mechanism is proposed to integrate the two subproblems' solutions. Finally, we conduct extensive experiments on the real dataset of Foshan, a modern Chinese city. The results demonstrate that compared with benchmarks, our approach scales well to city-scale problem instances with fine-grained periods, hundreds of regions, and hundreds of police officers.**

***Index Terms*— Patrol planning, city-scale, decomposition, grafting, approximation algorithm.**

I. Introduction

DURING the development of a modern city, police patrolling plays an essential role in improving the public safety and security of the city. Empirical studies have shown that the presence of police can significantly improve people's feelings of safety (FoS) [1]. For example, Fig.1 shows the association between police presence and people's FoS. From Fig. 1, we can find that the presence of police, *no matter how many*, can improve people's FoS significantly [1]. On the other hand, the emergency incidents such as criminal and traffic accidents are time sensitive, and the response time of the incidents (defined as the interval from the time when the incident occurs to the arrival of a police officer) should be within time guarantee [2], [3].

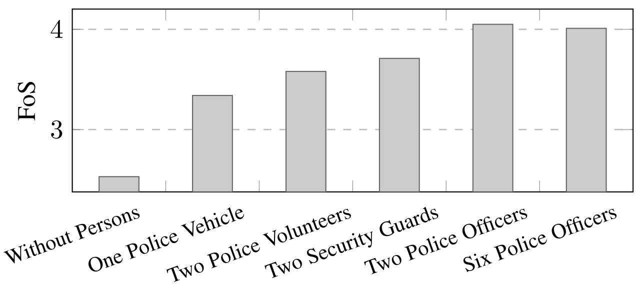

Fig. 1. Empirical evidence of police presence on the improvement of FoS [12].

In general, the Municipal Public Security Bureau (MPSB) wishes to schedule the police such that 1) when an incident occurs, there should be police nearby that can respond to it within a threshold time, and 2) when there are no incidents, the police should patrol in disperse to improve the FoS of the whole city. However, the MPSBs of many cities (e.g., the MPSB of Foshan, a typical modern city in China) still rely on static patrolling plans that assign police officers to specific regions. Since the police's main task is to patrol the assigned regions, they only patrol the assigned regions and seldom communicate with each other for coordination. For example, in Fig. 2a, the police always patrol several specific important checkpoints (labeled by ★), such as government departments, shopping malls and schools. However, the incidents (labeled by •) do not spread evenly and could not be fully covered by these fixed checkpoints. Moreover, in the mobile environments, the volume flow of people (VFoP) varies with time, e.g., schools are more crowded in the morning and afternoon, while shopping malls are more crowded during the evening. Fig. 2b shows the VFoP distributions at 16:00 and 20:00. Thus, existing static checkpoints-oriented patrol planning is inefficient to handle the VFoP dynamics and cannot respond to incidents timely.

The patrolling problem has been studied in many domains. For example, the police are routed to travel the network of regions efficiently in the patrol routing domain [4]–[9], the equilibrium patrolling strategy is designed in the security

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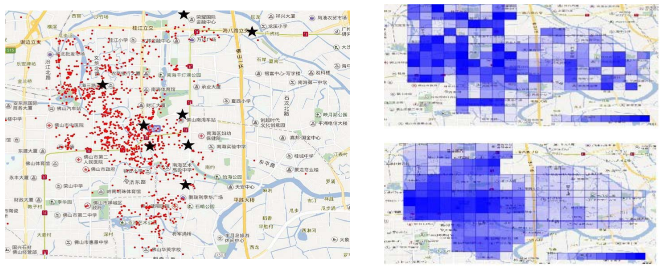


(a) Static Patrolling Policy

(b) Dynamics of VFoP

Fig. 2. Real World Data: (a) ★ indicates static checkpoints and • indicates incident distributions; (b) the top indicates the VFoP at 16:00 and the bottom indicates the VFoP at 20:00.

game domain [10]–[13], the scalable [14] and robust methods [15], [16] are proposed in the multi-robot domain, and the police are optimally allocated to regions [17]–[20] and dispatched to incidents [3], [21]–[23] in the emergency response domain. While the topic has been extensively studied, we provide some new insights that may suggest to reassess practical city police patrolling scenarios. For example, when the incidents occur, there should be police nearby that can respond within a threshold time, and when there are no incidents, they should be police patrolling around the city to enhance the police visibility rate (PVR) and improve public safety [3], [17]. This practical requirement makes 1) existing exact algorithms [8], [22]–[24] cannot apply to city-scale instances with fine-grained periods, hundreds of regions and hundreds of police officers, and 2) existing scalable algorithms [6], [7], [14], [19], [25] cannot establish a bound from optimum in terms of PVR. Moreover, the security game-based randomized policy [26], [27] cannot satisfy the hard constraint of response time guarantee.

To address the above issues, our first contribution is to propose an integer program (IP) to model the city-scale patrolling (CSP) problem of maximizing PVR with the constraint of incident response time guarantee. To solve such an NP-hard problem efficiently, our second contribution is the decomposition and grafting method. The original CSP is decomposed into two weakly-coupled subproblems. A polynomial time approximation and a polynomial time optimal algorithm are proposed for the decomposed subproblems respectively. The proposed decomposition is proved to achieve $1-\alpha$ approximation ratio with respect to the optimum, where α is the percentage of police used in MinP. To improve the PVR further, a grafting mechanism is proposed to integrate the two subproblems' solutions. Finally, we conduct extensive experiments on a real-world patrolling dataset of Foshan, a modern city in China. The experimental results show the advantages of the proposed algorithm over other benchmarks on scalability and solution quality.

The remainder of this paper is organized as follows. In Section 2, we provide a brief review of the literature on patrolling. In Section 3, we formulate the real-world CSP problem with the objective of PVR minimization and the constraint of incident response time guarantee. In Section 4, we propose an efficient decomposition and grafting-based approximation algorithm. In Section 5 we conduct a series of simulation experiments on real-world datasets to validate the proposed algorithm's effectiveness in maximizing PVR. Finally, we conclude our paper and discuss future work in Section 6.

II. Related Work

A. Police Resource Allocation

Curtin *et al.* [24] determine the optimal patrolling areas by a Integer Program (IP) to maximize incident coverag. Mukhopadhyay *et al.* [18], [19] study online police resource allocation variants for incident coverage where incidents arrive stochastically, depots are allocated in space, response vehicles are allocated in depots, and vehicles are assigned to incidents. In the Emergency Management Systems (EMSs) domain, base stations are optimally placed and emergency response resources are optimally allocated to these base stations [28]. EMSs work by first sampling a set of incident requests from historical data [23] and allocating emergency resources efficiently using operational research techniques, such as submodular maximization of satisfied incidents [17], Lagrangian relaxation [2], [25] and constraint programming [3]. Compared with these static police resource allocation algorithms without any PVR guarantee, we propose a mobile patrolling model in which the police are routed to patrol city regions spatial-temporally, rather than returning back to the base stations. An efficient algorithm is proposed for this novel patrolling variant that can establish a bound on PVR theoretically. For the stochastic emergency vehicle (EV) redeployment problem, Lei *et al.* [21] propose a two-stage traffic incident response model. In the first stage, the EVs are allocated to locations by the prior traffic incident information and in the second stage, the vehicles are redeployed to tackle with the stochastic service demands and returning time of EVs. They solve this two-stage EV redeployment problem by using a two-level integer programming, which cannot scale to city-scale instances. Park *et al.* [20] also propose an EV incident response model by considering the additional management of the stochastic occurrence of next incidents. These stochastic models cannot satisfy the hard constraint of covering IRs studied in this paper but can be useful to extend our work to stochastic model.

