Automatica 47 (2011) 2197–2210

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Adaptive compensation for infinite number of actuator failures or faults ☆

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ARTICLE INFO

Article history:
Received 30 August 2010
Received in revised form
12 January 2011
Accepted 27 April 2011
Available online 1 September 2011

Keywords:
Adaptive control
Modular design
Actuator failure/fault compensation
Fault tolerant control
Backstepping

ABSTRACT

It is both theoretically and practically important to investigate the problem of accommodating infinite number of actuator failures or faults in controlling uncertain systems. However, there is still no result available in developing adaptive controllers to address this problem. In this paper, a new adaptive failure/fault compensation control scheme is proposed for parametric strict feedback nonlinear systems. The techniques of nonlinear damping and parameter projection are employed in the design of controllers and parameter estimators, respectively. It is proved that the boundedness of all closed-loop signals can still be ensured in the case with infinite number of failures or faults, provided that the time interval between two successive changes of failure/fault pattern is bounded below by an arbitrary positive number. The performance of the tracking error in the mean square sense with respect to the frequency of failure/fault pattern changes is also established. Moreover, asymptotic tracking can be achieved when the total number of failures and faults is finite.

1. Introduction

In control systems, actuators may undergo complete failures or partial loss of effectiveness (PLOE) faults during operation. These failures or faults may cause instability and end up with catastrophic accidents if they are not well handled. Accommodating such failures/faults is important to ensure the safety of the systems, especially for life-critical systems such as aircrafts, spacecrafts, nuclear power plants and so on. Recently, increasing demands for safety and reliability in modern industrial systems have motivated more and more researchers to investigate the problem of actuator failure/fault accommodation.

Some effective approaches have been developed on synthesizing controllers to address the problem. They can be roughly classified into two categories, i.e. passive and active ones. Typical passive designs aim at achieving insensitivity of the system to certain presumed failures or faults by adopting robust control techniques; see for instance in Niemann and Stoustrup (2005), Veillette, Medanic, and Perkins (1992), Yang, Wang, and Soh (2001) and Zhao and Jiang (1998). Since fixed controllers are used and fault detection/diagnostic (FDD) is not required in these results, the design methods are computationally attractive. However, they have the drawback that the designed controllers are often conservative to handle large failure/fault pattern changes. In contrast to passive designs, the parameters and/or the structure of the controllers are adjustable with active approaches. A number of active approaches have been proposed, such as pseudo-inverse method (Gao & Antsaklis, 1991), eigenstructure assignment (Ashari, Sedigh, & Yazdanpanah, 2005), multiple model (Boskovic & Mehra, 2002b; Maybeck & Stevens, 1991), model predictive control (Kale & Chipperfield, 2005), neural networks/fuzzy logic based scheme (Diao & Passino, 2001; Zhang, Parisini, & Polycarpou, 2004; Zhang & Qin, 2008) and sliding mode control based scheme (Corradini & Orlando, 2007). Different from the ideas of redesigning the nominal controllers for the post-failure plants in these schemes, the virtual actuator method (Richter, Schlage, & Lunze, 2007, 2008) hides the effects of the failures from the nominal controller to preserve the nominal controller in the loop.

Apart from these, adaptive control is also an active method well suited for actuator failure/fault compensation (Ahmed-Zaid, Ioannou, Gousman, & Rooney, 1991; Bodson & Groszkiewicz, 1997) because of its prominent adapting ability to handle the structural and parametric uncertainties and variations in the systems. As opposed to most of the active approaches, many adaptive control design schemes can be applied with neither control restructuring nor FDD processing. Moreover, not only the uncertainties caused by the failures or faults, but also the unknown system parameters are estimated online for updating the controller parameters adaptively. In Tao, Chen, and Joshi (2002); Tao, Joshi, and Ma (2001), the authors proposed a class

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Gang Tao under the direction of Editor Miroslav Krstic.

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doi:10.1016/j.automatica.2011.08.022

of adaptive control methods for linear systems with complete actuator failures. As we know, backstepping technique (Krstic, Kanellakopoulos, & Kokotovic, 1995) has been widely used to design adaptive controllers for uncertain nonlinear systems due to its advantages. The results in Tao et al. (2002, 2001) have been successfully extended to nonlinear systems in Tang and Tao (2009), Tang, Tao, and Joshi (2003, 2007) and Zhang and Chen (2009) by adopting the backstepping technique. In Zhang, Xu, Guo, and Chu (2010), a robust adaptive output feedback controller was designed based on the backstepping technique to stabilize nonlinear systems with uncertain complete failures involving parameterizable and unparameterizable time varying terms. To ensure a prescribed transient performance of the tracking error for nonlinear systems in the presence of uncertain complete failures and PLOE faults, a new adaptive backstepping based failure/fault compensation scheme was recently proposed in Wang and Wen (2010).

In most of the existing results on adaptive actuator fault tolerant control, such as Boskovic, Jackson, Mehra, and Nguyen (2009); Boskovic, Yu, and Mehra (1998), Tang and Tao (2009), Tang et al. (2003), Tao et al. (2002, 2001), Wang and Wen (2010) and Zhang and Chen (2009); Zhang et al. (2010), only the cases with finite number of failures and faults are considered. It is assumed that one actuator may only fail once and the faulty mode does not change afterward. This implies that there exists a finite time T_r such that no further failure or fault occurs on the system after T_r. In these cases, although some unknown parameters will experience jumps at the time instants when failures or faults occur, the jumping sizes are bounded and the total number of jumps are finite. Thus the possible increase of the considered Lyapunov function, which includes the estimation errors of the unknown parameters, is bounded, which enables the closed-loop stability to be established. However, we cannot show the system stability in the same way when the number of failures or faults is infinite, because the possible increase of the Lyapunov function mentioned earlier cannot be ensured bounded automatically when the parameters experience infinite number of jumps. This is indeed the main challenge to find an adaptive solution to the problem of compensating for infinite number of failures theoretically. On the other hand, it is possible that some actuator failures or faults occur intermittently in practice. Thus the actuators may unawarely change from a faulty mode to a normally working mode or another different faulty mode infinitely many times. For example, poor electrical contact can cause repeated unknown breaking down failures on the actuators in some control systems. Although it is of both theoretical and practical importance to consider the case with infinite number of failures or faults, there is still no solid result available in this area so far. In Tang et al. (2007), the authors only conjectured that their proposed scheme could possibly be applied to this case. It was remarked that all the signals might still be ensured bounded as long as the time interval between two sequential changes of failure status is not too small. Nevertheless, to the best of our knowledge, no rigorous analysis has been reported by them.

In this paper, we shall deal with the problem of compensating for possibly infinite number of actuator failures or faults in controlling uncertain nonlinear systems based on adaptive backstepping technique. Through tremendous studies, we find that it is difficult to show the boundedness of all the signals using the tuning function design approaches as in Tang and Tao (2009), Tang et al. (2003, 2007), Wang and Wen (2010) and Zhang and Chen (2009), mainly because the unbounded derivatives of the parameters caused by jumps need to be considered in computing the derivative of the Lyapunov function. In fact from our simulation studies, instability is observed when the tuning function scheme as summarized in Wang and Wen (2010, Sec. 3) is utilized to compensate for infinite number of relatively frequent actuator failures. To overcome the difficulty, we propose a modular design scheme. Actually, so far there is also no result available by using backstepping based modular design scheme to compensate for actuator failures or faults even for the case of finite number of failures/faults. With compared to the existing tuning function methods, our designs have the following features. The control module and parameter estimator module are designed separately; nonlinear damping term functions are introduced in the control design to establish an input-to-state property of an error system; impulses caused by failures or faults are considered in computing the derivatives of the unknown parameters and these parameters are shown to satisfy a finite mean variation condition; the parameter update law involves projection operation to ensure the boundedness of estimation errors; the properties of the parameter estimator, which are useful for stability analysis, are also obtained. It is proved that the boundedness of all the closed-loop signals can be ensured with our scheme, provided that the time interval between two successive changes of failure/fault pattern is bounded below by an arbitrary positive number. It is also established that the tracking error can be small in the mean square sense if the changes of failure/fault pattern are infrequent. This shows that the less frequent the failure/fault pattern changes, the better the tracking performance is. Moreover, asymptotic tracking can still be achieved with the proposed scheme in the case with finite number of failures and faults as the tuning function methods.

