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Brief paper

Adaptive actuator failure compensation control of uncertain nonlinear systems with guaranteed transient performance[☆]

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ABSTRACT

In order to accommodate actuator failures which are uncertain in time, pattern and value, we propose two adaptive backstepping control schemes for parametric strict feedback systems. Firstly a basic design scheme on the basis of existing approaches is considered. It is analyzed that, when actuator failures occur, transient performance of the adaptive system cannot be adjusted through changing controller design parameters. Then we propose a new controller design scheme based on a prescribed performance bound (PPB) which characterizes the convergence rate and maximum overshoot of the tracking error. It is shown that the tracking error satisfies the prescribed performance bound all the time. Simulation studies also verify the established theoretical results that the PPB based scheme can improve transient performance compared with the basic scheme, while both ensure stability and asymptotic tracking with zero steady state error in the presence of uncertain actuator failures.

1. Introduction

In practical control mechanisms, various system components such as actuators, sensors and processors may undergo abrupt failures individually or simultaneously during operation. The adverse effects due to the failures require being compensated to enhance the reliability and safety of the system. The research on accommodating such failures and maintaining acceptable system performance is particularly important for life-critical systems. For example, if an actuator is suddenly stuck and can no longer deflect a certain control surface in an aircraft, it may end with catastrophic events.

In this work, we focus on the problem of actuator failure accommodation. Many effective approaches have been developed to address this problem. They can be roughly classified into two categories: passive and active ones. Typical passive approaches (see Benosman & Lum, 2010; Liao, Wang, & Yang, 2002; Veillette, Medanic, & Perkins, 1992; Yang, Wang, & Soh, 2001; Zhao & Jiang, 1998), mainly based on robust control theory, use unchangeable controllers throughout the failure-free case and failure cases. The designed controller in passive methods is easily to be implemented since neither fault detection and diagnosis block nor controller reconfiguration is required. However, they are often conservative for changes of failure pattern or values and the achieved system performance based on worst case failure may not be satisfactory for each failure scenario. In contrast to the passive solution, active methods utilize control reconfiguration to adjust controllers in real time so that the impacts of the failures can be compensated and the stability as well as the acceptable performance of the system can be maintained. A number of reconfigurable control schemes have been proposed such as linear quadratic (Looze, Weiss, Eterno, & Barrett, 1985), multiple model (Boskovic, Jackson, Mehra, & Nguyen, 2009; Boskovic & Mehra, 2002b; Boskovic, Yu, & Mehra, 1998), model following (Bodson & Groszkiewicz, 1997), eigenstructure assignment (Jiang, 1994), sliding mode control based scheme (Corradini & Orlando, 2007), learning based approaches (Diao & Passino, 2001; Polycarpou, 2001; Zhang, Parisini, & Polycarpou, 2004; Zhang & Qin, 2008) and other estimation based designs (Fliess, Join, & Sira-Ramirez, 2008; Tsai, Lee, Cofie, Shienh, & Chen, 2006). Apart from these, adaptive control has also been proved effective in reconfigurable control of systems with actuator failures. In adaptive control systems, controllers are designed with the aid of adaptation mechanisms to handle large uncertain structural and parametric variation caused by failures. In fact, the adaptive control methodology is applied in most of the above cited results such as Bodson and Groszkiewicz (1997), Boskovic and Mehra (2002b), Boskovic et al. (2009, 1998), Diao and Passino (2001), Looze et al. (1985), Polycarpou (2001), Tsai et al. (2006), Zhang and Qin (2008) and Zhang et al. (2004). In Yang and Ye (2010), an indirect adaptive H_∞ fault tolerant controller is designed based on linear matrix

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inequality (LMI) for linear systems with known system parameters. In Tao, Joshi, and Ma (2001) and Tao, Chen, and Joshi (2002), an alternative class of adaptive design schemes, known as direct adaptive control, were proposed to solve tracking problems for linear systems with unknown system parameters in the presence of total loss of effectiveness (TLOE) of actuators. The results were further extended to nonlinear systems in Tang, Tao, and Joshi (2003, 2005, 2007) by using backstepping techniques. Compared to other approaches, direct adaptive control combines the following features: it is specially designed for systems with uncertainties in both system dynamics and actuator failures; it can provide theoretically provable asymptotic tracking and stabilization; explicit fault detection is not necessary and parameters of control reconfiguration are adaptively updated directly so that the controller structure is simple; available actuation redundancy can be used so that the control objectives are still achievable with some actuators suffering from TLOE. However to the best knowledge of authors, very few results in adaptive control are available on investigating how to guarantee the transient performance of the system, besides showing system stability and steady state tracking performance. Note that multiple model adaptive control, switching and tuning (MMST) approaches (see for instance Boskovic et al., 1998) can offer improved transient behaviors, but the bounds of failure magnitudes and the unknown parameters associated with failures are often needed in advance to construct a finite set of models which can cover the state space. Besides, a safe switching rule is required as mentioned in Anderson, Brinsmead, Liberzon, and Morse (2001) since an MMST closed loop is not intrinsically stable.

In this paper, we shall deal with the problem of guaranteeing transient performance in direct adaptive control of uncertain parametric strict feedback systems in the presence of actuator failures. To accommodate the effects due to actuator failures, we propose two adaptive backstepping control schemes for parametric strict feedback systems. Firstly a design scheme based on an existing approach in Tang et al. (2003) is considered. It is shown that the scheme can ensure both stability and asymptotic tracking as in Tang et al. (2003) and we name it as a basic scheme. Note that the backstepping technique (Krstic, Kanellakopoulos, & Kokotovic, 1995) provides a promising way to improve the transient performance of adaptive systems in terms of L_2 and L_∞ norms of the tracking error. However, the transient performance is tunable only if certain trajectory initialization can be performed, see for example Krstic et al. (1995) and Zhou, Wen, and Zhang (2004). Apparently, such trajectory initializations involving state-resetting actions are difficult at the time instants when actuator failures occur, because they are uncertain in occurrence time, pattern and value. Therefore, transient performance of the adaptive system cannot be adjusted through changing controller design parameters with the basic scheme. By employing prescribed performance bounds (PPB) originally presented in Bechlioulis and Rovithakis (2009), we propose a new controller design scheme. A prescribed performance bound can characterize the convergence rate and maximum overshoot of the tracking error. With certain transformation techniques, a new transformed system is obtained by incorporating the prescribed performance bound into the original nonlinear system. An adaptive controller, named as PPB based controller, is designed for the transformed system. It is established that the tracking error can be guaranteed within the prescribed error bound all the time as long as the stability of the transformed error system is ensured, without re-setting system states no matter whether actuator failures occur or not. Thus the transient performance is ensured and can be improved by varying certain design parameters. It is also shown that, with suitable modifications on the prescribed performance bound in Bechlioulis and Rovithakis (2009), the tracking error can converge to zero asymptotically.

The remaining part of the paper is organized as follows. In Section 2, the control problem is formulated. The design and analysis of a basic scheme based on existing approaches are given in Section 3. In Section 4, we present a new PPB based control scheme for guaranteed transient performance. Stability analysis is established. In Section 5, simulation studies verify the effectiveness of the two schemes and show that the PPB based scheme can dramatically improve transient performances compared with the basic design method. Finally, we conclude the paper in Section 6.

2. Plant models and problem formulation

We consider a class of multiple-input single-output nonlinear systems as follows,

$$\dot{\chi} = f_0(\chi) + \sum_{l=1}^{p} \theta_l f_l(\chi) + \sum_{i=1}^{m} b_i g_i(\chi) u_i \tag{1}$$

$$y = h(\chi) \tag{2}$$

where $\chi \in \Re^n$, $y \in \Re$ are the state and the output, $u_i \in \Re$ for $i = 1, 2, \dots, m$ is the ith input of the system, i.e. the output of the ith actuator, $f_l(\chi) \in \Re^n$ for $l = 0, 1, \dots, p$, $g_i(\chi) \in \Re^n$ for $i = 1, 2, \dots, m$ and $h(\chi)$ are known smooth nonlinear functions, θ_l for $l = 1, 2, \dots, p$ and b_i for $i = 1, \dots, m$ are unknown parameters and control coefficients.

We denote u_{ci} as the input of the ith ($i = 1, 2, \dots, m$) actuator. An actuator with its input equal to its output, i.e. $u_i = u_{ci}$, is regarded as a failure-free actuator. The types of actuator failures that may take place on the ith actuator can be modeled as follows,

$$u_i = \rho_i u_{ci} + u_{ki}, \quad \forall t \geq t_{iF} \tag{3}$$

$$\rho_i u_{ki} = 0, \quad i = 1, 2, \dots, m \tag{4}$$

where $\rho_i \in [0, 1)$, u_{ki} and t_{iF} are all unknown constants. (3) shows that the ith actuator fails suddenly from time t_{iF}. (4) implies the following three cases, in which two typical types of failures (TLOE and PLOE) are included,

(1) $\rho_i \neq 0$ and $u_{ki} = 0$,

In this case, $u_i = \rho_i u_{ci}$, where $0 < \rho_i < 1$. This indicates partial loss of effectiveness (PLOE). For example, $\rho_i = 70\%$ means that the ith actuator loses 30% of its effectiveness.