B. Patrol Routing

Let $G = \langle V, E, W, P\rangle$ model the network of areas, where V denotes the hot-spot areas need to patrol, E denotes the edges between hot-spots, W denotes the weight/cost/distance of the edge, and P denotes the profit of visiting hot-spots. With limited police resources, Chawathe [4] and Keskin *et al.* [6] design patrolling routes to maximize the coverage of hot-spots, i.e., maximizing the hot-spots covered/visited by the police. Lau *et al.* [8] schedule police efficiently such that each hot-spot is patrolled with specific frequency and the objective is to minimize the patrolling cost. Given a patrolling budget (e.g., the maximum cost of a route), Feillet *et al.* [29] and Zeng *et al.* [7] search for the route that visits the most beneficial hot-spots. Since routing variants are always NP-hard [5], existing optimal IP methods [8], [29] cannot scale to large instances with hundreds of hot-spot areas. Varakantham *et al.* [30] propose a branch-and-bound-based local search algorithm to find the optimal traveling path. Although there are other scalable methods that also have performance guarantee, they assume that the graph is densest [4] or the coverage function is submodular [7] or the objective function can be decomposed as the sum of multiple interdependent functions [31], or there is no temporal constraint between two nodes [9]. Compared with these work, this paper proposes a polynomial approximation algorithm without these unrealistic assumptions.

C. Security Games

To protect airports [32], urban transit systems [33], wildlife [34], [35], fair sites [36], [37], and important buildings [38], a Stackelberg security game between defenders and attackers is modeled, and the equilibrium patrolling strategy is computed for the defender [39]–[41]. Zhang *et al.* [13], [27] propose opportunistic security games where the criminal behavior is correlated to patrolling strategies, and Brown *et al.* [26] use the historical data to learn the traffic violation with/without police presence. These pure patrolling strategies exponentially increase with the number of police, and randomized policies are widely used to maximize the expected patrolling utility [42]. However, the randomized patrolling plan cannot satisfy the hard constraint of response time guarantee studied in this paper.

D. Multi-Robot Patrolling

The robot is scheduled to patrol a closed area to complete tasks, such as visiting each point frequently [43] and delivering goods. Multi-robots can be exploited to improve the patrolling efficiency [44], however, the complex coordination policies among robots make multi-robot patrolling intractable [15]. The centralized decomposition technique, which works by decomposing the exponential joint states into independent domains [44] and distributed multi-robot patrolling that works by a greedy manner can provide scalable and fault-tolerable patrolling strategies [14]. However, these heuristics do not have any performance guarantee. For the adversarial multi-robot patrolling problem, there is an adversary that can penetrate through the closed area and the robots wish to patrol randomly to maximize the probability of detecting penetration [16]. However, these graph theory-based patrolling methods cannot always find a solution for the bi-criteria patrolling problem studied in this paper.

III. Model

A. Daily Patrolling

Let there be n police officers $A = \{a_1, a_2, \ldots, a_n\}$ that can be used for daily patrolling.[1] Each day, there are 3 patrolling shifts, each of which lasts for 8 hours, e.g., the first shift ranges from 00:00 to 08:00. Each shift is discretized into periods, each period includes $\delta \in [0, 8]$ hour, and the parameter δ can be tuned such that $\frac{8}{\delta}$ is an integer. The daily patrolling includes periods $\{1, 2, \ldots, T = \frac{24}{\delta}\}$. To avoid overfatigue, each police officer only serves one shift, e.g., if a_i is assigned to serve the first shift, he only patrols during periods $\{1, 2, \ldots, \frac{8}{\delta}\}$.

[1] In this paper, a police unit can not only represents a single police officer but also represent a police team including a couple of police officers.

B. City Network

Let $G = \langle V, D\rangle$ denote the city network where $V = \{v_1, v_2, \ldots, v_m\}$ denotes m regions of the city. Each region $v_i \in V$ is represented by its center's coordinate ($longitude$, $latitude$). $D = \{d_{ij} \in \mathbb{Z}_{\geq 0}\}_{v_i, v_j \in V}$ denotes the distance between regions v_i to v_j, and the police will take d_{ij} periods to move from v_i to v_j. This distance parameter satisfies the triangle inequality, i.e., for any v_i, v_j and v_k, we have $d_{ik} < d_{ij} + d_{jk}$.

C. Data-Driven Incident Requests (IRs) and Volume Flow of People (VFoP) Sampling

We adopt a data-driven sampling model (introduced by [17]) to generate the set of IRs $R = \{r_1, r_2, \ldots, r_l\}$ and the daily VFoP $Q_j = \{q_j^1, q_j^2, \ldots, q_j^T\}$ of each region v_j. Let $\mathcal{R}(j, k, t)$ denote the distribution of the next arriving IR with type k occurring at region v_j during period t and $\mathcal{Q}(j, t)$ denote the distribution of VFoP of v_j at period t. The parameters of the two stochastic distributions $\mathcal{R}(j, k, t)$ and $\mathcal{Q}(j, t)$ are estimated from historical data. We sample incident requests R and VFoP Q_j from $\mathcal{R}(j, k, t)$ and $\mathcal{Q}(j, t)$ for D_{sample} days (e.g., one week). The IRs are incremental and the VFoP q_j^t is averaged over D_{sample} instances.

D. Objective

The MPSB is mainly concerned with the objective of PVR maximization under the hard constraint of IRs response time guarantee.

- **IRs response time guarantee.** Let $R = \{r_1, r_2, \ldots, r_l\}$ denote the set of IRs, each $r_k \in R$ is denoted by a tuple $\langle v_{r_k}, \tau_{r_k}, \theta_{r_k}\rangle$, where $v_{r_k} \in V$ is the region of occurrence, τ_{r_k} is the time of occurrence, θ_{r_k} is the maximal tolerable response time (which is computed as the time between τ_{r_k} and the arrival time of the police officer). Let $N_{r_k} = \{v_j | d_{r_k j} \leq \theta_{r_k}\}$ denote the available regions where the police can respond to r_k with time guarantee, e.g., at period τ_{r_k}, there must be police patrolling $v_j \in N_{r_k}$.[2] For ease of expression, we call a request is *covered* if it can be responded with time guarantee.
- **PVR maximization.** Let q_j^t denote the VFoP of the region v_j at period t. Let $z_j^t \in \{0, 1\}$ denote whether there is a police officer patrolling v_j at period t ($z_j^t = 1$) or not ($z_j^t = 0$). According to the empirical evidence presented in Fig. 1 that the presence of one police officer is enough to maximize people's FoS, we can define the PVR of v_j at period t as $\phi_j^t = z_j^t \cdot q_j^t$.

[2] For the scenario that the police might not patrol at the center of the region and might not respond to the IRs timely, we can partition the city into more fine-grained local regions such that the longest distance between any two regions satisfy the response time guarantee.