The remaining part of the paper is organized as follows. In Section 2, the control problem is formulated. The design of both controller and parameter update law is presented in Section 3. The analysis of stability and tracking performance are established in Section 4 followed by simulation studies in Section 5. Apart from the comparative study mentioned previously, the effectiveness of our proposed scheme is further verified through an aircraft application. Finally, we conclude the paper in Section 6.

1.1. Notations and definitions

◇ For a scalar function $x(t) \in \Re$,
- $|x|$, the absolute value of x.

◇ For a vector function $x(t) = [x_1, \ldots, x_n]^T \in \Re^n$,
- $|x|$, Euclidean norm $|x| = \sqrt{\sum_i^n |x_i|^2}$;
- $\|x\|_p$, L_p norm for $p \in [1, \infty)$ that $\|x\|_p = \left(\int_0^\infty |x(\tau)|^p \mathrm{d}\tau\right)^{1/p}$;
- $\|x\|_\infty$, L_∞ norm $\|x\|_\infty = \sup_{t\geq 0} |x(t)|$;
- $x(t) \in S_1(\mu)$, if $\int_t^{t+T} |x(\tau)|\mathrm{d}\tau \leq \bar{c}_1\mu T + \bar{c}_2$ for $\mu \geq 0$, where $\bar{c}_1, \bar{c}_2$ are some positive constants, and $\bar{c}_1$ is independent of μ.
- $x(t) \in S_2(\mu)$, if $\int_t^{t+T} x(\tau)^T x(\tau)\mathrm{d}\tau \leq (\bar{c}_1\mu^2 + \bar{c}_3\mu)T + \bar{c}_2$ for $\mu \geq 0$, where $\bar{c}_i$ for $i = 1, 2, 3$ are some positive constants, and $\bar{c}_1, \bar{c}_3$ are independent of μ. We say that x is of the order μ in the mean square sense if $x \in S_2(\mu)$.

◇ For a matrix $A \in \Re^{m\times n}$,
- $\|A\|$, induced matrix norm of matrix A corresponding to the vector norm $|\cdot|$, i.e.
$$\|A\| \triangleq \sup_{x\neq 0,\ x\in\Re^n} \frac{|Ax|}{|x|} = \sup_{|x|\leq 1} |Ax| = \sup_{|x|=1} |Ax|;$$
- $\|A\|_F$, Frobenius norm $\|A\|_F = \sqrt{\mathrm{tr}\{A^T A\}}$, i.e. the square root of the sum of the absolute squares of its elements.

2. Problem formulation

We consider a class of multiple-input single-output nonlinear systems that are transformable into the following parametric strict feedback form.

$$\dot{x}_i = x_{i+1} + \varphi_i(\bar{x}_i)^T\theta, \quad i = 1, 2, \ldots, \varrho - 1$$

$$\dot{x}_\varrho = \varphi_0(x, \xi) + \varphi_\varrho(x, \xi)^T\theta + \sum_{j=1}^{m} b_j\beta_j(x, \xi)u_j$$

$$\dot{\xi} = \Psi(x, \xi) + \Phi(x, \xi)\theta$$

$$y = x_1, \tag{1}$$

where $x = [x_1, x_2, \ldots, x_\varrho]^T, \xi \in \Re^{n-\varrho}$ are the states, $y \in \Re$ is the output and $u_j \in \Re$ for $j = 1, 2, \ldots, m$ is the jth input of the system, i.e. the output of the jth actuator. $\beta_j(x, \xi), \varphi_0(x, \xi) \in \Re, \varphi_\varrho(x, \xi), \varphi_i(\bar{x}_i) \in \Re^p$ for $i = 1, 2, \ldots, \varrho - 1$ are known smooth nonlinear functions with $\bar{x}_i = (x_1, x_2, \ldots, x_i)$. $\theta \in \Re^p$ is a vector of unknown parameters and b_j for $j = 1, \ldots, m$ are unknown control coefficients.

Remark 1. As presented in Tang et al. (2003, Sec. 3.1), suppose there is a class of nonlinear systems modeled as,

$$\dot{\chi} = f_0(\chi) + \sum_{l=1}^{p} \theta_l f_l(\chi) + \sum_{j=1}^{m} b_j g_j(\chi)u_j$$

$$y = h(\chi), \tag{2}$$

where $\chi \in \Re^n, y, u_j$ for $j = 1, \ldots, m$ are the states, output and jth input of the system, respectively, $f_l(\chi) \in \Re^n$ for $l = 0, 1, \ldots, p$, $g_j(\chi) \in \Re^n$ for $j = 1, \ldots, m$ and $h(\chi)$ are known smooth nonlinear functions, θ_l for $l = 1, \ldots, p$ and b_j are unknown parameters and control coefficients. If $g_j(\chi) \in \text{span}\{g_0(\chi)\}, g_0(\chi) \in \Re^n$ and the nominal system $\dot{\chi} = f_0(\chi) + F(\chi)\theta + g_0(\chi)u_0, y = h(\chi)$, where $u_0 \in \Re, F(\chi) = [f_1(\chi), f_2(\chi), \ldots, f_p(\chi)] \in \Re^{n\times p}, \theta = [\theta_1, \theta_2, \ldots, \theta_p]^T \in R^p$, is transformable into the parametric-strict-feedback form with relative degree ϱ, the nonlinear plant (2) can be transformed to the form of (1).

Suppose that the internal dynamics in actuators is negligible. We denote u_{cj} for $j = 1, \ldots, m$ as the input of the jth actuator, which is to be designed. An actuator with its input equal to its output, i.e. $u_j = u_{cj}$, is regarded as fault free. The actuator failures and faults of interest are modeled as follows,

$$u_j(t) = \rho_{jh}u_{cj} + u_{kj,h}, \quad t \in [t_{jh,s}, t_{jh,e}),\ h \in Z^+ \tag{3}$$

$$\rho_{jh}u_{kj,h} = 0, \quad j = 1, \ldots, m, \tag{4}$$

where $\rho_{jh} \in [0, 1), u_{kj,h}, t_{jh,s}, t_{jh,e}$ are all unknown constants and $0 \le t_{j1,s} < t_{j1,e} \le t_{j2,s} < \cdots < t_{jh,e} \le t_{j(h+1),s} < t_{j(h+1),e}$ and so forth. Eq. (3) indicates that the jth actuator fails from time $t_{jh,s}$ till $t_{jh,e}$. $t_{j1,s}$ denotes the time instant when the first failure or fault takes place on the jth actuator.

Similar to Wang and Wen (2010), (3)–(4) cover both PLOE type of faults and complete failures.