(2) $\rho_i = 0$ and $u_{ki} \neq 0$,

$\rho_i = 0$ indicates that u_i can no longer be influenced by the control inputs u_{ci}. The fact that u_i is stuck at an unknown value u_{ki} is known as total loss of effectiveness (TLOE). As described in Boskovic and Mehra (1999, 2002a), $u_i = u_{ci}(t_{iF}^-)$ is the Lock-in-Place case of TLOE. However, in the Hard-Over case of TLOE, u_i takes either the upper position limit $\bar{u}_{ci}$ or lower limit $\underline{u}_{ci}$, i.e. $u_{ki} = \bar{u}_{ci}$ or $u_{ki} = \underline{u}_{ci}$.

(3) $\rho_i = 0$ and $u_{ki} = 0$.

This case corresponds to the Float type of TLOE in Boskovic and Mehra (1999, 2002a).

Remark 1. Note that actuators working in the failure-free case can also be represented as (3) with $\rho_i = 1$, $u_{ki} = 0$ for $t \geq 0$.

Since fault repairing is sometimes hardly implemented in many practical online cases, for example during the flight of an apparatus, possible changes from normal case to any one of the failure cases are assumed unidirectional. That is, the values of ρ_i can change only from $\rho_i = 1$ to $\rho_i = 0$ or some values with $0 < \rho_i < 1$). The uniqueness of t_{iF} indicates that a failure occurs only once on the ith actuator. Hence there exists a finite T_r denoting the time instant of the last failure. Such an assumption on the finite number of actuator failures can be found in many previous results, such as Boskovic et al. (1998), Tang et al. (2003, 2005, 2007) and Tao et al. (2001, 2002).

The control objects in this paper are as follows,

• The effects of considered types of actuator failures can be compensated so that the global stability of the closed-loop system is ensured and asymptotic tracking can be achieved.

• Tracking error $e(t) = y(t) - y_r(t)$ can be preserved within certain given prescribed performance bounds (PPB). In addition, transient performance in terms of the convergence rate and maximum overshoot of $e(t)$ can be improved by tuning design parameters.

To achieve the control objectives, the following assumptions are applied.

Assumption 1. The plant (1)–(2) is so constructed that for any TLOE type of actuator failures up to $m-1$, the remaining actuators can still achieve a desired control objective.

Assumption 2. $g_i(\chi) \in \text{span}\{g_0(\chi)\}$, $g_0(\chi) \in \Re^n$, for $i = 1, 2, \ldots, m$ and the nominal system $\dot{\chi} = f_0(\chi) + F(\chi)\theta + g_0(\chi)u_0$, $y = h(\chi)$ with $u_0 \in \Re$, is transformable into the parametric-strict-feedback form with relative degree ϱ, where $F(\chi) = [f_1(\chi), f_2(\chi), \ldots, f_p(\chi)] \in \Re^{n\times p}$, $\theta = [\theta_1, \theta_2, \ldots, \theta_p]^T \in \Re^p$.

Remark 2. As discussed in Boskovic and Mehra (1999), Tang et al. (2003, 2005) and Tao et al. (2001, 2002), Assumption 1 is a basic assumption to ensure the controllability of the plant and the existence of a nominal solution for the actuator failure compensation problem. Nevertheless, all actuators are allowed to suffer from PLOE type of actuator failures simultaneously.

Assumption 2 corresponds to the first actuator structure condition in Tang et al. (2003) that the nonlinear actuator functions $g_i(\chi)$ for $i = 1, 2, \ldots, m$ have similar structures.

As presented in Tang et al. (2003), based on Assumption 2, there exists a diffeomorphism $[x, \xi]^T = T(\chi)$ where $x \in \Re^{\varrho}$, $\xi \in \Re^{n-\varrho}$ such that the plant (1)–(2) can be transformed to the following form by incorporating the actuator failure model (3).

$$
\begin{aligned}
\dot{x}_j &= x_{j+1} + \varphi_j^T(x_1, \ldots, x_j)\theta, \quad j = 1, 2, \ldots, \varrho - 1,\\
\dot{x}_\varrho &= \varphi_0(x, \xi) + \varphi_\varrho^T(x, \xi)\theta + \sum_{i=1}^{m} b_i\beta_i(x, \xi)(\rho_i u_{ci} + u_{ki}),\\
\dot{\xi} &= \Psi(x, \xi) + \Phi(x, \xi)\theta,\\
y &= x_1.
\end{aligned}
\tag{5}
$$

Note that the transformed system (5) is the plant to be stabilized and to which we will apply the backstepping technique. Three additional assumptions are required.

Assumption 3. The reference signal $y_r(t)$ and its first ϱth order derivatives $y_r^{(j)}(j = 1, \ldots, \varrho)$ are known, bounded, and piecewise continuous.

Assumption 4. $\beta_i(x, \xi) \neq 0$, the signs of b_i, i.e. $\text{sgn}(b_i)$, for $i = 1, \ldots, m$ are known.

Assumption 5. The subsystem $\dot{\xi} = \Psi(x, \xi) + \Phi(x, \xi)\theta$ is input-to-state stable with respect to x as the input.

3. Basic control design for adaptive failure compensation

The main purpose of designing basic controllers is to carry out comparisons with our prescribed performance bounds (PPB) based controllers to be proposed later. It will be noted that a basic controller, from its design approaches and performances, can be considered as a representative of currently available direct adaptive failure compensation controllers.

The design of u_{ci} is generated by following the procedures in Tang et al. (2003, Section 3.1) with slight modifications. Thus only some important steps are presented. Meanwhile, stability analysis will be sketched briefly.

We introduce ϱ error variables

$$z_1 = y - y_r \tag{6}$$

$$z_j = x_j - \alpha_{j-1} - y_r^{(j-1)} \quad \text{for } j = 2, \ldots, \varrho \tag{7}$$

where α_j is the virtual control determined at the jth step that

$$
\begin{aligned}
\alpha_j &= -z_{j-1} - c_j z_j - \omega_j^T\hat{\theta} + \sum_{k=1}^{j-1}\left(\frac{\partial\alpha_{j-1}}{\partial x_k}x_{k+1} + \frac{\partial\alpha_{j-1}}{\partial y_r^{(k-1)}}y_r^{(k)}\right)\\
&\quad + \frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\Gamma\tau_j + \sum_{k=2}^{j-1}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma\omega_j z_k, \quad \text{for } j = 2, \ldots, \varrho - 1
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
\alpha_\varrho &= -z_{\varrho-1} - c_\varrho z_\varrho - \varphi_0 - \omega_\varrho^T\hat{\theta} + \sum_{k=1}^{\varrho-1}\left(\frac{\partial\alpha_{\varrho-1}}{\partial x_k}x_{k+1}\right.\\
&\quad \left. + \frac{\partial\alpha_{\varrho-1}}{\partial y_r^{(k-1)}}y_r^{(k)}\right) + \frac{\partial\alpha_{\varrho-1}}{\partial\hat{\theta}}\Gamma\tau_\varrho + \sum_{k=2}^{\varrho-1}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma\omega_\varrho z_k + y_r^{(\varrho)}
\end{aligned}
\tag{9}
$$

where

$$\tau_1 = \omega_1 z_1 \tag{10}$$

$$\tau_j = \tau_{j-1} + \omega_j z_j, \quad \text{for } j = 2, \ldots, \varrho \tag{11}$$

$$\omega_j = \varphi_j - \sum_{k=1}^{j-1}\frac{\partial\alpha_{j-1}}{\partial x_k}\varphi_k, \quad \text{for } j = 1, \ldots, \varrho. \tag{12}$$

The control law and parameter update laws are obtained as follows,

$$u_{ci} = \text{sgn}(b_i)\frac{1}{\beta_i}\hat{\kappa}^T w, \quad \text{for } i = 1, 2, \ldots, m \tag{13}$$

$$\dot{\hat{\theta}} = \Gamma\tau_\varrho \tag{14}$$

$$\dot{\hat{\kappa}} = -\Gamma_\kappa w z_\varrho \tag{15}$$

where

$$\hat{\kappa} = [\hat{\kappa}_1, \hat{\kappa}_2^T]^T, \qquad \hat{\kappa}_2 = [\hat{\kappa}_{2,1}, \hat{\kappa}_{2,2}, \ldots, \hat{\kappa}_{2,m}]^T \tag{16}$$

and $w = [\alpha_\varrho, \beta^T]^T$, $\beta = [\beta_1, \beta_2, \ldots, \beta_m]^T$. $\hat{\kappa}$ and $\hat{\theta}$ are the estimates of κ and θ respectively. κ represents the desired vector that can be chosen if b_i and failures are known. The details of κ will be given in later discussions. $\hat{\kappa}_{2,j}$ for $j = 1, 2, \ldots, m$ denotes the jth entry of $\hat{\kappa}_2$. Γ, Γ_κ are positive definite matrices and c_j for $j = 1, 2, \ldots, m$ are positive constants, all chosen by users. The controllers designed are named as basic controllers since they can only ensure system stability and a tracking property similar to those in Tang et al. (2003), as analyzed below.