E. Problem Formulation

The decision variables are defined as follows.

- $x_i^\beta \in \{0, 1\}$: set to 1 if a_i serves the $\beta \in \{0, 1, 2\}$ shift;
- $y_{ij}^t \in \{0, 1\}$: set to 1 if a_i patrols the region υ_j at period t;
- $z_j^t \in \{0, 1\}$: set to 1 if there is a police officer patrolling υ_j at t.

We use an Integer Program (IP) to formulate the city-scale patrolling (**CSP**) problem of maximizing daily PVR, while satisfying the constraint of IRs coverage.

$$\max \sum_{\upsilon_j \in V} \sum_{1 \le t \le T} z_j^t \cdot q_j^t \tag{1}$$

$$\textbf{s.t.} \sum_{\beta=0,1,2} x_i^\beta = 1, \quad \forall a_i, \tag{2}$$

$$\begin{cases} y_{ij}^t + x_i^\beta - 1 < \dfrac{t - \frac{8}{\delta} x_i^\beta \cdot \beta}{M} + 1, \quad \forall a_i, \upsilon_j, t, \beta, \\ y_{ij}^t + x_i^\beta - 1 \le \dfrac{\frac{8}{\delta}(x_i^\beta \cdot \beta + 1) - t}{M} + 1, \quad \forall a_i, \upsilon_j, t, \beta, \\ \sum_{\upsilon_j \in V} y_{ij}^t \le 1, \quad \forall a_i, t, \end{cases} \tag{3}$$

$$t + d_{jj'} + 1 - t' \le M(2 - y_{ij}^t - y_{ij'}^{t'}), \quad \forall a_i, \upsilon_j, \upsilon_{j'}, t < t', \tag{4}$$

$$\sum_{a_i \in A} \sum_{\upsilon_j \in N_{r_k}} y_{ij}^{\tau_{r_k}} \ge 1, \quad \forall r_k, \tag{5}$$

$$z_j^t \le \sum_{a_i \in A} y_{ij}^t, \quad \forall \upsilon_j, t. \tag{6}$$

Eq.(1) is the objective of maximizing daily PVR, where q_j^t is the VFoP of υ_j at period t. Constraint (2) ensures that each police officer only serves one shift. In constraint (3), the top two constraints ensure that once a_i is assigned to serve the β shift, he must only patrol during this shift. For example, if a_i serves the second shift (i.e., $x_i^1 = 1$), the top two constraints ensure that $\forall \upsilon_j, 0 < t \le \frac{8}{\delta}, \frac{16}{\delta} < t \le T,\ y_{ij}^t = 0$. $M \in \mathbb{R}_{>0}$ is large enough to guarantee this constraint. The bottom constraint ensures that at each period, each police officer can only patrol one region. Constraint (4) ensures patrolling consecutiveness, i.e., if police a_i patrols regions υ_j and $\upsilon_{j'}$ at period t and $t'(> t)$ respectively, there must be long enough periods between t' and t such that a_i can move from υ_j to $\upsilon_{j'}$. The use of symbol "1" is to ensure that the patrolling time period is correct. For example, assume a police officer patrols υ_i at the beginning of time period t and moves to patrol υ_j immediately at the end of time period t, he will patrol region υ_j at the time period $t + d_{ij} + 1$. Constraint (5) ensures that each request r_k must be covered. Constraint (6) computes whether there is a police officer patrolling υ_j at period t.

Remarks: In this model, we assume that the police are enough to cover the IRs, a natural question is how to address the spare resource scenario where there are not enough police to cover all IRs. One possible way is to reduce the samples of IRs, for example, if the police are not enough to cover IRs of the D_{sample} days, say one week, we can reduce D_{sample} to one day such that the sampled IRs can be covered by the insufficient police resources, and the patrolling plan is updated once a day.

F. Complexity Analysis

We show that CSP problem is NP-hard by the reduction from an arbitrary *setcover* decision (**SCD**) problem.

Theorem 1: The CSP problem is NP-hard.

For the reason of space limitation, all the proofs in this paper are present in the appendix.

IV. An Efficient Approximation Algorithm

In this section, we present an approximation algorithm to solve such an NP-hard CSP problem. The key idea behind the proposed algorithm is that we decompose the original CSP problem into two weakly-coupled subproblems: 1) the *minimum police subproblem (MinP)*: how to determine the minimum number of police used to cover all IRs; and 2) the *maximum PVR subproblem (MaxP)*: how to schedule the remaining police to maximize PVR. We design polynomial approximate and optimal algorithms for these two subproblems in Sections 4.1 and 4.2, respectively. In Section 4.3, we provide the performance guarantee of this decomposition technique for the original CSP. To further improve PVR, we propose a grafting mechanism that integrates the two subproblems' solutions in Section 4.4.

A. Minimum Police Subproblem

1) Problem Formulation: We use an IP to formulate the MinP Subproblem of determining the minimum number of police necessary for covering all IRs, shown as follows.

$$\min \sum_{a_i \in A} \omega_i \tag{7}$$

$$\textbf{s.t.} \begin{cases} x_i^\beta \le \omega_i, \quad \forall a_i, \beta, \\ y_{ij}^t \le \omega_i, \quad \forall a_i, \upsilon_j, t, \\ \omega_i \in \{0, 1\}, \quad \forall a_i, \end{cases}$$

$$\textit{Constraints } (2) \sim (5). \tag{8}$$

The decision variable $\omega_i \in \{0, 1\}$ sets to 1 if the police a_i is selected. Other variables have the similar definitions defined in the original CSP. Constraint (8) ensures that a_i can patrol and respond to IRs iff he is selected.

Complexity Analysis: By a reduction from the *setcover* problem to the simplified MinP problem where there is only one period, we can show that MinP is NP-hard. To improve the scalability, we propose a polynomial approximation algorithm that uses the lower bound $O(\ln l)$ police of the optimum. The key idea behind the approximation algorithm is that we iteratively select a police officer to cover IRs, and for each selected police, we dispatch as many IRs to him as possible in a greedy manner.

2) Dynamic Programming-Based Greedy Approach: Let $R^\beta = \{r_k \in R | \frac{8}{\delta}\beta < \tau_{r_k} \le \frac{8}{\delta}(\beta + 1)\}$ denote the IRs in the βth shift. Assume that a police a_i is dispatched to the β shift, let $\tilde{R}^\beta$ denote the remaining IRs after the former police have finished patrolling. Now, we need to optimize a_i's patrolling plan to cover as many IRs in $\tilde{R}^\beta$ as possible. To achieve this goal, we design a dynamic programming (DP)-based patrolling plan for a_i. Let $\tilde{R}_i^\beta(t, \upsilon_j) = \{r_o \in \tilde{R}^\beta | \upsilon_j \in N_{r_o}, \tau_{r_o} = t\}$ denote the IRs near the region υ_j that a_i can cover at period t. Let

Algorithm 1 DP-Based Greedy Algorithm for MinP

Input : The IRs R.
Output: The police A_{MinP} and their patrolling plans $\mathcal{P}_{MinP}$.