(1) $\rho_{jh} \neq 0$ and $u_{kj,h} = 0$.
In this case, $u_j = \rho_{jh}u_{cj}$, where $0 < \rho_{jh} < 1$. This indicates PLOE faults. For example, $\rho_{jh} = 70\%$ means that the jth actuator loses 30% of its effectiveness.

(2) $\rho_{jh} = 0$.

In this case, u_j is stuck at an unknown value $u_{kj,h}$ such that it can no longer be influenced by the control inputs u_{cj}. This indicates complete failures, which are sometimes referred to as total loss of effectiveness (TLOE) type of faults. The detailed descriptions of TLOE faults can be found in Boskovic and Mehra (1999, 2002a).

It is important to be noted that actuators working in fault free case can also be represented as (3) with $\rho_{jh} = 1$ and $u_{kj,h} = 0$. Therefore, the model in (3) is applicable to describe the output of an actuator no matter it fails or not.

Remark 2. By comparing (3)–(4) to the failure/fault models considered in Boskovic et al. (2009, 1998), Tang and Tao (2009), Tang et al. (2003), Tao et al. (2002, 2001), Wang and Wen (2010) and Zhang and Chen (2009); Zhang et al. (2010), h is not restricted to be finite. This implies (i) a failed actuator may operate normally again from time $t_{jh,e}$ till $t_{j(h+1),s}$ when the next failure or fault occurs on the same actuator; (ii) the failure/fault values ρ_{jh} or $u_{kj,h}$ changes to a new one, i.e. $\rho_{j(h+1)}$ or $u_{kj,h+1}$, from the time $t_{jh,e}(=t_{j(h+1),s})$.

The control objectives in this paper are as follows,

- The effects of considered types of actuator failures or faults can be compensated for so that the closed-loop system is maintained stable all the time.
- The tracking error $z_1(t) = y(t) - y_r(t)$ is small in the mean square sense that $z_1(t) \in S_2(\mu)$, where $S_2(\mu)$ is defined in Section 1.1.
- If the total number of failures and faults is finite, asymptotic tracking can still be achieved, i.e. $\lim_{t\to\infty} z_1(t) = 0$.

To achieve the control objectives, the following assumptions are imposed.

Assumption 1. The plant (1) is so constructed that for any up to $m - 1$ actuators undergoing complete failures simultaneously, the remaining actuators can still achieve the desired control objectives.

Assumption 2. The reference signal $y_r(t)$ and its first ϱth order derivatives $y_r^{(i)}$ $(i = 1, \ldots, \varrho)$ are known, bounded, and piecewise continuous.

Assumption 3. $\beta_j(x, \xi) \neq 0$, the signs of b_j, i.e. $\text{sgn}(b_j)$, for $j = 1, \ldots, m$ are known.

Assumption 4. $0 < \underline{b}_j \le |b_j| \le \bar{b}_j, |u_{kj,h}| \le \bar{u}_{kj}$. For the PLOE faults, $\underline{\rho}_j \le \rho_{jh} < 1$. There exists a convex compact set $\mathcal{C} \subset \Re^p$ such that $\exists\bar{\theta}, \theta_0, |\theta - \theta_0| \le \bar{\theta}$ for all $\theta \in \mathcal{C}$. Note that $\underline{b}_j, \bar{b}_j, \underline{\rho}_j, \bar{u}_{kj}, \theta_0, \bar{\theta}$ are all known finite positive constants.

Assumption 5. The subsystem $\dot{\xi} = \Psi(x, \xi) + \Phi(x, \xi)\theta$ is input-to-state stable with respect to x as the input.

Remark 3.

- As similarly discussed in Boskovic and Mehra (1999), Tang et al. (2003), Tao et al. (2002, 2001), Wang and Wen (2010) and Zhang and Chen (2009), Assumption 1 is a basic assumption to ensure the controllability of the system and the existence of a nominal solution for the adaptive failure compensation problem. However, all actuators are allowed to suffer from PLOE faults simultaneously.
- In Assumption 4, $\underline{\rho}_j$ denotes the lower bound of ρ_{jh} on the jth actuator in the case of PLOE faults. The knowledge of $\underline{\rho}_j$ will be used in designing the controllers and the estimators. The control objectives can be achieved with such designs no matter complete failures or PLOE faults occur.

3. Adaptive control design for failure compensation

Design u_{cj} in parallel as follows

$$u_{cj} = \frac{\text{sgn}(b_j)}{\beta_j}u_0, \tag{5}$$

where u_0 will be generated by performing backstepping technique. Based on (5) and the considered failures/faults modeled as in (3)–(4), the ϱth equation of the plant (1) has different forms in fault free and faulty cases.

- Fault Free Case

$$\dot{x}_\varrho = \varphi_0 + \varphi_\varrho^T\theta + \sum_{j=1}^{m} |b_j| u_0. \quad (6)$$

- Faulty Case.

We denote T_h for $h \in Z^+$ as the time instants at which the failure/fault pattern of the plant changes. Suppose that during time interval (T_h, T_{h+1}), there are q_h $(1 \le q_h \le m-1)$ actuators $j_1, j_2, \ldots, j_{qh}$ undergoing complete failures and the failure/fault pattern will be fixed until time T_{h+1}. We have

$$\dot{x}_\varrho = \varphi_0 + \varphi_\varrho^T\theta + \sum_{j\neq j_1, j_2, \ldots, j_{qh}} \rho_{jh}|b_j|u_0 + \sum_{j=j_1, j_2, \ldots, j_{qh}} b_j u_{kj,h}\beta_j. \quad (7)$$

From (1), (6) and (7), a unified model of $\dot{x}$ for both cases is constructed as

$$\dot{x}_i = x_{i+1} + \varphi_i^T\theta, \quad i = 1, 2, \ldots, \varrho - 1$$
$$\dot{x}_\varrho = \varphi_0 + \varphi_\varrho^T\theta + bu_0 + \beta^T k, \quad (8)$$

where

$$b = \begin{cases} \sum_{j=1}^{m} |b_j|, & \text{Fault Free} \\ \sum_{j\neq j_1, \ldots, j_{q_h}} \rho_{jh}|b_j|, & \text{Faulty} \end{cases} \quad (9)$$

$$\beta = [\beta_1, \ldots, \beta_m]^T \in \Re^m, \quad (10)$$

$$k = \begin{cases} [0, \ldots, 0]^T \in \Re^m, & \text{Fault Free} \\ [0, \ldots, b_{j_1}u_{kj_1,h}, 0, \ldots, \\ \quad b_{j_{qh}}u_{kj_{qh},h}, 0, \ldots, 0]^T \in \Re^m & \text{Faulty.} \end{cases} \quad (11)$$

Define that $\zeta = \min_{1\le j\le m}\{\underline{\rho}_j \underline{b}_j\}$, $k_j = e_{m,j}^T k$, where $e_{i,j}$ denotes the jth coordinate vector in $\Re^i$. From Assumption 1, there is at least one actuator free from complete failures, we have $b \ge \zeta$. Note that b, k_j for $j = 1, \ldots, m$ are time varying parameters that may jump. We further define $\vartheta = [b, \theta^T, k^T]^T \in \Re^{p+m+1}$, the property of ϑ is established in the following lemma.

