

# TOWARDS OPTIMUM LINEAR TRANSFORMATION UNDER ZERO-MEAN GAUSSIAN MIXTURES FOR DETECTION OF MOTOR IMAGERY EEG

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## ABSTRACT

Optimum linear transformation under mixture of zero-mean Gaussian conditions is an intriguing problem, especially in learning discriminative spatial components in motor imagery EEG for building brain computer interfaces. However, it is not well addressed in the past. In this paper, we study optimum linear transformation under mixture of zero-mean Gaussian. In particular, we formulate optimum transformation as a Bhattacharyya error bound minimization problem, and derive a numerical solution to estimate the bound from training samples. Based on the solution, we develop an algorithm for selecting optimum linear transformation. The proposed method is evaluated, in comparison with the state-of-the-art methods, using a publicly available data set of motor imagery EEG. The results attest to the superiority of the method for detecting motor imagery.

**Index Terms**— Linear transformation, classification, motor imagery EEG

## 1. INTRODUCTION

Motor imagery EEG based Brain-Computer Interface is attracting increasing attention from multi-disciplinary fields [1, 2]. In learning and classifying the signal of motor imagery EEG, linear transformation in the form of spatial filtering has been widely used to transform and reduce multi-channel EEG into a few variables that represent the brain signal of interest [3]. This transformation, as a means of improving signal to noise ratio, is important since motor imagery EEG is weak, vulnerable to noise corruptions and blurred spatially [4, 5].

The most successful linear transformation method so far is known as the common spatial pattern (CSP) technique [6] or its variants [7, 8, 9]. An experimental study [10] reported that CSP could lead to a higher accuracy of motor imagery EEG classification than independent component analysis. Designed for two-class classification, the CSP technique constructs linear transformation (spatial filters) that maximizes the variance of (usually band-pass filtered) EEG for one class while minimizing it for the other one [11]. Thus, from neurophysiology, CSP captures strong or attenuated rhythmic

activities that are associated with the Event-Related Desynchronization/Synchronization (ERD/ERS) effects of motor imagery [12].

However, inherent multiple manifestations of EEG may render CSP insufficient. This is especially prominent if one wants to differentiate motor imagery EEG from EEG in the so called *non-control* (NC) class (see [13]), which comprises any possible signals other than motor imagery. The NC class can produce rather complex patterns due to the sheer complexity of brain activities. On the other hand, CSP was designed for uni-modal Gaussians only. Thus, new techniques are called for that better account for high complexity and variability of EEG.

We propose a novel method for learning optimal linear transformation for detecting motor imagery EEG, i.e. classifying motor imagery EEG against NC EEG. First, we formulate the learning task into a problem of Bhattacharyya error bound minimization, where the NC class is described by a mixture of Gaussian (MOG) and the motor imagery class by a uni-modal Gaussian. We derive a numerical solution to estimate the error bound from training EEG samples, and use it to develop an algorithm for selecting optimal transformation. We evaluate this method in comparison with the state-of-the-arts, using a data set from BCI Competition IV [14].

## 2. METHOD

Denote a time sequence of multi-channel EEG sample by  $\mathbf{x}(t)$ , where  $\mathbf{x}$  is the vector consisting of electrical potentials at each EEG site on the scalp, and  $t$  is the time variable. A linear transformation of  $\mathbf{x}(t)$  using a matrix  $\mathbf{W}$  performs by

$$\mathbf{y}(t) = \mathbf{W}^T \mathbf{x}(t), \quad (1)$$

Here  $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_N]$  consists of  $N$  linear vectors. Effectively, each  $\mathbf{w}$  performs a spatial filtering that combines electrical potentials from different locations on the scalp.