1 Initialize $i=1$, $\tilde{R}^{\beta}=R^{\beta}$ and $R_{cover}=R^{0}$;
2 **while** $i \leq n$ && $R_{cover} \neq \emptyset$ **do**
3 Initialize $k^{*}=-1$ and $\beta^{*}=-1$;
4 **for** $0 \leq \beta \leq 2$ **do**
5 Using Eq.(9) to compute $\Omega_i^{\beta}(t_*, v_{j*}, \tilde{R}^{\beta})$;
6 **if** $\Omega_i^{\beta}(t_*, v_{j*}, \tilde{R}^{\beta}) > k^*$ **then**
7 $k^*=\Omega_i^{\beta}(t_*, v_{j*}, \tilde{R}^{\beta})$ and $\beta^*=\beta$;
8 Update $\tilde{R}^{\beta^*}=\tilde{R}^{\beta^*} \backslash R_i^{\beta^*}(t_*, v_{j*}, \tilde{R}^{\beta^*})$ and $R_{cover}=R_i^{\beta^*}(t_*, v_{j*}, \tilde{R}^{\beta^*})$;
9 **if** $R_{cover} \neq \emptyset$ **then**
10 Update $A_{MinP}=A_{MinP} \cup \{a_i\}$, $i=i+1$ and $\mathcal{P}_{MinP}=\mathcal{P}_{MinP} \cup \mathcal{P}_i^{\beta^*}(t_*, v_{j*}, \tilde{R}^{\beta^*})$.

$\Omega_i^{\beta}(t, v_j, \tilde{R}^{\beta})$ denote the maximum IRs covered by a_i if he patrols the region v_j at period t. We implement the following DP recurrence for a_i:

$$\Omega_i^{\beta}(t, v_j, \tilde{R}^{\beta}) = |\tilde{R}_i^{\beta}(t, v_j)| + \max_{v_{j'}: t+d_{jj'} < \frac{8}{\delta}(\beta+1)} \Omega_i^{\beta}(t+d_{jj'}+1, v_{j'}, \tilde{R}^{\beta} \backslash \tilde{R}_i^{\beta}(t, v_j)). \tag{9}$$

In Eq.(9), after covering the IRs $\tilde{R}_i^{\beta}(t, v_j)$ by patrolling v_j at period t, a_i always searches for the next optimal accessible region $v_{j'}$ within the βth shift, i.e., $t+d_{jj'} < \frac{8}{\delta}(\beta+1)$. We have the following initial conditions for Eq.(9): $\forall a_i, v_j, \tilde{R}^{\beta}$, $\Omega_i^{\beta}(\frac{8}{\delta}(\beta+1), v_j, \tilde{R}^{\beta}) = |\tilde{R}_i^{\beta}(\frac{8}{\delta}(\beta+1), v_j)|$. Finally, a_i's optimal patrolling plan is returned by $\max_{\beta \in \{0,1,2\}} \Omega_i^{\beta}(t_*, v_{j*}, \tilde{R}^{\beta})$, where

$$\Omega_i^{\beta}(t_*, v_{j*}, \tilde{R}^{\beta}) = \max_{\frac{8}{\delta}\beta < t \leq \frac{8}{\delta}(\beta+1), v_j \in V} \Omega_i^{\beta}(t, v_j, \tilde{R}^{\beta}). \tag{10}$$

The corresponding patrolling plan is denoted by $\mathcal{P}_i^{\beta}(t_*, v_{j*}, \tilde{R}^{\beta})$ and the covered IRs are denoted by $R_i^{\beta}(t_*, v_{j*}, \tilde{R}^{\beta})$.

We refer to such a DP-based algorithm as the greedy algorithm and present it in Algorithm 1. For each police a_i, if he is assigned to the βth shift, he always find the optimal patrol plan $\Omega_i^{\beta}(t_*, v_{j*}, \tilde{R}^{\beta})$ (step 5). To optimize the patrol efficiency, he is always assigned to the shift where he can satisfy the maximal IRs (steps 4$\sim$7). Algorithm 1 returns the number of police A_{MinP} used for covering IRs and their patrolling plan P_{MinP}.

Theorem 2: For a MinP with n police officers, m regions, T periods and l IRs, Algorithm 1 runs in time $O(nm^2T^2)$ and uses at most $O(\ln l)$ police of the optimum.

B. Maximum PVR Subproblem

After determining the police A_{MinP} used for MinP problem, we can schedule the remaining police $A_{MaxP}=A \setminus A_{MinP}$ for PVR maximization, which can be formulated by the following IP:

$$\begin{aligned} &\max \ \textstyle\sum_{v_j \in V} \sum_{1 \leq t \leq T} z_j^t q_j^t \\ &\textbf{s.t. } Constraints\ (2) \sim (4), (6). \end{aligned} \tag{11}$$

The objective (11) and constraints (2)$\sim$(4), (6) have been defined in the original CSP problem, where the available police officers A are replaced by A_{MaxP}. In the following, we first show the suboptimality of the "greedy" approach of successively determining the most beneficial plan.

1) Suboptimality of the Greedy Approach: The greedy idea proposed in MinP subproblem might be extended to the MaxP subproblem: using dynamic programming to patrol the most beneficial path with the maximal PVR for each police officer. However, we use an example to show the greedy approach is suboptimal.

Example 1: In Fig.3(a), there are two periods t_1 and t_2 and three regions v_1, v_2 and v_3. Assume that it is feasible to move between v_1 and v_2, v_2 and v_3, i.e., $t_2-t_1 > d_{12}$ and $t_2-t_1 > d_{23}$. Each period-region vertex v_{tj} is associated with a value indicating its VFoP, e.g., for the vertex v_{11}, the value 9 indicates that at period t_1, there are 9 units of VFoP at the region v_1. Assume that there are two police officers a_1 and a_2. In Fig.3(b), under the greedy approach, the patrolling plan of a_1 is $(t_1, v_1) \rightarrow (t_2, v_2)$ that achieves the largest PVR, the patrolling path of a_2 then is $(t_1, v_3) \rightarrow (t_2, v_3)$, i.e., at the t_1 period, a_1 (resp. a_2) is patrolling v_1 (resp. v_3), and at the t_2 period, a_1 (resp. a_2) is patrolling v_2 (resp.v_3). The greedy approach achieves 24 units of PVR. However, the optimal patrolling plans are $(t_1, v_1) \rightarrow (t_2, v_1)$ for a_1 and $(t_1, v_3) \rightarrow (t_2, v_2)$ for a_2, which achieves 27 units of PVR.

From the above example, we can see that the greedy algorithm can cause an arbitrarily large loss of PVR. Next, by collaborating with the network flow technique, we propose an optimal approach.

2) Network Flow-Based Optimal Approach: This optimal approach includes two stages: 1) given that the police officers have been assigned to each shift, we first propose the network flow-based patrolling in each shift, and analyse some desirable properties, and 2) based on these properties, we provide an efficient algorithm to assign the police to shifts.

Network Flow-Based Patrolling in One Shift: We first provide an interpretation of MaxP problem in terms of a flow network, which can be denoted by a weighted directed graph $G_f = (V_f, E_f)$. Each period-region pair (t, v_j) is denoted by a vertex $v_{tj} \in V_f$, and a feasible movement from one vertex v_{tj} to another vertex $v_{t'j'}$ (i.e., $t'-t > d_{jj'}$) is represented by a direct edge in E_f from v_{tj} to $v_{t'j'}$. Given two connected vertices v_{ti} and $v_{t'j}$, we model the 'cost' between them by the negation of the VFoP of the latter vertex $v_{t'j}$. A minimum cost flow is a maximum flow, such that the sum of its edges' weights is the minimum.