3.1. Stability analysis

For the basic controllers developed, we establish the following result.

Theorem 1. *Consider the closed-loop adaptive system consisting of the plant (1)–(2), the controller (13), the parameter update laws (14)–(15) in the presence of possible actuator failures (3) and (4) under Assumptions 1–5. The boundedness of all the signals are ensured and the asymptotic tracking is achieved, i.e.* $\lim_{t\to\infty}[y(t) - y_r(t)] = 0$.

Proof. As presented in Remark 1, there are a finite number of time instants T_k for $k = 1, 2, \ldots, r$ $(r \leq m)$ at which one or more of the actuators fail. T_r is referred as the last time of failure in Remark 1. Suppose during time interval $[T_{k-1}, T_k)$, where $k = 1, \ldots, r+1$, $T_0 = 0$, $T_{r+1} = \infty$, there are p_k $(p_k \geq 1)$ failed actuators $j_1, j_2, \ldots, j_{p_k}$ and the failure pattern will not change until time T_k. Among these p_k failed actuators, q_{tot_k} actuators $j_{1,1}, j_{1,2}, \ldots, j_{1,q_{\text{tot}_k}}$ suffer from TLOE and q_{par_k} actuators $j_{2,1}, j_{2,2}, \ldots, j_{2,q_{\text{par}_k}}$ undergo PLOE. We define a set $P_k = \{j_1, j_2, \ldots, j_{p_k}\}$ and two subsets of P_k that $Q_{\text{tot}_k} = \{j_{1,1}, j_{1,2}, \ldots, j_{1,q_{\text{tot}_k}}\}$ and $Q_{\text{par}_k} = \{j_{2,1}, j_{2,2}, \ldots, j_{2,q_{\text{par}_k}}\} = P_k \setminus Q_{\text{tot}_k}$. We define a positive definite function V_{k-1} during $[T_{k-1}, T_k)$ as

$$V_{k-1} = \frac{1}{2}z^T z + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta} + \sum_{i=1, i\notin Q_{\text{tot}_k}}^{m} \frac{\rho_i |b_i|}{2}\tilde{\kappa}^T \Gamma_\kappa^{-1}\tilde{\kappa} \tag{17}$$

where $z = [z_1, z_2, \ldots, z_\varrho]^T$. If b_i, ρ_i and u_{kh} for $i = 1, 2, \ldots, m$, $h \in Q_{\text{tot}_k}$ are known, κ is a desired constant vector which can be chosen to satisfy that

$$\sum_{i=1, i\notin Q_{\text{tot}_k}}^{m} |b_i|\rho_i \kappa^T w = \alpha_\varrho - \sum_{h\in Q_{\text{tot}_k}} b_h \beta_h u_{kh}$$
$$\Rightarrow \kappa_1 = \frac{1}{\sum_{i=1, i\notin Q_{\text{tot}_k}}^{m} |b_i|\rho_i}, \kappa_{2,h} = \frac{-b_h u_{kh}}{\sum_{i=1, i\notin Q_{\text{tot}_k}}^{m} |b_i|\rho_i},$$
$$\text{for } h \in Q_{\text{tot}_k} \quad \text{and} \quad \kappa_{2,h} = 0, \quad h \in \{1, 2, \ldots, m\}\setminus Q_{\text{tot}_k}. \tag{18}$$

From the design through (6)–(15), the time derivative of V_{k-1} is computed as

$$\dot{V}_{k-1} = -\sum_{j=1}^{\varrho} c_j z_j^2, \quad k = 1, 2, \ldots, r+1. \tag{19}$$

We define $V_{k-1}(T_k^-) = \lim_{\Delta t\to 0^-} V_{k-1}(T_k + \Delta t)$ and $V_{k-1}(T_{k-1}^+) = \lim_{\Delta t\to 0^+} V_{k-1}(T_{k-1}+\Delta t) = V_{k-1}(T_{k-1})$. If we let a function $V(t) = V_{k-1}(t)$, for $t \in [T_{k-1}, T_k)$, $k = 1, \ldots, r+1$, $V(t)$ is thus a piecewise continuous function. From (19), we have V_{k-1} is non-increasing during the time interval $[T_{k-1}, T_k)$ and $V_{k-1}(T_k^-) \leq V_{k-1}(T_{k-1}^+)$. When $k = 1$, $V_0(t) \leq V_0(0)$ for $t \in [0, T_1)$, the boundedness of $z(t)$, $\tilde{\theta}(t)$ and $\tilde{\kappa}(t)$ for $t \in [0, T_1)$ is ensured since the initial value $V_0(0)$ is finite. $V_0(T_1^-) \leq V_0(0)$. When $k > 1$, $V_{k-1}(t)$ is bounded if $V_{k-1}(T_{k-1}^+)$ is bounded. Observing (17), at the time instant $t = T_k$, $V_{k-1}(T_k^-)$ is changed to $V_k(T_k^+) = V_{k-1}(T_k^-) + \Delta V_k$, where ΔV_k is due to the changes on the coefficients in front of $\kappa^T \Gamma_\kappa \kappa$ and possible jumpings on κ and ΔV_k is finite. This implies that the initial value $V_k(T_k^+)$ for $[T_k, T_{k+1})$ is bounded if the final value $V_{k-1}(T_k^-)$ for $[T_{k-1}, T_k)$ is bounded. The above facts conclude the boundedness of $z(t)$, $\tilde{\theta}(t)$, $\tilde{\kappa}(t)$ for $t \in [0, \infty)$ and $z(t) \in L^2$. From (13), control signals u_{ci} for $i = 1, 2, \ldots, m$ are also bounded. From (6)–(7) and Assumption 3, $x(t)$ is bounded. From Assumption 5, $\xi(t)$ is bounded with respect to $x(t)$ as the input. The closed-loop stability is then established. Noting $\dot{z} \in L^\infty$, it follows that $\lim_{t\to\infty} z(t) = 0$. From (6), the asymptotic tracking is achieved, i.e. $\lim_{t\to\infty}[y(t) - y_r(t)] = 0$. □

3.2. Transient performance analysis

We firstly define two norms $L_{2[a,b]}$ and $L_{\infty[a,b]}$ as follows.

$$\|x(t)\|_{2[a,b]} = \left(\int_a^b |x(t)|^2 \mathrm{d}t\right)^{1/2} \tag{20}$$

$$\|x(t)\|_{\infty[a,b]} = \sup_{t\in[a,b]} |x(t)|. \tag{21}$$

We then derive the bounds for the tracking error $z_1(t)$ in terms of both $L_{2[T_{k-1},t_k]}$ and $L_{\infty[T_{k-1},t_k]}$ norms, where $k = 1, \ldots, r+1$, $t_k \in (T_{k-1}, T_k)$ with $T_0 = 0$, $T_{r+1} = \infty$. From (19), we have

$$\dot{V}_{k-1} \leq -c_1 z_1^2 \leq 0. \tag{22}$$

It follows that

$$\begin{aligned}\|z_1(t)\|^2_{2[T_{k-1},t_k]} &= \int_{T_{k-1}}^{t_k} z_1(t)^2 \mathrm{d}t \leq -\frac{1}{c_1}\int_{T_{k-1}}^{t_k} \dot{V}_{k-1}(t)\mathrm{d}t \\ &= -\frac{1}{c_1}\left[V_{k-1}(T_{k-1}) - V_{k-1}(t_k)\right] \leq \frac{1}{c_1} V_{k-1}(T_{k-1})\end{aligned} \tag{23}$$

and

$$z_1(t)^2 \leq 2V_{k-1}(t) \leq 2V_{k-1}(T_{k-1}), \quad t \in [T_{k-1}, T_k). \tag{24}$$

Define that $\|\tilde{\theta}(T_{k-1})\|^2_{\Gamma^{-1}} = \tilde{\theta}^T(T_{k-1})\Gamma^{-1}\tilde{\theta}(T_{k-1})$ and $\|\tilde{\kappa}(T_{k-1})\|^2_{\Gamma_\kappa^{-1}} = \tilde{\kappa}^T(T_{k-1})\Gamma_\kappa^{-1}\tilde{\kappa}(T_{k-1})$. From (23) and (24), we have

$$\begin{aligned}\|z_1(t)\|_{2[T_{k-1},t_k]} \leq \frac{1}{\sqrt{2c_1}}\Bigg[& z^T z(T_{k-1}) + \|\tilde{\theta}(T_{k-1})\|^2_{\Gamma^{-1}} \\ &+ \sum_{i=1, i\notin Q_{\text{tot}_k}}^{m} \rho_i |b_i| \|\tilde{\kappa}(T_{k-1})\|^2_{\Gamma_\kappa^{-1}}\Bigg]^{\frac{1}{2}}\end{aligned} \tag{25}$$

$$\begin{aligned}\|z_1(t)\|_{\infty[T_{k-1},t_k]} \leq \Bigg[& z^T z(T_{k-1})^2 + \|\tilde{\theta}(T_{k-1})\|^2_{\Gamma^{-1}} \\ &+ \sum_{i=1, i\notin Q_{\text{tot}_k}}^{m} \rho_i |b_i| \|\tilde{\kappa}(T_{k-1})\|^2_{\Gamma_\kappa^{-1}}\Bigg]^{\frac{1}{2}}.\end{aligned} \tag{26}$$

Based on these results, we have the following discussions.