Lemma 1. *The derivative of $\vartheta(t)$ satisfies that $\dot{\vartheta}(t) \in S_1(\mu)$, where $S_1(\mu)$ is defined in Section 1.1, i.e.*

$$\int_t^{t+T} |\dot{\vartheta}(\tau)| d\tau \le C_1\mu T + C_2, \quad \forall t, T \quad (12)$$

with $C_1, C_2 > 0$, μ is defined as

$$\mu = \frac{1}{T^*}, \quad (13)$$

where T^* denotes the minimum value of time intervals between any successive changes of failure/fault pattern. C_1 is independent of μ.

Proof. From Assumption 4, the upper bounds of the jumping sizes on b and k_j can be calculated. If b or k_j jumps at time instant t, we obtain that

$$|b(t^+) - b(t^-)| \le \sum_{j=1}^{m} \bar{b}_j - \zeta, \quad (14)$$

$$|k_j(t^+) - k_j(t^-)| \le 2\bar{b}_j \bar{u}_{kj}. \quad (15)$$

Define $\bar{K} = \max_{1\le j\le m}\{\sum_{k=1}^m \bar{b}_k - \zeta, 2\bar{b}_j\bar{u}_{kj}\}$. Clearly, $\bar{K}$ is finite. Denote T_h, where $h \in Z^+$, as the time instant when the failure/fault pattern changes. The failure/fault pattern will be fixed during time interval (T_h, T_{h+1}). Because of the definition of T^*, $T_{h+1} - T_h \ge T^*$ is satisfied for all T_h, T_{h+1}. We know that $|\dot{\vartheta}(t)| \le \bar{\bar{K}} \sum_h \delta(t - T_h)$, where $\delta(t - T_h)$ is the shifted unit impulse function and $\bar{\bar{K}} = \sqrt{p+m+1}\bar{K}$. Consider the integral interval $t \sim t+T$ in the following cases:

◇ $T < T^*$ and $T_{h-1} < t \le T_h \le t+T < T_{h+1}$, which corresponds to the case that there is one and only one time of failure/fault pattern change during $[t, t+T]$. Thus we have

$$\int_t^{t+T} |\dot{\vartheta}(\tau)| d\tau \le \bar{\bar{K}}. \quad (16)$$

◇ $T < T^*$, $t > T_h$ and $t+T < T_{h+1}$, which corresponds to the case that the failure/fault pattern is fixed during $[t, t+T]$. We have

$$\int_t^{t+T} |\dot{\vartheta}(\tau)| d\tau = 0. \quad (17)$$

◇ $T \ge T^*$, $t \le T_h$ and $t+T \ge T_{h+N}$, where N is the largest integer that is less than or equal to T/T^*. This refers to the case that there are at most $N+1$ times of failure/fault pattern changes occurring during $[t, t+T]$. We then obtain

$$\int_t^{t+T} |\dot{\vartheta}(\tau)| d\tau = \bar{\bar{K}}(N+1) \le \bar{\bar{K}}\frac{1}{T^*}T + \bar{\bar{K}}. \quad (18)$$

◇ $T \ge T^*$, $t \le T_h$ and $t+T < T_{h+N}$, where N is the same as the above case. This refers to the case that there are at most N times of failure/fault pattern changes occurring during $[t, t+T]$. We then have

$$\int_t^{t+T} |\dot{\vartheta}(\tau)| d\tau = \bar{\bar{K}}N \le \bar{\bar{K}}\frac{1}{T^*}T. \quad (19)$$

Clearly, the above four cases include all the possibilities of t and $t+T$. From (16)–(19), if it is defined that $C_1, C_2 = \bar{\bar{K}}$, (12) follows and C_1 is independent of μ. Therefore $\dot{\vartheta} \in S_1(\mu)$. Note that μ decreases as T^* increases. □

3.1. Design of u_0

This subsection is devoted to constructing u_0 by performing backstepping technique on the model (8). We introduce the error variables

$$z_i = x_i - y_r^{(i-1)} - \alpha_{i-1}, \quad i = 1, \ldots, \varrho \quad (20)$$

where $\alpha_0 = 0$ and α_i is the stabilizing function generated at the ith step given by,

$$\alpha_i = -z_{i-1} - (c_i + s_i)z_i - w_i^T\hat{\theta} + \sum_{k=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_k}x_{k+1} + \frac{\partial\alpha_{i-1}}{\partial y_r^{(k-1)}}y_r^{(k)}\right), \quad i = 1, \ldots, \varrho - 1 \quad (21)$$

$$\alpha_\varrho = \frac{1}{\hat{b}}\bar{\alpha}_\varrho - \frac{1}{\zeta}(c_\varrho + s_\varrho)z_\varrho \quad (22)$$

$$\bar{\alpha}_\varrho = -z_{\varrho-1} - \varphi_0 - w_\varrho^T\hat{\theta} - \beta^T\hat{k} + \sum_{k=1}^{\varrho-1}\left(\frac{\partial\alpha_{\varrho-1}}{\partial x_k}x_{k+1} + \frac{\partial\alpha_{\varrho-1}}{\partial y_r^{(k-1)}}y_r^{(k)}\right), \quad (23)$$

where $\hat{b}$, $\hat{\theta}$ and $\hat{k}$ are the estimates of b, θ and k, respectively. w_i and the nonlinear damping functions s_i are designed as

$$w_i = \varphi_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\varphi_i, \quad i = 1, \ldots, \varrho \quad (24)$$

$$s_i = \kappa_i|w_i|^2 + g_i\left|\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}^T\right|^2, \quad i = 1, \ldots, \varrho - 1 \quad (25)$$

$$s_\varrho = \kappa_\varrho\left[|w_\varrho|^2 + \left|\frac{y_r^{(\varrho)} + \bar{\alpha}_\varrho}{\hat{b}}\right|^2 + |\beta|^2\right] + g_\varrho\left|\frac{\partial\alpha_{\varrho-1}}{\partial\hat{\theta}}^T\right|^2. \quad (26)$$

Remark 4. Different from the existing tuning function designs such as in Tang et al. (2003, 2007) and Wang and Wen (2010), the use of nonlinear damping functions here is to construct a controller such that an input-to-state property of an error system given later in (67) with respect to $\tilde{\vartheta}$ and $\dot{\hat{\theta}}$ as the inputs will be established in Section 4.

Finally, u_0 is designed as

$$u_0 = \alpha_\varrho + \frac{y_r^{(\varrho)}}{\hat{b}}. \tag{27}$$

3.2. Design of parameter update law

In this subsection, preliminary design of certain filters is first presented and some boundedness properties of related signals are also established. Then the design of adaptive law involving the details of parameter projection design is provided. Further, the properties of the estimator which are useful in the analysis of system stability and the performance of tracking error in the mean square sense will also be shown.

3.2.1. Preliminary design

Eq. (8) can be written in parametric x-model as

$$\dot{x} = f(x) + F^T(x,u)\vartheta, \tag{28}$$

where $f(x) = [x_2, x_3, \ldots, x_\varrho, \varphi_0]^T$ and

$$F^T(x,u) = \begin{bmatrix} 0, & \varphi_1^T, & 0_{1\times m} \\ 0, & \varphi_2^T, & 0_{1\times m} \\ \vdots & \vdots & \vdots \\ u_0, & \varphi_\varrho^T, & \beta^T \end{bmatrix} \in \Re^{\varrho\times(p+m+1)}. \tag{29}$$

We introduce two filters

$$\dot{\Omega}^T = A(x,t)\Omega^T + F^T(x,u), \quad \Omega \in \Re^{(p+m+1)\times\varrho} \tag{30}$$

$$\dot{\Omega}_0 = A(x,t)(\Omega_0 + x) - f(x), \quad \Omega_0 \in \Re^\varrho \tag{31}$$

where $A(x,t)$ is chosen as

$$A(x,t) = A_0 - \gamma F^T(x,u)F(x,u)P, \tag{32}$$

with $\gamma > 0$ and A_0 is an arbitrary constant matrix such that $PA_0 + A_0^T P = -I, P = P^T > 0$. We now have the following lemmas.