Given that there are K^{β} police officers assigned to the βth shift, we construct a flow network $s-s'(\beta, K^{\beta})$ as follows.

- Create a source vertex s and a sink vertex s';
- Create K^{β} vertices $\{s_1, s_2, \ldots, s_{K^{\beta}}\}$, and an edge from s to each s_i with the capacity equals to 1 and zero cost;

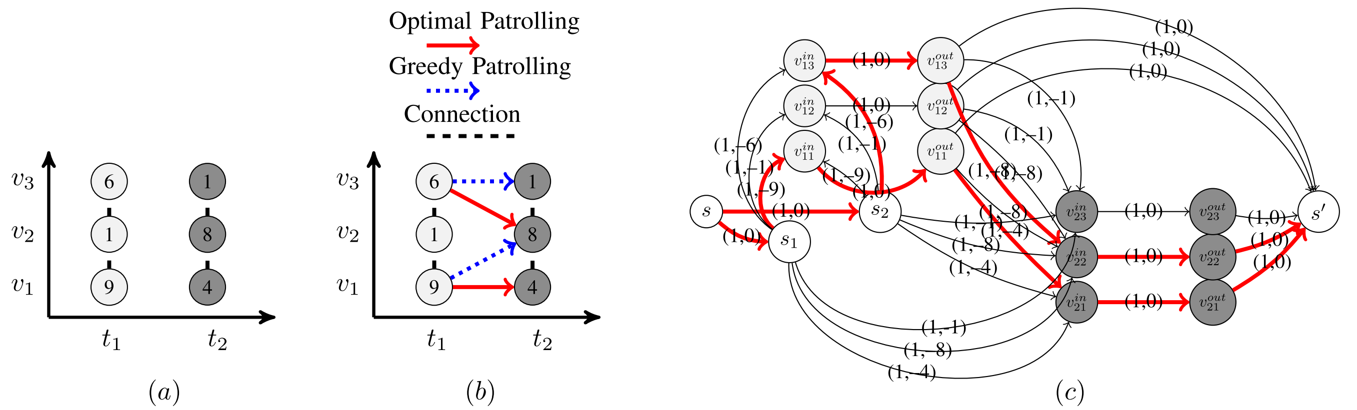


Fig. 3. An example of network flow-based patrolling.

Algorithm 2 Network Flow-Based Patrolling for MaxP

Input : The number of police K^β, VFoP Q_j and $D_{m\times m}$.
Output: Patrolling plans $\mathcal{P}^\beta_{MaxP}(K^\beta)$
Build a flow network $s-s'(\beta, K^\beta)$;
Using minimum cost flow algorithm [45] to compute the K^β vertex-disjoint patrolling plan $\mathcal{P}^\beta_{MaxP}(K^\beta)$.

- For each period-region pair (t, v_j), create an *in* period-region vertex v^{in}_{tj} and an *out* period-region vertex v^{out}_{tj}, and an edge from v^{in}_{tj} to v^{out}_{tj} with the capacity equals to 1 and zero cost;
- Create an edge from each vertex s_i to each *in* period-region vertex v^{in}_{tj} with the capacity equals to 1 and the cost equals to the negation VFoP of the region v_j at period t;
- Create an edge from *out* period-region vertex v^{out}_{tj} to other *in* period-region vertex $v^{in}_{t'j'}$ if $t'-t->d_{jj'}$, with the capacity set to 1 and the cost set to the negation VFoP of the region $v_{j'}$ at period t;
- Create an edge from each *out* period-region vertex v^{out}_{tj} to the sink vertex s' with the capacity equals to 1 and zero cost.

Fig.3(c) is the corresponding flow network of the example shown in Fig.3(a). The optimal flow with the maximal PVR is highlighted in red.

Given the constructed s–s' flow network, a polynomial-time scaling minimum cost flow algorithm [45] can be implemented for the flow computation. Each vertex either has the one unit capacity *in-degree* or one unit capacity *out-degree*, thus each flow starting from source vertex s to the target vertex s' must be vertex-disjoint. In a flow network, two paths with the common start and end vertices that have no other vertices in common are vertex-disjoint paths. Each flow then can be regarded as a police's patrolling plan. Finally, the network flow-based patrolling approach is proposed in Algorithm 2.

3) Properties of Algorithm 2: We analyse the polynomial time, optimal and submodular properties of Algorithm 2.

a) Polynomial time: For a MaxP with m regions, T periods and K police officers, there are $2mT+K$ vertices and m^2T^2 edges in the constructed flow network s–s'. By using the polynomial scaling flow network algorithm [45], Algorithm 2 runs in $O(m^2T^2)$.

b) Optimality: We prove that in the constructed flow network $s-s'$, if the optimal patrolling solution is $\mathcal{P}^{opt}$, Algorithm 2 will return a maximum flow whose negation has the same value as $\mathcal{P}^{opt}$'s. This implies that the algorithm always returns an optimal patrolling solution. Let $p_i = \{(t^1_i, v_{t^1_i}), \ldots, (t^T_i, v_{t^T_i})\}$ denote a police a_i's patrolling plan, where each period-region tuple (t, v_t) indicates that a_i is patrolling the region v_t at period t.

Definition 1 (Disjoint Patrolling Plans): Two patrolling plans $p_i = \{(t^1_i, v_{t^1_i}), \ldots, (t^T_i, v_{t^T_i})\}$ and $p_j = \{(t^1_j, v_{t^1_j}), \ldots, (t^T_j, v_{t^T_j})\}$ are disjoint iff $\nexists t^l_i = t^k_j$: $v_{t^l_i} = v_{t^k_j}$. A patrolling solution $\mathcal{P} = \{p_1, p_2, \ldots, p_K\}$ is disjoint iff $\forall p_i, p_j \in \mathcal{P}$, p_i and p_j are disjoint.

Let $\mathcal{P}^{opt} = \{p^{opt}_1, p^{opt}_2, \ldots, p^{opt}_K\}$ denote the optimal patrolling solution of the K police officers. Let $\Phi_{\mathcal{P}^{opt}}$ denote the PVR of $\mathcal{P}^{opt}$ and Φ_f denote the PVR returned by the Algorithm 2, we will prove that Φ_f have the same value with that of $\mathcal{P}^{opt}$.

Theorem 3: Φ_f returns the same value with $\Phi_{\mathcal{P}^{opt}}$.

c) Monotone and submodular properties: We provide another desirable property of Algorithm 2 that is useful for assigning police to shift in Section 4.2.4 and for approximation ratio analysis of the CSP in Section 4.3. Let Φ_f denote the PVR returned by Algorithm 2, we show that the objective function Φ_f satisfies monotone and submodular properties with respect to the set of police A. Let U be a non-empty finite set and f be a function $f : 2^U \to \mathbb{R}$, where 2^U denotes the power set of U. The function f is ***monotone*** if $f(\mathcal{A}) \le f(\mathcal{B})$ for all $\mathcal{A} \subseteq \mathcal{B} \subseteq U$ and ***submodular*** if $f(\mathcal{A} \cup s) - f(\mathcal{A}) \ge f(\mathcal{B} \cup s) - f(\mathcal{B})$ for all $\mathcal{A} \subseteq \mathcal{B} \subseteq U$ and $s \in U \backslash \mathcal{B}$.