For motor imagery EEG analysis, usually a band-pass filter is applied before hand to capture rhythmic process in EEG associated with motor imagery. As a result, the mean value of EEG samples becomes zero. Then the EEG samples from the motor imagery class  $\omega_p$  can be described by a zero-mean

uni-Gaussian distribution function  $p(\mathbf{x}_i|\omega_p)$  characterized by a covariance matrix  $\psi_p$ . The NC class  $\omega_n$  can be modelled by a MOG function  $p(\mathbf{x}|\omega_n)$  consisting of  $M$  sub-states

$$p(\mathbf{x}|\omega_n) \sim \sum_{j=1}^M \tilde{P}(\chi_j) \mathcal{N}(0, \tilde{\psi}_j) \quad (2)$$

where  $\tilde{P}(\chi_j)$  is the prior probability of each sub-state in NC, and  $\mathcal{N}()$  denotes the corresponding Gaussian distribution function with zero mean and covariance matrix  $\tilde{\psi}_j$ .

After linear transformation by  $W$ , the distributions of the two classes become

$$p(\mathbf{y}|\omega_p) \sim \mathcal{N}(0, \psi_{p(y)}), \quad (3)$$

$$p(\mathbf{y}|\omega_n) \sim \sum_{j=1}^M \tilde{P}(\chi_j) \mathcal{N}(0, \tilde{\psi}_{j(y)}), \quad (4)$$

where  $\psi_{p(y)} = \mathbf{W}^T \psi_p \mathbf{W}$  and  $\tilde{\psi}_{j(y)} = \mathbf{W}^T \tilde{\psi}_j \mathbf{W}$

The objective of optimal linear transformation is to find the  $W$  which results in the most separable classes, i.e. lowest Bayesian classification error rate for  $\mathbf{y} \in \omega_p$  vs.  $\mathbf{y} \in \omega_n$ . Instead of using the original Bayesian classification error rate which is difficult to compute, we introduce an estimate of upper bound, i.e. the Bhattacharyya bound [15].

$$\epsilon_B(\mathbf{y}) = \sqrt{P(\omega_p)P(\omega_n)} \int \sqrt{p(\mathbf{y}|\omega_p)p(\mathbf{y}|\omega_n)} d\mathbf{y}. \quad (5)$$

Now we derive an estimate of  $\epsilon_B$  from training samples. First, ignoring the constant positive factor  $\sqrt{P(\omega_p)P(\omega_n)}$ , the bound reduces to

$$\epsilon_B = \int \sqrt{p(\mathbf{y}|\omega_p)p(\mathbf{y}|\omega_n)} d\mathbf{y} \equiv \int \mu_1(\mathbf{y})\mu_2(\mathbf{y}) d\mathbf{y} \quad (6)$$

where  $\mu_1(\mathbf{y}) = \sqrt{p(\mathbf{y}|\omega_p)}$  and  $\mu_2(\mathbf{y}) = \sqrt{p(\mathbf{y}|\omega_n)}$ .

Straightforward expanding of  $\mu_1(\mathbf{y})$  gives

$$\mu_1(\mathbf{y}) = \left\{ \frac{(2\pi)^{-\frac{n_c}{2}}}{|2\psi_{p(y)}|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}\mathbf{y}^T (2\psi_{p(y)})^{-1} \mathbf{y}\right) \right\} \cdot \left\{ (2\pi)^{\frac{n_c}{4}} 2^{\frac{n_c}{2}} |\psi_{p(y)}|^{\frac{1}{4}} \right\} \quad (7)$$

The expression in the first curly bracket can be viewed as a probability density function

$$\mathcal{P}(\mathbf{y}) = \mathcal{N}(0, 2\psi_{p(y)}), \quad (8)$$

The expression in the second curly bracket, multiplied by  $\mu_2(\mathbf{y})$ , can be written as

$$\mathcal{Q}(\mathbf{y}) = (2\pi)^{\frac{n_c}{4}} 2^{\frac{n_c}{2}} |\psi_{p(y)}|^{\frac{1}{4}} \cdot \sqrt{\sum_{j=1}^M \tilde{P}(\chi_j) \mathcal{N}(0, \tilde{\psi}_{j(y)})}. \quad (9)$$