(1) When $k = 1$, (25)–(26) gives the bounds of the $L_{2[0,t_1]}$ and $L_{\infty[0,t_1]}$ norms $(t_1 < T_1)$ for the tracking error $z_1(t)$ before the first failure occurs. From the definition in (7), the initial value $z(0)$ may increase by increasing c_1, Γ, Γ_κ. By performing trajectory initialization, i.e. setting $z(0) = 0$ (see for instance Krstic et al. (1995) and Zhou et al. (2004)), the transient performance of $z_1(t)$ in the sense of these two norms during $[0, T_1)$ can be improved by increasing c_1 and/or Γ, Γ_κ.

(2) However, it is impossible to perform trajectory initialization at each T_{k-1} for $k > 1$ because the failure time, type and value are all unknown. Thus the initial value $V_{k-1}(T_{k-1})$ during $[T_{k-1}, T_k)$ for $k > 1$ may be increased by increasing c_1, Γ, Γ_κ. Moreover, it cannot be guaranteed from 1) that the final value $V_0(T_1^-)$ during $[0, T_1)$ is smaller with larger c_1, Γ, Γ_κ. Hence a larger $V_0(T_1^-)$ may result in a larger initial value $V_1(T_1)$ for the next interval. Therefore, the conclusion on improving transient performance in terms of either the $L_{2[T_{k-1,t_k}]}$ or $L_{\infty[T_{k-1,t_k}]}$ norm by adjusting c_1, Γ, Γ_κ cannot be drawn for $z_1(t)$ with $t \geq T_1$.

To guarantee transient performance of the tracking error, especially when failures take place, an alternative approach based on prescribed performance bounds proposed in Bechlioulis and Rovithakis (2009) is employed to design adaptive compensation controllers.

4. Prescribed performance bounds (PPB) based control design

The objective in this section is to ensure the transient performance in the sense that the tracking error $e(t) = y(t) -$

$y_r(t)$ is preserved within a specified PPB all the time no matter when actuator failures occur, in addition to stability and steady state tracking properties. Similar to Bechlioulis and Rovithakis (2009), the characterization of a prescribed performance bound is required. To do this, a decreasing smooth function $\eta(t)$: $R_+ \to R_+ \setminus \{0\}$ with $\lim_{t\to\infty}\eta(t) = \eta_\infty > 0$ is firstly chosen as a performance function. For example, $\eta(t) = (\eta_0 - \eta_\infty)e^{-at} + \eta_\infty$ where $\eta_0 > \eta_\infty$ and $a > 0$. Then by satisfying the condition that

$$-\underline{\delta}\eta(t) < e(t) < \bar{\delta}\eta(t), \quad \forall t \geq 0 \tag{27}$$

where $0 < \underline{\delta}, \bar{\delta} \leq 1$ are prescribed scalars, the objective of guaranteeing transient performance can be achieved.

Remark 3. (1) From (27), $\bar{\delta}\eta(0)$ and $-\underline{\delta}\eta(0)$ serve as the upper bound of the maximum overshoot and lower bound of the undershoot (i.e. negative overshoot) of $e(t)$, respectively. The decreasing rate of $\eta(t)$ introduces a lower bound on the convergence speed of $e(t)$.

(2) If an actuator failure occurs when $\eta(t)$ approaches η_∞ closely enough, $-\underline{\delta}(\eta_\infty+\epsilon) < e(t) < \bar{\delta}(\eta_\infty+\epsilon)$ will be satisfied, where $\epsilon > 0$ is sufficiently small. This implies that there will be no occurrence of unacceptable large overshooting due to such an actuator failure.

(3) No trajectory initialization action is required, hence the transient performance of the system can be guaranteed without a priori knowledge of the failure time, type and value. In fact, by changing the design parameters of function $\eta(t)$ and the positive scalars $\underline{\delta}$, $\bar{\delta}$, the transient performance in terms of the convergence rate and maximum overshoot of tracking error $e(t)$ can be improved.

4.1. Transformed system

Solving the control problem satisfying the "constrained" error condition (27) can be transformed to solving a problem with boundedness of signals as the only requirements. Moreover, to achieve asymptotic tracking, asymptotic stabilization of the transformed system to be constructed is essential. To do these, we design a smooth and strictly increasing function $S(\nu)$ with the following properties:

(i)

$$-\underline{\delta} < S(\nu) < \bar{\delta} \tag{28}$$

(ii)

$$\lim_{\nu\to+\infty} S(\nu) = \bar{\delta}, \ \lim_{\nu\to-\infty} S(\nu) = -\underline{\delta} \tag{29}$$

(iii)

$$S(0) = 0. \tag{30}$$

From properties (i) and (ii) of $S(\nu)$, performance condition (27) can be expressed as

$$e(t) = \eta(t)S(\nu). \tag{31}$$

Because of the strict monotonicity of $S(\nu)$ and the fact that $\eta(t) \neq 0$, the inverse function

$$\nu = S^{-1}\left(\frac{e(t)}{\eta(t)}\right) \tag{32}$$

exists. We call ν as a transformed error. If $-\underline{\delta}\eta(0) < e(0) < \bar{\delta}\eta(0)$, and $\nu(t)$ is ensured bounded for $t \geq 0$ by our designed controller, we will have that $-\underline{\delta} < \frac{e(t)}{\eta(t)} < \bar{\delta}$. Furthermore, from property (iii) of $S(\nu)$, asymptotic tracking (i.e. $\lim_{t\to\infty} e(t) = 0$) can be achieved if $\lim_{t\to\infty}\nu(t) = 0$ is followed.

In this paper, we design $S(\nu)$ as

$$S(\nu) = \frac{\bar{\delta}e^{(\nu+r)} - \underline{\delta}e^{-(\nu+r)}}{e^{(\nu+r)} + e^{-(\nu+r)}} \tag{33}$$

where $r = \frac{\ln(\underline{\delta}/\bar{\delta})}{2}$. It can be easily shown that $S(\nu)$ has the properties (i)–(iii). The transformed error $\nu(t)$ is solved as

$$\nu = S^{-1}(\lambda(t)) = \frac{1}{2}\ln(\bar{\delta}\lambda(t) + \bar{\delta}\underline{\delta}) - \frac{1}{2}\ln(\underline{\delta}\bar{\delta} - \underline{\delta}\lambda(t)) \tag{34}$$

where $\lambda(t) = e(t)/\eta(t)$. We compute the time derivative of ν as

$$\begin{aligned}\dot{\nu} &= \frac{\partial S^{-1}}{\partial \lambda}\dot{\lambda} = \frac{1}{2}\left[\frac{1}{\lambda+\underline{\delta}} - \frac{1}{\lambda-\bar{\delta}}\right]\left(\frac{\dot{e}}{\eta} - \frac{e\dot{\eta}}{\eta^2}\right)\\ &= \zeta\left(\dot{e} - \frac{e\dot{\eta}}{\eta}\right) = \zeta\left(\dot{y} - \dot{y}_r - \frac{e\dot{\eta}}{\eta}\right)\end{aligned} \tag{35}$$

where ζ is defined as

$$\zeta = \frac{1}{2\eta}\left[\frac{1}{\lambda+\underline{\delta}} - \frac{1}{\lambda-\bar{\delta}}\right]. \tag{36}$$