Lemma 1: The function Φ_f is monotone and submodular with respect to the set of police officers A.

4) Assignment of Police to Shifts: We have above addressed the MaxP problem where the police officers have been assigned to the shifts. There are three shifts and we need to optimally assign the police A_{MaxP} to shifts, so that the total PVR is maximized. We propose an iterative polynomial time

Algorithm 3 Assignment of Police to Shifts

```
Input  : The police officers A_MaxP
Output: Police assignment to shifts K^β, β = 0, 1, 2.
1 Assigning A_MaxP to the shifts randomly, denoted the assignment by K^0, K^1 and K^2;
2 while TRUE do
3     β_in=arg max_{β=0,1,2} G_f(β, K^β);
4     β_out=arg min_{β=0,1,2} L_f(β, K^β);
5     if G_f(β_in, K^{β_in}) > L_f(β_out, K^{β_out}) then
6         K^{β_in}=K^{β_in} + 1 and K^{β_out}=K^{β_out} − 1;
7     else
8         Return FALSE.
```

Algorithm 4 MinP-MaxP Patrolling Plan Grafting

```
Initialize P_i^G = ∅, ∀a_i ∈ A_MinP;
for a_i ∈ A_MinP do
    for p_i^{jj'} ∈ P_i do
        if p_i^{jj'} is a feasible MinP subplan then
            Using Eq.(12) to compute p_i^{jj',G};
            P_i^G = P_i^G ∪ p_i^{jj',G};
Return grafting plan 𝒫_MaxP = 𝒫_MaxP ⋃_{a_i∈A_MinP} P_i^G.
```

algorithm. Given a shift β and the number of police officers K^β assigned, let $\Phi_f(\beta, K^\beta)$ denote the optimal PVR returned by Algorithm 2. Let $G_f(\beta, K^\beta){=}\Phi_f(\beta, K^\beta{+}1){-}\Phi_f(\beta, K^\beta)$ denote the gain of PVR by adding one police officer to the shift β and $L_f(\beta, K^\beta){=}\Phi_f(\beta, K^\beta)$–$\Phi_f(\beta, K^\beta$–$1)$ denote the loss of PVR by removing one police officer from the shift β. The optimal police assignment to shifts is shown in Algorithm 3. In step 1, the police A_{MaxP} are randomly assigned to shifts. In Steps 3$\sim$6, a police officer is iteratively moved from one shift to another with the aim of improving PVR. Steps 3 and 4 search the shift (β_{in}) that has the highest gain and the shift (β_{out}) that has the lowest loss, respectively. If the gain is larger, Step 5 moves from one police officer from shift β_{out} to β_{in} to improve PVR. This iteration proceeds until there is no such beneficial movement (Step 2).

We show the convergence of Algorithm 3 and how fast it converges to the optimal solution.

Theorem 4: Algorithm 3 takes at most $O(|A_{MaxP}|)$ iterations to converge to the optimal solution, where each iteration includes polynomial time computations of Algorithm 2.

C. Approximation Ratio of CSP

In this section, we provide the approximation ratio Φ_{app}/Φ_{opt} of the proposed decomposition method, where Φ_{app} and Φ_{opt} are PVRs returned by the proposed decomposition method and the optimum of CSP. Before presenting the approximation ratio result, we first provide a useful lemma derived from Algorithm 2.

Lemma 2: For the MaxP subproblem, given two police sets A_h and A_l, where $|A_h| = h$, $|A_l| = l(\leq h)$, we have that $\Phi_f(A_l) \geq \frac{l}{h}\Phi_f(A_h)$. $\Phi_f(A_i)$ indicates the PVR returned by Algorithm 2 with police A_i, where $|A_i| = i$.

Theorem 5: Let $\alpha = \frac{|A_{MinP}|}{|A|}$ denote the ratio of police used in MinP subproblem, $\frac{\Phi_{app}}{\Phi_{opt}} \geq 1 - \alpha$.

D. Improving PVR by Grafting

In this section, to further improve PVR, we propose a grafting mechanism that integrates the two subproblems' patrolling plans. The motivation of plan grafting is that during MinP patrolling, there might be free periods between responding to two consecutive IR. For example, a police officer is scheduled to patrol regions v_j and $v_{j'}$ consecutively at period 5 and 20, and these $20 - 5 = 15$ free periods during MinP patrolling can be exploited to graft the MaxP patrolling plans.

Given a police officer $a_i \in A_{MinP}$, let $P_i = \{p_i^{jj'}\}_{1\leq t_j < t_{j'} \leq T}$ denote his patrolling plan. Each subplan $p_i^{jj'} = \langle t_j, v_j, t_{j'}, v_{j'}\rangle$ indicates that at period t_j, a_i is patrolling v_j and moves to patrol $v_{j'}$ at period $t_{j'}$. The grafting mechanism works by two steps: 1) identifying the feasible MinP subplan $p_i^{jj'}$ that has enough free periods between t_j and $t_{j'}$; and 2) optimizing the MaxP plan to graft on $p_i^{jj'}$.

Definition 2 (Feasible MinP Subplans): Given a police a_i that is used for MinP and his patrolling plan P_i, his subplan $p_i^{jj'} = \langle t_j, v_j, t_{j'}, v_{j'}\rangle$ is feasible for grafting MaxP plans iff $\exists v_u \in V,\ t_j{+}d_{ju}{+}d_{uj'}{<}t_{j'}$.

This feasible condition ensures that there must be enough periods between t_j and $t_{j'}$ such that a_i can patrol from v_j to v_u and return back to $v_{j'}$ before $t_{j'}$.

Optimize the Grafting Plan of MaxP: For each feasible MinP subplan $p_i^{jj'} = \langle t_j, v_j, t_{j'}, v_{j'}\rangle$, we wish to schedule a_i to patrol between periods t_j and $t_{j'}$ optimally to maximize PVR. Let $\Psi_i(t, v_u, p_i^{jj'})$ denote the PVR of the optimal grafting plan if a_i is patrolling v_u at period t ($t_j < t < t_{j'}$). We propose the following DP-based plan grafting:

$$\Psi_i(t, v_u, p_i^{jj'}) = \max \begin{cases} \max_{v_\omega \in V\setminus v_u : t+d_{u\omega}+1+d_{\omega j'} < t_{j'}} (1 - z_u^t) q_u^t \\ \quad + \Psi_i(t + d_{u\omega} + 1, v_\omega, p_i^{jj'}), \text{ if } t+1+d_{uj'} < t_{j'}; \\ \Psi_i(t+1, v_u, p_i^{jj'}) + (1 - z_u^t) q_u^t, \text{ if } t+1+d_{uj'} < t_{j'}; \\ \Psi_i(t_{j'}, v_{j'}, p_i^{jj'}) + (1 - z_u^t) q_u^t, \text{ if } t+1+d_{uj'} = t_{j'}. \end{cases} \tag{12}$$

The first two terms have the same definitions with Eq.(9), where $z_u^t \in \{0, 1\}$ indicates that whether there is police patrolling at the region v_u at the period t ($z_u^t = 1$), or not ($z_u^t = 0$). The third term means that if it is infeasible to return back to $v_{j'}$ before $t_{j'}$ after one more period patrolling at v_u, a_i should return back to $v_{j'}$ immediately. Finally, let $p_i^{jj',G}$ denote the optimal MaxP plan grafted on MinP subplan $p_i^{jj'}$ and $\Psi_i(t_j, v_j, p_i^{jj'})$ denote the PVR of $p_i^{jj',G}$. Based on Eq.(12), we propose the MinP-MaxP plan grafting mechanism in Algorithm 4.