Lemma 2. *For a time varying system $\dot{\psi} = A(x(t),t)\psi$, the state transition matrix $\Phi_A(t,t_0)$ satisfies that*

$$\|\Phi_A(t,t_0)\| \le \bar{\bar{k}}_0 e^{-r_0(t-t_0)}, \tag{33}$$

where $\bar{\bar{k}}_0$ and r_0 are some positive constants.

Proof. Defining a positive definite quadratic function $V = \psi^T P\psi$. It satisfies that $\dot{V} \le -\psi^T\psi$ and $\lambda_{\min}(P)\psi^T\psi \le V \le \lambda_{\max}(P)\psi^T\psi$, where $\lambda_{\max}(P), \lambda_{\min}(P)$ are the maximum and minimum eigenvalue of P, respectively. Thus the equilibrium point $\psi = 0$ is exponentially stable from Theorem 4.10 in Khalil (1996). Moreover, $\|\Phi_A(t,t_0)\| \le \bar{\bar{k}}_0 e^{-r_0(t-t_0)}$ for $\bar{\bar{k}}_0$, $r_0 > 0$ can be shown by following similar procedures in proving Theorem 4.11 in Khalil (1996). □

Lemma 3. *The state Ω of the filter (30) satisfies that $\|\Omega\|_\infty \le C_3$ irrespectively of the boundedness of its input F^T, where C_3 is a positive constant given by*

$$C_3 = \sqrt{\varrho}\max\left\{\|\Omega(0)\|_F, \sqrt{\frac{p+m+1}{2\gamma}}\right\}. \tag{34}$$

Proof. Similarly to the proof on Pages 250–251 in Krstic et al. (1995), we obtain that

$$\begin{aligned}\frac{d}{dt}\mathrm{tr}\{\Omega P\Omega^T\} &= -\|\Omega\|_F^2 - 2\gamma\left\|FP\Omega^T - \frac{1}{2\gamma}I_{p+m+1}\right\|_F^2 \\ &\quad + \frac{1}{2\gamma}\mathrm{tr}\{I_{p+m+1}\} \\ &\le -\|\Omega\|_F^2 + \frac{p+m+1}{2\gamma}.\end{aligned} \tag{35}$$

From (35) and the fact that $\lambda_{\min}(P)\|\Omega\|_F^2 \le \mathrm{tr}\{\Omega P\Omega^T\}$, it follows that $\Omega \in L_\infty$ and

$$\|\Omega\|_\infty \le \sqrt{\varrho}\|\Omega\|_F \le \sqrt{\varrho}\max\left\{\|\Omega(0)\|_F, \sqrt{\frac{p+m+1}{2\gamma}}\right\}. \quad \square \tag{36}$$

Combining (28), (31), and defining $\mathcal{Y} = \Omega_0 + x$, we have

$$\dot{\mathcal{Y}} = A\mathcal{Y} + F^T\vartheta. \tag{37}$$

Introduce that $\varepsilon = \mathcal{Y} - \Omega^T\vartheta$. From (30) and (37), the derivative of ε is computed as

$$\begin{aligned}\dot{\varepsilon} &= A\mathcal{Y} + F^T\vartheta - (A\Omega^T + F^T)\vartheta - \Omega^T\dot{\vartheta} \\ &= A\varepsilon - \Omega^T\dot{\vartheta}.\end{aligned} \tag{38}$$

Then, the following results are obtained.

Lemma 4.

(i) If μ is finite, ε is bounded;
(ii) $\varepsilon \in S_1(\mu)$ and $\varepsilon \in S_2(\mu)$.

Proof.

• Proof of (i). The solution of (38) is

$$\varepsilon(t) = \Phi_A\varepsilon(0) - \int_0^t \Phi_A(t,\tau)\Omega^T(\tau)\dot{\vartheta}(\tau)d\tau. \tag{39}$$

From Lemmas 2 and 3, we have

$$\begin{aligned}|\varepsilon(t)| &\le \bar{\bar{k}}_0 e^{-r_0 t}|\varepsilon(0)| + \bar{\bar{k}}_0\|\Omega\|_\infty \int_0^t e^{-r_0(t-\tau)}|\dot{\vartheta}(\tau)|d\tau \\ &= \varepsilon_1 + \varepsilon_2,\end{aligned} \tag{40}$$

where $\varepsilon_1 = \bar{\bar{k}}_0 e^{-r_0 t}|\varepsilon(0)|$ and $\varepsilon_2 = \bar{\bar{k}}_0\|\Omega\|_\infty \int_0^t e^{-r_0(t-\tau)}\cdot|\dot{\vartheta}(\tau)|d\tau$, respectively.

From Lemma 1 and the definition of ε_2, one can show that $\varepsilon_2 \le C_4\mu + C_5$, where

$$C_4 = \frac{\bar{\bar{k}}_0 C_1\|\Omega\|_\infty e^{r_0}}{1-e^{-r_0}}, \qquad C_5 = \frac{\bar{\bar{k}}_0 C_2\|\Omega\|_\infty e^{r_0}}{1-e^{-r_0}}, \tag{41}$$

by following similar procedures in proving that $\Delta(t,t_0) \le c$ on Pages 84–85 in Ioannou and Sun (1996). Thus we conclude that ε_2 is bounded as long as μ is finite. Consequently, ε is bounded.

• Proof of (ii).

By integrating (40) over $[t, t+T]$, we have

$$\begin{aligned}\int_t^{t+T}|\varepsilon(\tau)|d\tau &\le \int_t^{t+T}\bar{\bar{k}}_0 e^{-r_0\tau}|\varepsilon(0)|d\tau \\ &\quad + \bar{\bar{k}}_0\|\Omega\|_\infty\int_t^{t+T}\int_0^\tau e^{-r_0(\tau-s)}\cdot|\dot{\vartheta}(s)|ds d\tau \\ &= \frac{\bar{\bar{k}}_0|\varepsilon(0)|}{r_0} + \bar{\bar{k}}_0\|\Omega\|_\infty\int_t^{t+T}e^{-r_0\tau}\int_0^t e^{r_0 s}|\dot{\vartheta}(s)|ds d\tau\end{aligned}$$

$$+\bar{\bar{k}}_0\|\Omega\|_\infty\int_t^{t+T}\mathrm{e}^{-r_0\tau}\int_t^\tau \mathrm{e}^{r_0 s}|\dot\vartheta(s)|\mathrm{d}s\mathrm{d}\tau$$
$$\le \frac{\bar{\bar{k}}_0|\varepsilon(0)|}{r_0}+\frac{\bar{\bar{k}}_0\|\Omega\|_\infty}{r_0}\int_0^t \mathrm{e}^{-r_0(t-s)}|\dot\vartheta(s)|\mathrm{d}s$$
$$+\bar{\bar{k}}_0\|\Omega\|_\infty\int_t^{t+T}\mathrm{e}^{-r_0\tau}\int_t^\tau \mathrm{e}^{r_0 s}|\dot\vartheta(s)|\mathrm{d}s\mathrm{d}\tau, \quad (42)$$

where the last inequality is obtained by using $\mathrm{e}^{-r_0t}-\mathrm{e}^{-r_0(t+T)}\le \mathrm{e}^{-r_0t}$.