Owing to the property (i) of $S(\nu)$ and (31), ζ is well defined and $\zeta \neq 0$. We now incorporate the prescribed performance bound into the original nonlinear system (5). By replacing the equation of $\dot{x}_1$ with $\dot{\nu}$, (5) can be transformed to

$$\dot{\nu} = \zeta\left(x_2 + \varphi_1^T\theta - \dot{y}_r - \frac{e\dot{\eta}}{\eta}\right) \tag{37}$$

$$\dot{x}_j = x_{j+1} + \varphi_j^T\theta, \quad j = 2, \ldots, \varrho - 1 \tag{38}$$

$$\dot{x}_\varrho = \varphi_0 + \varphi_\varrho^T\theta + \sum_{i=1}^m b_i\beta_i(\rho_i u_{ci} + u_{ki}) \tag{39}$$

$$\dot{\xi} = \Psi(x,\xi) + \Phi(x,\xi)\theta. \tag{40}$$

4.2. Controller design

Compared with the basic design, the major difference lies in the first two steps in performing the backstepping procedure. Thus the details of Step 1 and Step 2 are elaborated. Define

$$z_1 = \nu \tag{41}$$

$$z_j = x_j - \alpha_{j-1} - y_r^{(j-1)}, \quad j = 2, \ldots, \varrho. \tag{42}$$

Step 1. From (37) and (41) and the definition of z_2 in (42), we have

$$\dot{z}_1 = \zeta\left(z_2 + \alpha_1 + \varphi_1^T\theta - \frac{e\dot{\eta}}{\eta}\right). \tag{43}$$

To stabilize (43), α_1 is designed as

$$\alpha_1 = -\frac{c_1 z_1}{\zeta} - \varphi_1^T\hat{\theta} + \frac{e\dot{\eta}}{\eta} \tag{44}$$

where c_1 is a positive constant and $\hat{\theta}$ is an estimate of θ. We define a positive definite function $\bar{V}_1$ as

$$\bar{V}_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{45}$$

where $\tilde{\theta} = \hat{\theta} - \theta$, Γ is a positive definite design matrix. Then

$$\dot{\bar{V}}_1 = -c_1 z_1^2 + \zeta z_1 z_2 + \tilde{\theta}^T\Gamma^{-1}(\dot{\hat{\theta}} - \Gamma\varphi_1 z_1\zeta). \tag{46}$$

We choose the first tuning function τ_1 as

$$\tau_1 = \varphi_1 z_1 \zeta. \tag{47}$$

It follows that

$$\dot{\bar{V}}_1 = -c_1 z_1^2 + \zeta z_1 z_2 + \tilde{\theta}^T \Gamma^{-1}(\dot{\hat{\theta}} - \Gamma \tau_1). \tag{48}$$

Step 2. We firstly clarify the arguments of the function α_1. By examining (44) along with (34) and (36), we see that α_1 is a function of x_1, y_r, η, $\dot{\eta}$ and $\hat{\theta}$. Differentiating (42) for $j = 2$, with the help of (38) and the definition that $z_3 = x_3 - \alpha_2 - y_r^{(2)}$, we obtain

$$\begin{aligned}\dot{z}_2 &= \dot{x}_2 - \dot{\alpha}_1 - y_r^{(2)} \\ &= z_3 + \alpha_2 + \varphi_2^T\theta - \frac{\partial \alpha_1}{\partial x_1}(x_2 + \varphi_1^T\theta) - \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r - \frac{\partial \alpha_1}{\partial \eta}\dot{\eta} \\ &\quad - \frac{\partial \alpha_1}{\partial \dot{\eta}}\eta^{(2)} - \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}}.\end{aligned} \tag{49}$$

With the second tuning function τ_2 chosen as

$$\tau_2 = \tau_1 + \omega_2 z_2 \tag{50}$$

where

$$\omega_2 = \varphi_2 - \frac{\partial \alpha_1}{\partial x_1}\varphi_1. \tag{51}$$

The second stabilization function α_2, if $z_3 = 0$, is designed as

$$\begin{aligned}\alpha_2 &= -\zeta z_1 - c_2 z_2 - \left(\varphi_2 - \frac{\partial \alpha_1}{\partial x_1}\varphi_1\right)^T \hat{\theta} + \frac{\partial \alpha_1}{\partial x_1} x_2 \\ &\quad + \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r + \sum_{k=1}^{2} \frac{\partial \alpha_1}{\partial \eta^{(k-1)}}\eta^{(k)} + \frac{\partial \alpha_1}{\partial \hat{\theta}}\Gamma \tau_2.\end{aligned} \tag{52}$$

Denote $\bar{x}_j = (x_1, \ldots, x_j)$, $\bar{\eta}^{(j)} = (\eta, \dot{\eta}, \ldots, \eta^{(j)})$ and $\bar{y}_r^{(j-1)} = (y_r, \dot{y}_r, \ldots, y_r^{(j-1)})$. Note that in the backstepping procedure, α_j for $j \geq 2$, is a function of $\bar{x}_j$, $\bar{\eta}^{(j)}$, $\bar{y}_r^{(j-1)}$, $\hat{\theta}$.

Define a positive definite function at this step as

$$\bar{V}_2 = \bar{V}_1 + \frac{1}{2} z_2^2. \tag{53}$$

From (48), (49) and (52), the time derivative of V_2 can be computed as

$$\begin{aligned}\dot{\bar{V}}_2 &= -c_1 z_1^2 - c_2 z_2^2 + z_2 z_3 + \tilde{\theta}^T \Gamma^{-1}(\dot{\hat{\theta}} - \Gamma\tau_2) \\ &\quad - \frac{\partial \alpha_1}{\partial \hat{\theta}}(\dot{\hat{\theta}} - \Gamma \tau_2) z_2\end{aligned} \tag{54}$$

Step j where $j = 3, \ldots, \varrho$

$$\begin{aligned}\alpha_j &= -z_{j-1} - c_j z_j - \omega_j^T \hat{\theta} + \sum_{k=1}^{i} \frac{\partial \alpha_{j-1}}{\partial \eta^{(k-1)}}\eta^{(k)} + \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \\ &\quad \times \Gamma \tau_j + \sum_{k=2}^{j-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}\Gamma \omega_j z_k + \sum_{k=1}^{j-1}\left(\frac{\partial \alpha_{j-1}}{\partial x_k}x_{k+1} + \frac{\partial \alpha_{j-1}}{\partial y_r^{(k-1)}} y_r^{(k)}\right), \\ &\quad j = 2, \ldots, \varrho - 1\end{aligned} \tag{55}$$

$$\begin{aligned}\alpha_\varrho &= -z_{\varrho-1} - c_\varrho z_\varrho - \varphi_0 - \omega_\varrho^T \hat{\theta} \\ &\quad + \sum_{k=1}^{\varrho-1}\left(\frac{\partial \alpha_{\varrho-1}}{\partial x_k}x_{k+1} + \frac{\partial \alpha_{\varrho-1}}{\partial y_r^{(k-1)}}y_r^{(k)}\right) + \sum_{k=1}^{\varrho}\frac{\partial \alpha_{\varrho-1}}{\partial \eta^{(k-1)}}\eta^{(k)} \\ &\quad + \frac{\partial \alpha_{\varrho-1}}{\partial \hat{\theta}}\Gamma \tau_\varrho + \sum_{k=2}^{\varrho-1}\frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}\Gamma \omega_\varrho z_k + y_r^{(\varrho)}\end{aligned} \tag{56}$$

$$\tau_j = \tau_{j-1} + \omega_j z_j \tag{57}$$

$$\omega_j = \varphi_j - \sum_{k=1}^{j-1}\frac{\partial \alpha_{j-1}}{\partial x_k}\varphi_k, \quad j = 2, \ldots, \varrho. \tag{58}$$

Control laws and parameter update laws are determined at the ϱth step as

$$u_{ci} = \operatorname{sgn}(b_i)\frac{1}{\beta_i}\hat{\kappa}^T w, \quad \text{for } i = 1, \ldots, m \tag{59}$$

$$\dot{\hat{\theta}} = \Gamma \tau_\varrho \tag{60}$$

$$\dot{\hat{\kappa}} = -\Gamma_\kappa w z_\varrho. \tag{61}$$

Note that u_{ci}, $\dot{\hat{\theta}}$ and $\dot{\hat{\kappa}}$ are designed in the same form as in (13)–(15) with the signals α_ϱ, τ_ϱ and constructed $w = [\alpha_\varrho, \beta]$ changed appropriately.

4.3. Stability analysis

For an arbitrary initial tracking error $e(0)$, we can select $\eta(0)$, $\bar{\delta}$ and $\underline{\delta}$ to satisfy that $-\underline{\delta}\eta(0) < e(0) < \bar{\delta}\eta(0)$. As discussed in Remark 3, the transient performance of $e(t)$ can be improved by tuning the design parameters $\bar{\delta}$, $\underline{\delta}$ and parameters of $\eta(t)$ including its speed of convergence, η_∞ at a steady state as long as $e(t)$ is preserved within a specified PPB as described in (27). Observing the generated transformed error $\nu = S^{-1}\left(\frac{e(t)}{\eta(t)}\right)$ and the injective property of $S(\nu)$, we conclude that (27) is satisfied if $\nu(t) \in L^\infty$ with the designed controllers in the previous subsection. Moreover, $\lim_{t\to\infty}\nu(t) = 0$ is essential to achieve asymptotic tracking. Therefore, the asymptotic stabilization of the transformed system (37)–(40) is sufficient to attain the control objectives. The main results of PPB based control design are established in the following theorem.