Therefore, the Bhattacharyya bound becomes

$$\epsilon_B = \int \mathcal{P}(\mathbf{y}) \mathcal{Q}(\mathbf{y}) d\mathbf{y}. \quad (10)$$

Since  $\mathcal{P}(\mathbf{y})$  is a probability distribution function, the Bhattacharyya bound can be viewed as an expectation

$$\epsilon_B = E[\mathcal{Q}(\mathbf{y})], \text{ with } \mathbf{y} \sim \mathcal{P}(\mathbf{y}) \quad (11)$$

To generate samples from  $\mathcal{P}(\mathbf{y})$ , we can simply manipulate the training samples of  $\{\hat{\mathbf{x}}_i | \hat{\mathbf{x}}_i \in \omega_p, i = 1, \dots, n_p\}$  by

$$(\mathbf{W}^T \sqrt{2}\hat{\mathbf{x}}) \sim \mathcal{N}(0, 2\mathbf{W}^T \psi_p \mathbf{W}) = \mathcal{P}(\mathbf{y}) \quad (12)$$

Therefore,

$$\epsilon_B = \lim_{n_p \rightarrow \infty} \frac{1}{n_p} \sum_{i=1}^{n_p} \mathcal{Q}(\mathbf{W}^T \sqrt{2}\hat{\mathbf{x}}_i) \quad (13)$$

With Eq. 13, we can estimate the Bhattacharyya bound from training samples. Due to the complexity of the expression, computing the global optimum  $W$  is beyond the scope of this paper. In this work we consider a simplified problem: construct a set of discriminative linear transformations and select the optimum one that maximizes  $\epsilon_B$  in Eq. 13.

Heuristically, we use CSP to construct a raw set of discriminative linear transformations. First, we pair the motor imagery class  $\omega_p$  with each sub-state of the NC class, and perform CSP to obtain discriminative linear projection vectors [11]. The linear projection vectors are collected from each pair above to form a raw set, from which we enumerate combinations of the linear vectors. Denote a combined projection matrix  $\mathbf{W}_{\mathcal{K}}$  determined by the set of selected vectors' indices  $\mathcal{K}$ . The optimum linear transformation can be obtained at

$$\mathcal{W}_{\text{opt}} = \underset{\mathcal{W}_{\mathcal{K}}}{\text{argmin}} \frac{1}{n_p} \sum_{i=1}^{n_p} \mathcal{Q}(\mathbf{W}_{\mathcal{K}}^T \sqrt{2}\hat{\mathbf{x}}_i) \quad (14)$$

### 3. EXPERIMENTAL RESULTS

#### 3.1. Experimental Setting

The proposed method is evaluated using the BCI Competition IV dataset 1 [14] ([http://ida.first.fhg.de/projects/bci/competition\\_iv/desc\\_1.html](http://ida.first.fhg.de/projects/bci/competition_iv/desc_1.html)). Briefly speaking, it consists of motor imagery EEG from 4 human subjects using a 59-channel EEG device plus 3 artificially generated ones. Only human EEG data are used here. During data collection, the subjects were guided by the computer to perform one of two pre-defined 4-second long motor imagery tasks. The motor imagery tasks were interleaved with the NC class, which comprised 2s of blank screen plus 2s of fixation period. The objective of motor imagery detection is then to predict from the EEG sequence if, at each time point, the subject is performing a motor imagery task or staying in NC state.

We incorporate the proposed method into a filter-band framework so as to address the variability of responsible rhythms of motor imagery from one person to another [6]. Briefly, we use a maximum mutual information criterion to select the most responsive spatio-spectral features. (Readers interested in the framework are referred to our past work [16] for details.) Based on these features, class labels (the motor imagery class as 1 and the NC class as 0) are predicted by a support vector regression machine from LibSVM toolbox(SVR) [17]. Besides, we tentatively use the following method to build the mixture of Gaussian model for the non-control state EEG. We extract different time intervals relative to the beginning of a motor imagery task and assign the EEG samples therein to different sub-states of the NC class. In particular, the EEG samples in the time period of [-3 -1] seconds is sub-state 1, while the rest samples constitute sub-state 2.