From the Proof of (i), we have

$$\int_t^{t+T}|\varepsilon(\tau)|\mathrm{d}\tau \le \frac{\bar{\bar{k}}_0|\varepsilon(0)|+C_4\mu+C_5}{r_0}+\bar{\bar{k}}_0\|\Omega\|_\infty \times\int_t^{t+T}\mathrm{e}^{-r_0\tau}\int_t^\tau \mathrm{e}^{r_0 s}|\dot\vartheta(s)|\mathrm{d}s\mathrm{d}\tau. \quad (43)$$

By changing the sequence of integration, (43) becomes

$$\int_t^{t+T}|\varepsilon(\tau)|\mathrm{d}\tau \le \frac{\bar{\bar{k}}_0|\varepsilon(0)|+C_4\mu+C_5}{r_0}+\bar{\bar{k}}_0\|\Omega\|_\infty \times\int_t^{t+T}\mathrm{e}^{r_0 s}|\dot\vartheta(s)|\int_s^{t+T}\mathrm{e}^{-r_0\tau}\mathrm{d}\tau\mathrm{d}s$$
$$\le \frac{\bar{\bar{k}}_0|\varepsilon(0)|+C_4\mu+C_5}{r_0}+\frac{\bar{\bar{k}}_0\|\Omega\|_\infty}{r_0}\int_t^{t+T}|\dot\vartheta(s)|\mathrm{d}s. \quad (44)$$

From Lemma 1, we obtain that

$$\int_t^{t+T}|\varepsilon(\tau)|\mathrm{d}\tau \le C_6\mu T+C_7, \quad (45)$$

where $C_6=\bar{\bar{k}}_0C_1\|\Omega\|_\infty/r_0$ and

$$C_7=\frac{\bar{\bar{k}}_0|\varepsilon(0)|+C_4\mu+C_5+\bar{\bar{k}}_0C_2\|\Omega\|_\infty}{r_0}. \quad (46)$$

Therefore, $\varepsilon\in S_1(\mu)$.

From (40), it follows that $\|\varepsilon\|_\infty \le \bar{\bar{k}}_0|\varepsilon(0)|+C_4\mu+C_5$. By utilizing Hölder's inequality, we obtain that

$$\int_t^{t+T}\varepsilon(\tau)^T\varepsilon(\tau)\mathrm{d}\tau \le \|\varepsilon\|_\infty\int_t^{t+T}|\varepsilon(\tau)|\mathrm{d}\tau$$
$$=\|\varepsilon\|_\infty(C_6\mu T+C_7)$$
$$=(C_8\mu^2+C_9\mu)T+C_{10}, \quad (47)$$

where $C_8=C_4C_6, C_9=C_6(\bar{\bar{k}}_0|\varepsilon(0)|+C_5)+C_4C_7, C_{10}=C_7(\bar{\bar{k}}_0|\varepsilon(0)|+C_4\mu+C_5)$.

Hence $\varepsilon\in S_2(\mu)$ is concluded. □

3.2.2. Design of adaptive law

Now we introduce the "prediction" of y as $\hat y=\Omega^T\hat\vartheta$, where $\hat\vartheta=[\hat b,\hat\theta^T,\hat k^T]^T$. The "prediction error" $\epsilon=y-\hat y$ can be written as

$$\epsilon=\Omega^T\tilde\vartheta+\varepsilon, \quad (48)$$

where $\tilde\vartheta=\vartheta-\hat\vartheta$.

Design the update law for $\hat\vartheta$ by following standard parameter estimation algorithm Krstic et al. (1995) as

$$\dot{\hat\vartheta}=\mathrm{Proj}\{\Gamma\Omega\epsilon\},\quad \Gamma=\Gamma^T>0 \quad (49)$$

where Proj{·} is a smooth projection operator to ensure that

$$\hat\vartheta(t)=(\hat\vartheta_1,\ldots,\hat\vartheta_{p+m+1})^T\in\Pi_0,\quad \forall t. \quad (50)$$

In (50), the set Π_0 is defined similarly as in Example 1 of Pomet and Praly (1992), i.e.

$$\Pi_0=\left\{\hat\vartheta\left|\begin{matrix}|\hat\vartheta_i-\nu_i|<\sigma_i, & i=1,p+2,\ldots,p+m+1\\ |\hat\theta-\theta_0|<\bar\theta, & \hat\theta=[\hat\vartheta_2,\ldots,\hat\vartheta_{p+1}]^T\end{matrix}\right.\right\}. \quad (51)$$

Note that θ_0 and $\bar\theta$ are given in Assumption 4 and ν_i,σ_i are given as

$$\nu_1=\left(\zeta+\sum_{j=1}^m\bar b_j\right)/2,$$
$$\nu_i=0,\quad i=p+2,\ldots,p+m+1; \quad (52)$$
$$\sigma_1=\nu_1-\zeta,$$
$$\sigma_i=\bar b_j\bar u_{k(i-p-1)},\quad i=p+2,\ldots,p+m+1. \quad (53)$$

By doing these, $\zeta\le\hat b\le\sum_{j=1}^m\bar b_j$, $|\hat k_j|\le\bar b_j\bar u_{kj}$ and $\hat\theta\in\mathcal{C}$ all the time. Based on Krstic et al. (1995) and Pomet and Praly (1992), the detailed design of projection operator is given below.

Choosing a C^2 function $\mathcal{P}(\hat\vartheta)$: $\Re^{p+m+1}\to\Re$ as

$$\mathcal{P}(\hat\vartheta)=\sum_{i=1,p+2,\ldots,p+m+1}\left|\frac{\hat\vartheta_i-\nu_i}{\sigma_i}\right|^q+\left(\frac{|\hat\theta-\theta_0|}{\bar\theta}\right)^q-1+\varsigma, \quad (54)$$

where $0<\varsigma<1$ and $q\ge2$ are two real numbers. We then define the set Π as

$$\Pi=\left\{\hat\vartheta\,|\,\mathcal{P}(\hat\vartheta)\le0\right\}. \quad (55)$$

Clearly, Π approaches Π_0 as ς decreases and q increases. Similar to (E.3) in Krstic et al. (1995), we consider the following convex set

$$\Pi_\varsigma=\left\{\hat\vartheta\left|\mathcal{P}(\hat\vartheta)\le\frac{\varsigma}{2}\right.\right\}, \quad (56)$$

which contains Π for the purpose of constructing a smooth projection operator as

$$\mathrm{Proj}(\tau)=\begin{cases}\tau, \mathcal{P}(\hat\vartheta)\le0 \quad\text{or}\quad \frac{\partial\mathcal{P}}{\partial\vartheta}(\hat\vartheta)\tau\le0\\ \tau-c(\hat\vartheta)\Gamma\frac{\frac{\partial\mathcal{P}}{\partial\hat\vartheta}(\hat\vartheta)\frac{\partial\mathcal{P}}{\partial\hat\vartheta}(\hat\vartheta)^T}{\frac{\partial\mathcal{P}}{\partial\hat\vartheta}(\hat\vartheta)^T\Gamma\frac{\partial\mathcal{P}}{\partial\hat\vartheta}(\hat\vartheta)}\tau, \quad \text{if not}\end{cases} \quad (57)$$

where $\hat\vartheta(0)\in\Pi$ and

$$c(\hat\vartheta)=\min\left\{1,\frac{2\mathcal{P}(\hat\vartheta)}{\varsigma}\right\}. \quad (58)$$

It is helpful to be noted that

$$c(\hat\vartheta)=\begin{cases}0, & \mathcal{P}(\hat\vartheta)=0\\ 1, & \mathcal{P}(\hat\vartheta)=\frac{\varsigma}{2}.\end{cases} \quad (59)$$

The properties of projection operator (57) are rendered in the following lemma.