Theorem 2. *Consider the closed-loop adaptive system consisting of the plant (1)–(2), the PPB based controller (59) with the parameter update laws (60)–(61) in the presence of possible actuator failures (3) and (4) under Assumptions 1–5. The boundedness of all the signals and tracking error $e(t) = y(t) - y_r(t)$ asymptotically approaching zero are ensured. Furthermore, the transient performance of the system in the sense that $e(t)$ is preserved within a specified PPB all the time, i.e. $-\underline{\delta}\eta(t) < e(t) < \bar{\delta}\eta(t)$ with $t \geq 0$ is guaranteed.*

Proof. From (43) and (44), it is obtained that

$$\dot{z}_1 = -c_1 z_1 + \zeta z_2 - \zeta \varphi_1^T \tilde{\theta}. \tag{62}$$

From (49), (52) and (57), we have

$$\begin{aligned}\dot{z}_2 &= -c_2 z_2 - \zeta z_1 + z_3 - \omega_2^T \tilde{\theta} + \frac{\partial \alpha_1}{\partial \hat{\theta}}\Gamma(\tau_2 - \tau_\varrho) \\ &= -c_2 z_2 - \zeta z_1 + z_3 - \omega_2^T\tilde{\theta} - \sum_{k=3}^{\varrho}\frac{\partial \alpha_1}{\partial \hat{\theta}}\Gamma \omega_k z_k.\end{aligned} \tag{63}$$

From the design along (55)–(58) for $j = 3, \ldots, \varrho - 1$, it can be shown that

$$\begin{aligned}\dot{z}_j &= -c_j z_j - z_{j-1} + z_{j+1} - \omega_j^T \tilde{\theta} + \sum_{k=2}^{j-1}\frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}\Gamma \omega_j z_k \\ &\quad - \sum_{k=j+1}^{\varrho}\frac{\partial \alpha_1}{\partial \hat{\theta}}\Gamma \omega_k z_k.\end{aligned} \tag{64}$$

Similar to the proof of Theorem 1, suppose that there are $(r+1)$ time intervals $[T_{k-1}, T_k)$ $(k = 1, \ldots, r+1)$ along $[0, \infty)$. $T_0 = 0$, T_1 and T_r refer to the first and last time that failures occur respectively,

$T_{r+1} = \infty$. During $[0, T_1)$, from (39), (42), (56) and (59), the derivative of z_ϱ is computed as

$$\dot{z}_\varrho = \varphi_0 + \varphi_\varrho^T \tilde{\theta} + \sum_{i=1}^{m} |b_i| \hat{\kappa}^T w - \dot{\alpha}_{\varrho-1} - y_r^{(\varrho)}$$
$$= \varphi_0 + \varphi_\varrho^T \theta + \sum_{i=1}^{m} |b_i| (\kappa + \tilde{\kappa})^T w - \dot{\alpha}_{\varrho-1} - y_r^{(\varrho)} \quad (65)$$

where $\tilde{\kappa} = \hat{\kappa} - \kappa$. If b_i is known, κ is a desired constant vector which can be chosen to satisfy

$$\sum_{i=1}^{m} |b_i| \kappa^T w = \alpha_\varrho$$
$$\Rightarrow \kappa_1 = \frac{1}{\sum_{i=1}^{m} |b_i|}, \kappa_{2,k} = 0 \quad \text{for } k = 1, \ldots, m. \quad (66)$$

Substituting (66) into (65), we have

$$\dot{z}_\varrho = -c_\varrho z_\varrho - z_{\varrho-1} - \omega_\varrho^T \tilde{\theta} + \sum_{k=2}^{\varrho-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma \omega_\varrho z_k + \sum_{i=1}^{m} |b_i| \tilde{\kappa}^T w. \quad (67)$$

We define the error vector $z(t) = [z_1, z_2, \ldots, z_\varrho]^T$, $\omega_1 = \zeta \varphi_1$. From (62)–(64) and (67), the derivative of $z(t)$ during $[0, T_1)$ is summarized as

$$\dot{z} = A_z z - \Omega^T \tilde{\theta} + \begin{bmatrix} 0_{(\varrho-1)\times 1} \\ \sum_{i=1}^{m} |b_i| \tilde{\kappa}^T w \end{bmatrix} \quad (68)$$

where

$$A_z = \begin{bmatrix} -c_1 & \zeta & 0 & \cdots & 0 \\ -\zeta & -c_2 & 1 + \sigma_{2,3} & \cdots & \sigma_{2,\varrho} \\ 0 & -1 - \sigma_{2,3} & -c_3 & \cdots & \sigma_{3,\varrho} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -\sigma_{2,\varrho} & \cdots & -1 - \sigma_{\varrho-1,\varrho} & -c_\varrho \end{bmatrix} \quad (69)$$

$$\sigma_{j,k} = -\frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma \omega_k \quad (70)$$

$$\Omega = [\omega_1, \omega_2, \ldots, \omega_n]. \quad (71)$$

It can be shown that $A_z + A_z^T = -2\text{diag}\{c_1, c_2, \ldots, c_\varrho\}$. Define a positive definite $V_0(t)$ for $t \in [0, T_1)$ as

$$V_0 = \frac{1}{2} z^T z + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \sum_{i=1}^{m} \frac{|b_i|}{2} \tilde{\kappa}^T \Gamma_\kappa^{-1} \tilde{\kappa}. \quad (72)$$

Differentiating V_0, we obtain

$$\dot{V}_0 = -\sum_{j=1}^{\varrho} c_j z_j^2. \quad (73)$$

Thus we have $V_0(T_1^-) \le V_0(0)$, where $V_0(T_1^-)$ is defined as the same as in Section 3.1. Assume also that during the time interval $[T_{k-1}, T_k)$ with $k = 2, \ldots, r$, subsets Q_{tot_k} and Q_{par_k} correspond to the actuators undergoing TLOE and PLOE respectively. The derivative of $z(t)$ during $[T_{k-1}, T_k)$ can then be written as

$$\dot{z} = A_z z - \Omega^T \tilde{\theta} + \begin{bmatrix} 0_{(\varrho-1)\times 1} \\ \sum_{i=1, i \notin Q_{\text{tot}_k}}^{m} \rho_i |b_i| w^T \tilde{\kappa} \end{bmatrix}. \quad (74)$$

Define V_{k-1} during $[T_{k-1}, T_k)$ in the same form of (17). $\dot{V}_{k-1} = -\sum_{j=1}^{\varrho} c_j z_j^2$ can also be achieved. Then by following the similar procedure in Section 3.1, it can be shown that z, $\tilde{\theta}$, $\tilde{\kappa}$, $x(t)$ and u_{ci} are bounded and $z(t) \in L^2$. From the fact that $\nu = z_1$, $\nu(t)$ is bounded. ζ is bounded from (36) and (27) is thus satisfied. The closed-loop stability is then established. Noting $\dot{z} \in L^\infty$, it follows that $\lim_{t\to\infty} z(t) = 0$. From (30), $\lim_{t\to\infty} e(t) = 0$ which implies that asymptotic tracking can still be retained. □

5. Simulation studies

To compare the PPB based control scheme with the basic control method, we use the same twin otter aircraft longitudinal nonlinear dynamics model as in Tang et al. (2003).

$$\dot{V} = \frac{F_x \cos(\alpha) + F_z \sin(\alpha)}{m}$$
$$\dot{\alpha} = q + \frac{-F_x \sin(\alpha) + F_z \cos(\alpha)}{mV}$$
$$\dot{\theta} = q$$
$$\dot{q} = \frac{M}{I_y} \quad (75)$$

where

$$F_x = \bar{q} S C_x + T_x - mg \sin(\theta)$$
$$F_z = \bar{q} S C_z + T_z + mg \cos(\theta)$$
$$M = \bar{q} c S C_m \quad (76)$$

and $\bar{q} = \frac{1}{2} \rho V^2$, C_x, C_z and C_m are polynomial functions

$$C_x = C_{x1} \alpha + C_{x2} \alpha^2 + C_{x3} + C_{x4}(d_1 \delta_{e1} + d_2 \delta_{e2})$$
$$C_z = C_{z1} \alpha + C_{z2} \alpha^2 + C_{z3} + C_{z4}(d_1 \delta_{e1} + d_2 \delta_{e2}) + C_{z5} q$$
$$C_m = C_{m1} \alpha + C_{m2} \alpha^2 + C_{m3} + C_{m4}(d_1 \delta_{e1} + d_2 \delta_{e2}) + C_{m5} q. \quad (77)$$