To evaluate the proposed method, a 5-fold cross-validation is performed that divides the data into 5 continuous segments. Each time, 4 segments are used for training the system, while the rest one is for testing. Two statistics of performance are investigated: 1. the mean square error (MSE) between the true class labels and the predicted ones; 2. the Area Under the ROC curve (AUC). AUC is equal to the probability that a classifier will rank a randomly chosen positive instance higher than a randomly chosen negative one [18]. Especially, here we pay attention to low false positive rates, which are of particular interest to detecting motor imagery EEG [13].

We also compare the proposed method (referred to *multi-modal* hereafter) with the traditional uni-modal approach (referred to as *uni-modal*) and the winning algorithm of the competition(referred to as *ComptWin*). The uni-modal method is the filter-bank common spatial pattern technique [16], which uses CSP for linear transformation. Note that the ComptWin method was developed by the present authors.

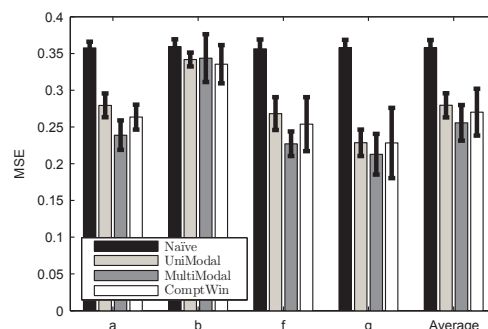
### 3.2. Results

Figure 1 illustrates the comparative MSE result. Evidently, all the methods yielded significantly lower MSE compared to the naïve method: a baseline method which constantly predicted the class label as 0. Compared with both the uni-modal approach or the competition winner method, the proposed method (*multi-modal*) reduced the averaged MSE by 0.02 to 0.03.

The advantage of the proposed method is more prominent in AUC, if low false positive rate is regarded. As shown in Table 1, *multi-modal* yielded a dramatically higher AUC at 0.04, averaged over all subjects, compared with 0.029 by *uni-modal* and 0.02 by *ComptWin*.

### 4. CONCLUSION

Optimum spatial filtering or linear transformation under zero-mean multi-dimensional Gaussian mixture conditions is an



**Fig. 1.** Comparison in MSE between the proposed method, uni-modal approach and the competition winner algorithm.

**Table 1.** Area under the ROC curve (AUC) for low false positive rate (FPR) range (FPR<0.1). The maximum possible AUC is 0.1 (if and only if the true positive rate TPR  $\equiv$  1) and the chance value is 0.005 (if TPR  $\equiv$  FPR).

subject	MI	ComptWin	Uni-Modal	Multi-Modal
'a'	MI-1	0.0162	0.0102	<b>0.0353</b>
	MI-2	0.0267	0.0470	<b>0.0499</b>
'b'	MI-1	0.0176	0.0233	<b>0.0287</b>
	MI-2	0.0073	0.0235	<b>0.0255</b>
'c'	MI-1	0.0260	0.0243	<b>0.0460</b>
	MI-2	0.0106	0.0240	<b>0.0443</b>
'd'	MI-1	0.0292	0.0380	<b>0.0455</b>
	MI-2	0.0370	0.0428	<b>0.0474</b>
Avg	Both	0.0213	0.0291	<b>0.0403</b>

intriguing problem, especially in learning optimum linear transformation for classification of motor imagery EEG. In contrast to traditional uni-modal Gaussian methods, we have proposed a novel method that can select optimum linear transformations under mixture of Gaussian conditions. Experimental study has clearly indicated that the proposed method can lead to a significantly better signal detection results especially in terms of AUC, in comparison with the state-of-the-art methods.

### 5. REFERENCES

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