Lemma 5.

(i) $\mathrm{Proj}(\tau)^T\Gamma^{-1}\mathrm{Proj}(\tau)\le\tau^T\Gamma^{-1}\tau,\ \forall\hat\vartheta\in\Pi_\varsigma$.

(ii) *Let $\Gamma(t),\tau(t)$ be continuously differentiable and $\dot{\hat\vartheta}=\mathrm{Proj}(\tau), \hat\vartheta(0)\in\Pi_\varsigma$. Then on its domain of definition, the solution $\hat\vartheta(t)$ remains in Π_ς.*

(iii) $-\tilde\vartheta^T\Gamma^{-1}\mathrm{Proj}(\tau)\le-\tilde\vartheta\Gamma^{-1}\tau, \forall\hat\vartheta\in\Pi_\varsigma,\ \theta\in\Pi$.

Proof. The proof is similar to the proof of Lemma E.1 in Krstic et al. (1995). □

Based on these, we have the following results, which will be useful in the analysis of system stability and the performance of tracking error in the mean square sense.

Lemma 6. *The estimator (49) has the following properties.*
(i) $\epsilon \in S_2(\mu)$;
(ii) $\dot{\hat{\vartheta}} \in S_2(\mu)$.

Proof. We define a positive definite function

$$V_\vartheta = \frac{1}{2}\tilde{\vartheta}^T \Gamma^{-1}\tilde{\vartheta}. \tag{60}$$

From Lemma 5(iii), we have

$$\begin{aligned}\dot{V}_\vartheta &= \tilde{\vartheta}^T\Gamma^{-1}(\dot{\vartheta} - \dot{\hat{\vartheta}}) \le -\tilde{\vartheta}^T\Gamma^{-1}(\Gamma\Omega\epsilon) + \tilde{\vartheta}^T\Gamma^{-1}\dot{\vartheta} \\ &= -(\epsilon - \varepsilon)^T\epsilon + \tilde{\vartheta}^T\Gamma^{-1}\dot{\vartheta} \\ &\le -\epsilon^T\epsilon + |\varepsilon^T\epsilon| + \tilde{\vartheta}^T\Gamma^{-1}\dot{\vartheta}.\end{aligned} \tag{61}$$

• Proof of (i).
By integrating both sides of (61) and using Hölder's inequality, we obtain

$$\begin{aligned}\int_t^{t+T}\dot{V}_\vartheta \mathrm{d}\tau &\le -\int_t^{t+T}\epsilon^T\epsilon\mathrm{d}\tau + \|\epsilon\|_\infty\int_t^{t+T}|\varepsilon|\mathrm{d}\tau \\ &\quad + \|\tilde{\vartheta}\|_\infty\|\Gamma^{-1}\|_\infty\int_t^{t+T}|\dot{\vartheta}|\mathrm{d}\tau \\ &\le -\int_t^{t+T}\epsilon^T\epsilon\mathrm{d}\tau + \|\epsilon\|_\infty(C_6\mu T + C_7) \\ &\quad + \|\tilde{\vartheta}\|_\infty\frac{1}{\lambda_{\min}(\Gamma)}(C_1\mu T + C_2).\end{aligned} \tag{62}$$

Thus

$$\begin{aligned}&\int_t^{t+T}\epsilon(\tau)^T\epsilon(\tau)\mathrm{d}\tau \\ &\le \frac{1}{2\lambda_{\min}(\Gamma)}\left(\tilde{\vartheta}(t)^T\tilde{\vartheta}(t) - \tilde{\vartheta}(t+T)^T\tilde{\vartheta}(t+T)\right) \\ &\quad + \|\epsilon\|_\infty(C_6\mu T + C_7) + \frac{\|\tilde{\vartheta}\|_\infty}{\lambda_{\min}(\Gamma)}(C_1\mu T + C_2) \\ &\le (C_{11}\mu^2 + C_{12}\mu)T + C_{13},\end{aligned} \tag{63}$$

where $C_{11} = C_8$ and

$$C_{12} = C_9 + \frac{C_1\|\tilde{\vartheta}\|_\infty}{\lambda_{\min}(\Gamma)}, \qquad C_{13} = C_{10} + \frac{\|\tilde{\vartheta}\|_\infty^2 + 2C_2\|\tilde{\vartheta}\|_\infty}{2\lambda_{\min}(\Gamma)}. \tag{64}$$

From Lemma 5(iii), $\hat{\vartheta}(t)$ remains in Π_ς if $\hat{\vartheta}(0) \in \Pi_\varsigma$. From Assumption 4 and the definition of Π_ς, we know that $\vartheta \in \Pi_\varsigma$. Therefore $\tilde{\vartheta}$ is bounded by utilizing the projection operator, $\epsilon \in S_2(\mu)$.

• Proof of (ii).
From Lemma 5(i) and Hölder's inequality, we have

$$\begin{aligned}\int_t^{t+T}\dot{\hat{\vartheta}}^T\dot{\hat{\vartheta}}\mathrm{d}\tau &\le \int_t^{t+T}\epsilon^T\Omega^T\Gamma^2\Omega\epsilon \\ &\le \lambda_{\max}(\Gamma)^2\left\|\,\|\Omega\|_F^2\right\|_\infty\int_t^{t+T}\epsilon^T\epsilon\mathrm{d}\tau.\end{aligned} \tag{65}$$

Thus from (63),

$$\int_t^{t+T}\dot{\hat{\vartheta}}(\tau)^T\dot{\hat{\vartheta}}(\tau)\mathrm{d}\tau \le (C_{14}\mu^2 + C_{15}\mu)T + C_{16}, \tag{66}$$

where $C_{1i} = C_{1i-3}\lambda_{\max}(\Gamma)^2\|\,\|\Omega\|_F^2\,\|_\infty$ for $i = 4, 5, 6$. Therefore, $\dot{\hat{\vartheta}} \in S_2(\mu)$ is concluded. □

4. Stability analysis

We will first prove the input-to-state stability of an error system with $\tilde{\vartheta}$ and $\dot{\hat{\theta}}$ as the inputs. An error system obtained by applying the design procedure (20)–(27) to system (8) is given by

$$\dot{z} = A_z(z, \hat{\vartheta}, t)z + W_\vartheta(z, \hat{\vartheta}, t)^T\tilde{\vartheta} + Q_\theta(z, \hat{\vartheta}, t)^T\dot{\hat{\theta}}, \tag{67}$$

where

$$A_z = \begin{bmatrix} -(c_1+s_1) & 1 & 0 & \cdots & 0 \\ -1 & -(c_2+s_2) & 1 & \ddots & \vdots \\ 0 & -1 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & -1 & -\frac{b}{\zeta}(c_\varrho + s_\varrho)\end{bmatrix}, \tag{68}$$

$$W_\vartheta^T = \begin{bmatrix} 0 & w_1^T & 0_{1\times m} \\ 0 & w_2^T & 0_{1\times m} \\ \vdots & \vdots & \vdots \\ \dfrac{\bar{\alpha}_\varrho + y_r^{(\varrho)}}{\hat{b}} & w_\varrho^T & \beta^T\end{bmatrix}, \tag{69}$$

$$Q_\theta^T = \left[0, -\frac{\partial\alpha_1}{\partial\hat{\theta}}, \ldots, \frac{-\partial\alpha_{\varrho-1}}{\partial\hat{\theta}}\right]^T. \tag{70}$$

For the error system (67)–(70), the following input-to-state property holds.