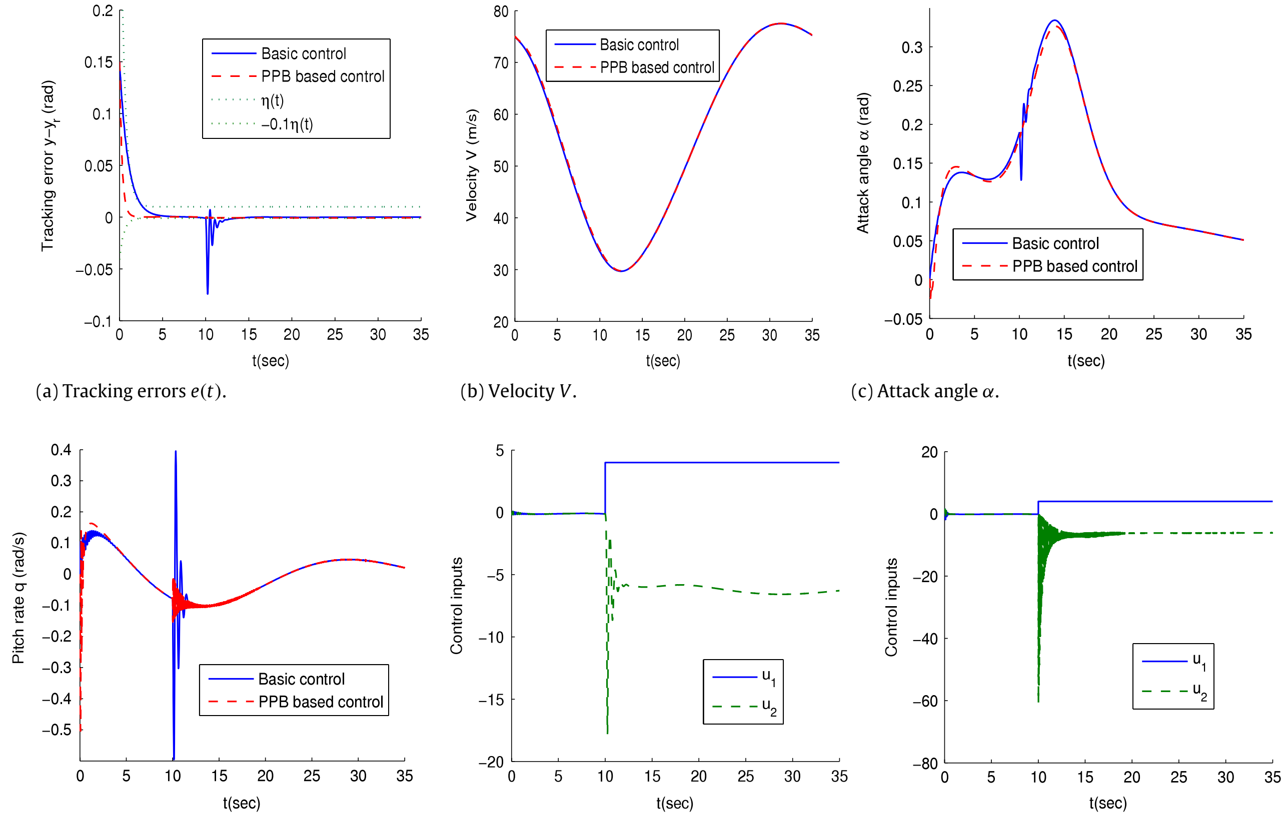
In (75), V is the velocity, α is the attack angle, θ is the pitch angle and q is the pitch rate. They are chosen as states $\chi_1, \chi_2, \chi_3, \chi_4$ respectively. In (77), δ_{e1}, δ_{e2} are the elevator angles of an augmented two-piece elevator chosen as two actuators u_1 and u_2. The rest of the notations are illustrated in the following table.

m	The mass
I_y	The moment of inertia
ρ	the air density
S	The wing area
c	The mean chord
T_x	The components of the thrust along the body x
T_z	The components of the thrust along the body z

The control objective is to ensure that the closed-loop system is stable and the pitch angle $y = \chi_3$ can asymptotically track a given signal y_r in the presence of actuator failures with guaranteed transient performance of $e(t) = y(t) - y_r(t)$. As explained in Tang et al. (2003), there exists a diffeomorphism $[\xi, x]^T = T(\chi) = [T_1(\chi), T_2(\chi), \chi_3, \chi_4]$ that (75) can be transformed into the parametric-strict-feedback form as in (5).

$$\dot{\chi}_3 = \chi_4$$
$$\dot{\chi}_4 = \varphi(\chi)^T \vartheta + \sum_{i=1}^{2} b_i \chi_1^2 (\rho_i u_{ci} + u_{ki})$$
$$\dot{\xi} = \Psi(\xi, x) + \Phi(\xi, x) \vartheta \quad (78)$$

where $\vartheta \in R^4$ is an unknown constant vector and $\varphi(\chi) = [\chi_1^2 \chi_2, \chi_1^2 \chi_2^2, \chi_1^2, \chi_1^2 \chi_4]^T$, $x = [\chi_3, \chi_4]^T$. Input-to-state stability of zero dynamics is shown in Tang et al. (2003). Relative degree $\varrho =$



(a) Tracking errors $e(t)$. (b) Velocity V. (c) Attack angle α.

(d) Pitch rate q. (e) Control inputs with basic design method. (f) Control inputs with PPB based control method.

Fig. 1. Simulation results under failure case 1.

2. The aircraft parameters in the simulation study are set based on the data sheet in Miller and William (1999): $m = 4600$ kg, $I_y = 31027$ kg m^2, $S = 39.02$ m^2, $c = 1.98$ m, $T_x = 4864$ N, $T_z = 212$ N, $\rho = 0.7377$ kg/m^3 at the altitude of 5000 m, and for the 0° flap setting. In addition, $d_1 = 0.6$, $d_2 = 0.4$, $C_{x1} = 0.39$, $C_{x2} = 2.9099$, $C_{x3} = -0.0758$, $C_{x4} = 0.0961$, $C_{z1} = -7.0186$, $C_{z2} = 4.1109$, $C_{z3} = -0.3112$, $C_{z4} = -0.2340$, $C_{z5} = -0.1023$, $C_{m1} = -0.8789$, $C_{m2} = -3.852$, $C_{m3} = -0.0108$, $C_{m4} = -1.8987$, $C_{m5} = -0.6266$ are unknown constants. The reference signal y_r is set as $y_r = \mathrm{e}^{-0.05t}\sin(0.2t)$. The initial states and estimates are set as $\chi(0) = [75, 0, 0.15, 0]^T$, $\hat{\vartheta}(0) = [0, 0, -0.04, 0]$.

Design the control inputs with PPB through the procedures as given in Section 4.2. By noting that in (59) β_1 and β_2 are the same as χ_1^2, the control laws are designed as $u_{ci} = sgn(b_i)\frac{1}{\chi_1^2}\hat{\kappa}[\alpha_2, \chi_1^2]$, for $i = 1, 2$. A prescribed performance bound (PPB) is given by choosing $\eta(t) = 0.4\mathrm{e}^{-2t} + 0.01$, $\underline{\delta} = 0.1$ and $\bar{\delta} = 1$. Other design parameters are chosen as $c_1 = c_2 = 1$, $\Gamma = 0.005I$ and $\Gamma_\kappa = [1, 0; 0, 0.01]$. The initial value of $\hat{\kappa}$ are set as $\hat{\kappa}(0) = [-1.2, 0]$. Two failure cases are considered respectively,

(1) Case 1: actuator u_1 is stuck at $u_1 = 4$ from $t = 10$ s, thus undergoes a TLOE type of failure.

The tracking error $e(t) = y(t) - y_r(t)$ is plotted in Fig. 1(a). To show the improved transient performance with a PPB based proposed scheme, the tracking error performance using the basic design method with the same design parameters is also plotted for comparison. The comparisons on the performances of velocity, attack angle, pitch rate as well as control inputs using the PPB based control scheme and the basic design method are given in Fig. 1(b)–(f), respectively.