TABLE I

RUNTIME AND PVR ON THE NUMBER OF POLICE. N/A = RUNTIME EXCEEDED 500 SECONDS. TO MAKE THE TABLE CLEAR, WE OMIT THE STATISTICAL ERRORS, AND EACH CELL IS STATISTICALLY SIGNIFICANT AT 95% CONFIDENCE LEVEL

Number of Police	Runtime(s)						PVR					
	Dp-Nf	AD	GLS	Dp-Cpl	Cpl-Cpl	Cpl	Dp-Nf	AD	GLS	Dp-Cpl	Cpl-Cpl	Cpl
2	0.023	0.015	0.014	2.221	2.47	12.11	6.38	5.89	5.61	6.38	6.38	6.38
4	0.022	0.019	0.016	5.112	5.71	67.8	10.64	10.28	10.13	10.64	10.85	11.23
6	0.021	0.021	0.018	5.115	6.03	418.16	14.86	12.99	12.15	14.86	14.86	17.22
8	0.028	0.017	0.019	5.077	6.39	N/A	20.25	16.74	18.32	20.25	20.25	N/A
10	0.029	0.021	0.02	18.87	20.42	N/A	26.45	21.6	22.77	26.45	26.45	N/A
12	0.03	0.024	0.025	60.34	62.19	N/A	29.54	28.59	26.83	29.54	29.54	N/A
14	0.032	0.027	0.028	63.04	65.71	N/A	37.34	30.95	26.04	37.34	37.34	N/A
16	0.033	0.029	0.029	381.3	384.5	N/A	38.1	32.6	32.44	38.1	38.1	N/A
18	0.036	0.031	0.03	N/A	N/A	N/A	41.87	35.27	35.13	N/A	N/A	N/A

Proposition 1: For a CSP with n police officers, m regions and T periods, the running time of Algorithm 4 is $O(nm^2T^3)$.

V. EXPERIMENTAL EVALUATION

Experiment Setup: We conduct simulation experiments on a real-world dataset of Foshan, a modern city in China. The city is divided into grids, consisting of 20×15 regions, and each day is discredized into 3 shifts, each includes 16 periods, each period sustains 0.5 hour (i.e.,$\delta = 0.5$).[3] The distance between any two adjacent regions is set to 1 period. The dataset contains approximately 24,000 IRs of the year 2017, including criminal, public security, dispute and other kinds IRs. Threshold response time varies with the request type, e.g., for criminal and public security requests, there must be police patrolling at the location of the IRs, and for dispute and other kinds of requests, the response time is restricted within 1 period. Each IR contains the time, location and type information. For each request type, the IR arrival of each region follows an independent and identical Poisson distribution. We generate request samples over one week. Moreover, we use Baidu Map (https://map.baidu.com/) to generate the average VFoP of each region each period over one week, and for convenience, the VFoP is scaled in the range [0, 1]. We verify the scalability and solution quality of the proposed algorithm on these simulations. All computations are performed on a 64-bit PC with a dual-core 3.60 GHz CPU and 16 GB memory. All results are averaged over 40 instances. Each record is statistically significant at 95% confidence level, and for clarity, we omit the statistical error bars on the figures.

A. Tests of the Performance in the Small-Scale Scenarios

Comparing Algorithms: Let **Dp-Nf** denote our proposed algorithm that decomposes the CSP into the MinP and MaxP two sub-problems, and solves them by the dynamic programming and network flow techniques, respectively. We compare **Dp-Nf** with five benchmarks:

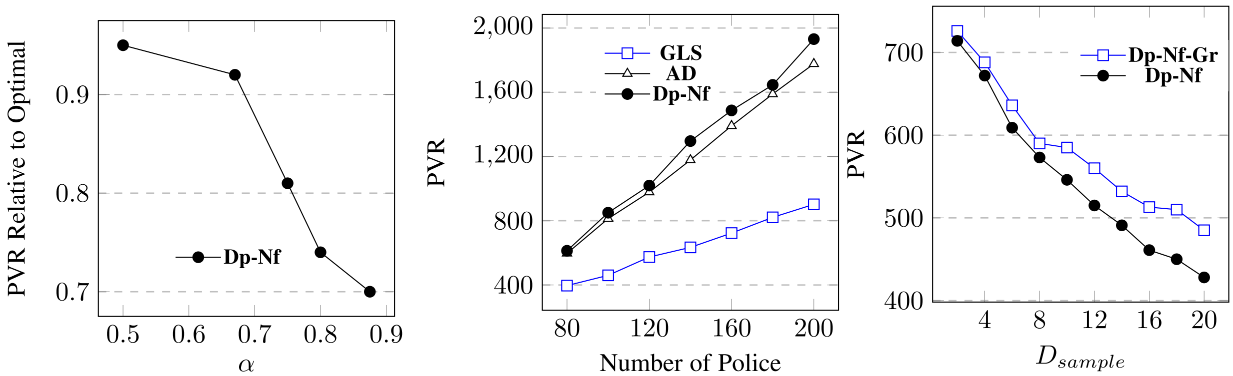
- **Cpl**, which solves the CSP by the optimal solver CPLEX (version 12.6);
- **Cpl-Cpl**, which decomposes the CSP into MinP and MaxP, and solves both of them by the CPLEX [3];
- **Dp-Cpl**, which decomposes the CSP into MinP and MaxP, and solves the former by the dynamic programming and the latter by the CPLEX;
- Greedy Local Search (**GLS**), which decomposes the CSP into MinP and MaxP: in the MinP, each police patrols the regions greedily by moving to the nearby regions that can cover the maximum IRs and in the MaxP, each police searches for the nearby regions with the maximum VFoP [9], [17], [19];
- Abstraction and Division (**AD**), which divides the city into 25 sub-divisions (each with 4×3 scale). The police are allocated to sub-divisions proportionally to their IRs. For each sub-division, three periods are abstracted in one period, and the original CSP is solved by the Cpl [15], [27].

The performance is measured by the solution quality in terms of PVR and the runtime in seconds.

Results: Table I compares the runtime and PVR of algorithms under various numbers of police. In Table I, we observe that with the runtime cap of 500 seconds, the Cpl cannot scale-up to the instances with 8 police, and Cpl-Cpl and Dp-Cpl can not scale-up to the instances with 18 police. Dp-Nf, AD, and GLS can return the solutions within one second for various instance scales. The Cpl-Cpl and Dp-Cpl take almost the same runtime, indicating that the MaxP subproblem is the bottleneck for computation. On the other hand, in terms of PVR, we can observe that 1) in the worst case where there are 6 police, DP-Nf can achieve above 85% PVR relative to the optimal solution Cpl, and 2) Dp-Nf achieves more PVR than AD and GLS in small-scale scenarios.