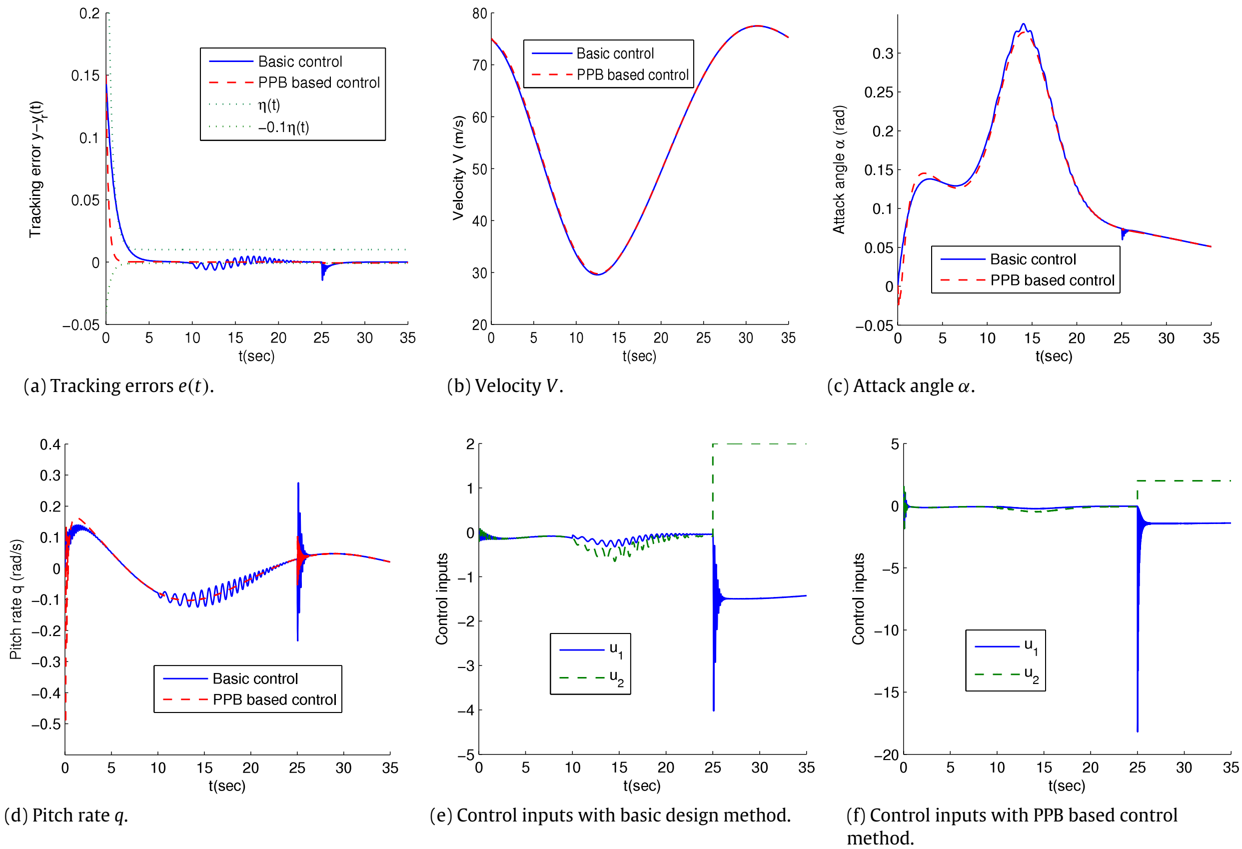
(2) Case 2: actuator u_1 loses 50% of its effectiveness from $t = 10$ s. and actuator u_2 is stuck at $u_2 = 2$ from $t = 25$ s.

The comparisons on the performances of tracking error, velocity, attack angle, pitch rate and control inputs are given in Fig. 2(a)–(f), respectively.

It can be seen that all signals are bounded and asymptotic tracking can be ensured under both cases. From Fig. 1(a) and 2(a), the tracking error is shown to converge at a faster rate in the initial phase before failures occur using the PPB based control method. At the time instant when failures occur, the large overshoot on tracking error with the basic design method can be reduced by preserving the tracking error within a prescribed bound with the PPB based control method.

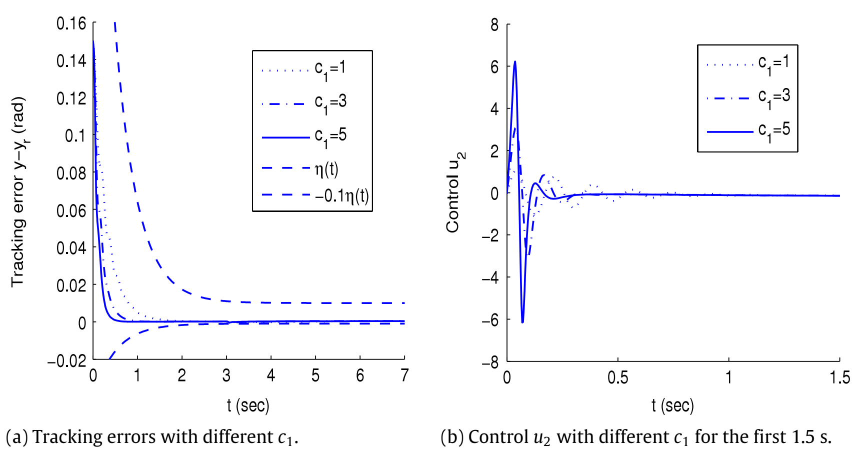
Remark 4. From (36) and (37), it can be seen that the term $1/\eta$ is involved in the derivative of ν. Thus a small η could make the signal ν as well as the tracking error $e(t)$ less smooth. Although decreasing η_0 and η_∞ can improve the transient performance of $e(t)$ in terms of the maximum overshoot as discussed in Remark 3, there is a compromise in choosing these two parameters.

About the issue on how to choose the free design parameters c_j, Γ, and Γ_κ, there is still no quantitative measure in terms of certain cost functions when the PPB based control method is utilized. Also no explicit relationship between the performance of tracking error and these parameters has been obtained under the failure case. However, we may choose these parameters by following the well established rule of the basic design scheme under the failure free case, as in Krstic et al. (1995) and Zhou, Wen, and Wang (2009), etc. According to the discussions in Section 3.2, with the basic design method, the transient performance of the tracking error in the sense of both $L_{2[0,t_1]}$ and $L_{\infty[0,t_1]}$ norms ($t_1 < T_1$, where T_1 denotes the time instant when the first failure takes place) can be improved by increasing c_1, Γ, Γ_κ. However, their increases may increase the magnitudes of the control signals. Thus a compromise might be reached.



(a) Tracking errors $e(t)$.
(b) Velocity V.
(c) Attack angle α.
(d) Pitch rate q.
(e) Control inputs with basic design method.
(f) Control inputs with PPB based control method.

Fig. 2. Simulation results under failure case 2.



(a) Tracking errors with different c_1.
(b) Control u_2 with different c_1 for the first 1.5 s.

Fig. 3. Comparisons of tracking errors and control u_2 with different c_1.

For the choice of these free parameters with PPB based control, we now use an example to illustrate how the choice of c_1 affects the L_2 performance of the tracking error. Consider the same plant as in (75)–(77) under the failure case that actuator u_1 loses 90% of its effectiveness from $t = 3$ s. All parameters and the initial states are the same as those given above, except for c_1, Γ and Γ_κ. We change c_1 by setting $c_1 = 1, 3$ and 5 respectively with Γ and Γ_κ being fixed at $\Gamma = 0.01 \times I(4)$ and $\Gamma_\kappa = 0.01 \times I(2)$, the tracking error $y - y_r$ with different c_1 are compared in Fig. 3(a). Obviously, the $L_{2[0,t_1]}$ norms of the tracking error decrease as c_1 increases especially before the failure occur. We also present control u_2 with different c_1 for the first 1.5 s in Fig. 3(b). It can be seen that the magnitude of u_2 increases with increased c_1. Similar results would be followed if we change Γ and Γ_κ with a fixed c_1. The results once again show that a compromise may be reached in choosing novel free design parameters.

6. Conclusion

Two adaptive backstepping control schemes for parametric strict feedback systems in the presence of unknown actuator failures are presented in this paper. The actuator failures under consideration include total and partial loss of effectiveness (TLOE and PLOE). System stability and asymptotic tracking are shown to be maintained with both schemes. It is analyzed that transient performance of the adaptive system is not adjustable with the first control scheme proposed on the basis of an existing adaptive failure compensation approach. However, the transient performance can be improved and adjusted by preserving the tracking error within a prescribed performance bound (PPB) by the second control scheme. Simulation studies also verify the theoretical results.

Further research is needed to investigate the transient performance for a larger class of nonlinear systems and actuator failures. Moreover, the accommodation of external disturbances, unmodeled dynamics is still an open issue in the design and analysis of direct adaptive control systems in the presence of actuator failures. This is possible with appropriate modifications by following certain approaches such as those in Wen, Zhang, and Soh (1999) and Zhang, Wen, and Soh (1999). The relaxation of the assumption on known signs of control coefficients is also important from an application point of view, which is achievable based on some available methods, for example those in Bechlioulis and Rovithakis (2009) and Zhang, Wen, and Soh (2000). These will be interesting topics for future investigation.

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