Fig. 4a shows the Dp-Nf's PVR relative to the optimum Cpl with varying the parameter α (i.e., the percent of police used in the MinP) in small-scale instances. Being consistent with the theoretical approximation ratio 1-α, Dp-Nf performs worse with the increase of α. On one hand, when α is small, i.e., $\alpha = 0.5$, our proposed algorithm can achieve above 90% PVR relative to the optimum Cpl. On the other hand, in the worst case where $\alpha = 0.9$, i.e., 90% of police officers are used to serve the IRs and the remained 10% of police officers are used for PVR maximizing, Dp-Nf can still produce about 70% PVR relative to the optimum Cpl. This can conclude that for the instance where there are hundreds of police officers and only a small percent of police officers are used to serve IRs,

[3] In the appendix, we also test conduct the experiments with varying δ.



(a) PVR Relative to Optimal (b) Robustness $\eta = 0\ 2$ (c) Advantage of Grafting

Fig. 4. (a) PVR relative to optimum on α. (b) Robustness on PVR estimation uncertainty. (c) The advantage of grafting on samples.

TABLE II
THE PVR ON LARGE SCALE CSPS. EACH CELL IS STATISTICALLY SIGNIFICANT AT 95% CONFIDENCE LEVEL

Number of Police	80	100	120	140	160	180	200
Dp-Nf	**677**	**857**	**1095**	**1249**	**1508**	**1582**	**1905**
AD	579	783	985	1183	1379	1571	1761
GLS	384	472	558	644	728	813	897
Dp-Nf(Fine-Grained)	143	204	262	324	383	436	499
AD(Fine-Grained)	122	177	232	286	339	392	444
GLS(Fine-Grained)	108	136	161	185	209	234	257

our proposed algorithm can achieve a high percent of PVR relative to the optimal solution.

B. Tests of the Performance in the Large-Scale Scenarios

In the large scale instances, Cpl, Cpl-Cpl and Dp-Cpl cannot scale-up, we only validate the PVR performance of Dp-Nf with that of AD and GLS.

Results: Table II shows the PVR under various numbers of police under the normal scenario (i.e., the city area is divided into 20×15 regions) and the fine-grained scenario (i.e., the city area is divided into 40×30 regions). We first generate IRs and VFoP in the fine-grained scenario. For each 2×2 regions in the fine-grained scenario, we integrate them as a single region in the normal scenario. For such an integrated region, its IRs are the union of IRs of its sub-regions in the fine-grained scenario and the VFoP is the sum of VFoP associated with its sub-regions in the fine-grained scenario. The algorithm Dp-Nf (AD and GLS respectively) in the fine-grained scenario is denoted by Dp-Nf (Fine-Grained)(AD(Fine-Grained) and GLS(Fine-Grained) respectively). From Table II, we observe that 1) Dp-Nf produces the highest PVR, which is followed by AD and GLS. 2) AD produces much larger PVR than GLS. The potential reason that Dp-Nf is prior than AD is that the police officer are restricted to specific regions, which can degrade the PVR if he can patrol further to other regions. On the other hand, the potential reason that AD performs better than GLS is that each police is only allowed to serve one shift, and in the optimal solution, the police will not patrol along a long path. This kind of patrolling policy can ensure AD not lose too much PVR by only patrolling at his 4×3 sub-division. 3) In the fine-grained scenario, the police will lose some PVR. This can be explained by the fact that in the fine-grained scenario, each region has the smaller VFoP than that in the normal scenario.

C. Robustness

In the real-world, the estimation of VFoP value may not be perfect. Therefore, we analyze the performance of our Dp-Nr under the existence of noise of VFoP. Let q^t_j be the estimated VFoP derived from the historically data, ${}^t_j$ is the real value, which is drawn uniformly within $\tilde{q}^t_j \in q^t_j \cdot [1-\eta, 1+\eta]$. We use the estimated VFoP q^t_j to allocate police officers, and use $\tilde{q}^t_j$ to compute real VFoP. We compare total VFoP of GLS, AD and Dp-Nr under the uncertainty degree $\eta = 0.2$. The results are shown in Fig. 4b, from which we observe that Dp-Nr can always produce the largest VFoP. This observation shows the strong robustness of our proposed algorithm.

D. The Advantage of Grafting

Let **Dp-Nf-Gr** denote the algorithm that integrates the grafting mechanism with **Dp-Nf**. Fig. 4c shows the advantage of Dp-Nf-Gr with varying sampling days D_{sample}. When D_{sample} becomes larger, the advantage of Dp-Nf-Gr over Dp-Nf on improving PVR becomes more significant. This can be explained by the fact that when D_{sample} is large, say 7 days, there are many police officers needed in MinP for covering IRs, and there will be more feasible MinP sub-plans. By grating the MaxP patrolling solution, the Dp-Nf-Gr can well exploit these free periods of the sub-plans to improve PVR.

E. Case Study

In this section, we provide an intuitive example about how our algorithm behaves on the real-world map. This case study includes 5 police officers, and a few number of IRs such that

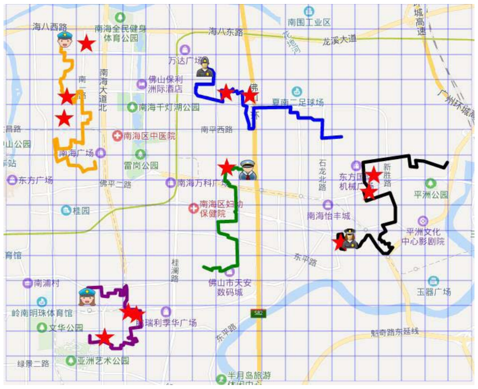


Fig. 5. Qualitative comparisons of police patrolling path.

these 5 police officers can cover. Fig. 5 shows the patrolling path of these 5 police officers, from which we notice that 1) the police officers can coordinate well with each other to patrol different regions; 2) each police officer always patrols around the regions where there are IRs, and 3) when there are no incidents, the police can patrol around further to improve the city PVR. For example, for the most right police officer, after handling the IR at the East International Machinery Square, she moves to patrol the Pingzhou Park where there is a large VFoP.

VI. Conclusion and Future Work

This paper studies the city-scale patrolling (CSP) problem with fine-grained periods, hundreds of regions, and hundreds of police officers. The NP-hardness complexity of CSP is analyzed. A decomposition technique is proposed to approximate CSP. A grating mechanism is proposed to further improve patrolling efficiency. Experimental results on a real dataset shows that the proposed algorithm scales well to the city-scale problem instances and achieves higher PVR than other benchmark algorithms.

For the future work, we find it is an interesting topic to study the stochastic patrol planning problem where the arrival of IRs, volume flow of population and traveling time are uncertain. We would like to integrate the offline learning and online planning for such stochastic variant. On the offline learning, we use the historical data (e.g., IRs occurrence and travel time among regions) to model city patrol scenarios. On the online planning, we can schedule the police officers to patrol regions based on the real-time traffic condition and the offline long-term estimated model.

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