Lemma 7. *If $\tilde{\theta}, \tilde{b}, \tilde{k}, \dot{\hat{\theta}} \in L_\infty$, then $z \in L_\infty$ and*

$$\begin{aligned}|z(t)| &\le \frac{1}{2\sqrt{c_0}}\left[\frac{1}{\kappa_0}(\|\tilde{\theta}\|_\infty^2 + \|\tilde{b}\|_\infty^2 + \|\tilde{k}\|_\infty^2) + \frac{1}{g_0}\|\dot{\hat{\theta}}\|_\infty^2\right]^{\frac{1}{2}} \\ &\quad + |z(0)|\mathrm{e}^{-c_0 t},\end{aligned} \tag{71}$$

where $\tilde{\theta} = \theta - \hat{\theta}$, $\tilde{b} = b - \hat{b}$ and $\tilde{k} = k - \hat{k}$ and c_0, κ_0 and g_0 are defined as

$$c_0 = \min_{1\le i\le\varrho} c_i, \qquad \kappa_0 = \left(\sum_{i=1}^{\varrho}\frac{1}{\kappa_i}\right)^{-1}, \qquad g_0 = \left(\sum_{i=1}^{\varrho}\frac{1}{g_i}\right)^{-1}. \tag{72}$$

Proof. Along the solutions of (67), we compute

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}|z|^2\right) &\le -c_0|z|^2 - \sum_{i=1}^{\varrho}\kappa_i\left|w_i z_i - \frac{1}{2\kappa_i}\tilde{\theta}\right|^2 \\ &\quad - \sum_{i=1}^{\varrho}g_i\left|\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}^T z_i + \frac{1}{2g_i}\dot{\hat{\theta}}\right|^2 \\ &\quad - \kappa_\varrho\left[\left(\frac{y_r^{(\varrho)} + \bar{\alpha}_\varrho}{\hat{b}}\right)z_\varrho - \frac{1}{2\kappa_n}\tilde{b}\right]^2 \\ &\quad - \kappa_\varrho\left|\beta z_\varrho - \frac{1}{2\kappa_\varrho}\tilde{k}\right|^2 + \left(\sum_{i=1}^{n}\frac{1}{4\kappa_i}\right)|\tilde{\theta}|^2 \\ &\quad + \left(\sum_{i=1}^{n}\frac{1}{4g_i}\right)|\dot{\hat{\theta}}|^2 + \frac{1}{4\kappa_n}(\tilde{b}^2 + |\tilde{k}|^2) \\ &\le -c_0|z|^2 + \frac{1}{4}\left[\frac{1}{\kappa_0}(|\tilde{\theta}|^2 + \tilde{b}^2 + |\tilde{k}|^2) + \frac{1}{g_0}|\dot{\hat{\theta}}|^2\right].\end{aligned} \tag{73}$$

Let $v = |z|^2$ and $\mathscr{L} = \left[\frac{1}{\kappa_0}(|\tilde{\theta}|^2 + \tilde{b}^2 + |\tilde{k}|^2) + \frac{1}{g_0}|\dot{\hat{\theta}}|^2\right]^{1/2}$, it follows that

$$\dot{v} \le -2c_0 v + \frac{1}{2}\mathscr{L}^2. \tag{74}$$

If $\tilde{\theta}, \tilde{b}, \tilde{k}$ and $\dot{\hat{\theta}} \in L_\infty$, $\mathscr{L} \in L_\infty$, then $v \in L_\infty$ and

$$\begin{aligned} v(t) &\le v(0)\mathrm{e}^{-2c_0 t} + \frac{1}{4c_0}\|\mathscr{L}\|_\infty^2 \\ &\le v(0)\mathrm{e}^{-2c_0 t} + \frac{1}{4c_0}\left[\frac{1}{\kappa_0}(\|\tilde{\theta}\|_\infty^2 + \|\tilde{b}\|_\infty^2 \right. \\ &\quad \left. + \|\tilde{k}\|_\infty^2) + \frac{1}{g_0}\|\dot{\hat{\theta}}\|_\infty^2\right]. \quad \square \end{aligned} \tag{75}$$

We are now at the position to present the main results of this paper in the following theorem.

Theorem 1. *Consider the closed-loop adaptive system consisting of the nonlinear plant (1), the controller (5), (27) and the parameter update law (49). Irrespective of actuator failures or faults modeled in (3)–(4) under Assumptions 1–5, we have the following results.*

(i) *All the signals of the closed-loop system are ensured bounded as long as μ is finite.*
(ii) *The tracking error $z_1 = y - y_r$ is small in the mean square sense that $z_1(t) \in S_2(\mu)$.*
(iii) *The asymptotic tracking can be achieved for a finite number of failures and faults, i.e. $\lim_{t\to\infty} z_1(t) = 0$.*

Proof.

• Proof of (i).

$\tilde{\vartheta}$ is bounded by utilizing the projection operator in (49). From Lemma 3, Ω is bounded. From Lemma 4, ε is bounded as long as μ is finite. Thus from (48), ϵ is bounded and so is $\dot{\hat{\vartheta}}$. Thus all the conditions in Lemma 7 are satisfied, then $z(t) \in L_\infty$. From Assumption 2, the definition of z_i in (20) and the design of α_i in (21)–(23), $x(t) \in L_\infty$. From Assumption 5, ξ is bounded with respect to $x(t)$ as the input. α_ρ is then bounded. From (5) and (27), control signals u_{cj} for $j = 1, 2, \ldots, m$ are also bounded. The closed-loop stability is then established.

• Proof of (ii).

Rewrite (67) as

$$\dot{z} = \bar{A}_z(z, \hat{\vartheta}, t)z + \bar{W}_\vartheta(z, \hat{\vartheta}, t)^T\tilde{\vartheta} + Q_\theta(z, \hat{\vartheta}, t)^T\dot{\hat{\theta}}, \tag{76}$$

where Q_θ is the same as in (70) and

$$\bar{A}_z = \begin{bmatrix} -(c_1+s_1) & 1 & 0 & \cdots & 0 \\ -1 & -(c_2+s_2) & 1 & \ddots & \vdots \\ 0 & -1 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & -1 & -\frac{\hat{b}}{\zeta}(c_\varrho + s_\varrho) \end{bmatrix}, \tag{77}$$

$$\bar{W}_\vartheta^T = \begin{bmatrix} 0 & w_1^T & 0_{1\times m} \\ 0 & w_2^T & 0_{1\times m} \\ \vdots & \vdots & \vdots \\ u_0 & w_n^T & \beta^T \end{bmatrix}. \tag{78}$$

Introduce the state χ^T as

$$\dot{\chi}^T = \bar{A}_z\chi^T + \bar{W}_\vartheta^T. \tag{79}$$

Similarly to Lemma 2, we obtain that $\|\Phi_{\bar{A}_z}(t, t_0)\| \le \bar{\bar{k}}_1\mathrm{e}^{-r_1(t-t_0)}$ where $\bar{\bar{k}}_1, r_1$ are positive constants. Thus $\chi \in L_\infty$ is shown from (79) and the boundedness of $\bar{W}_\vartheta$.

By defining η as

$$\eta = z - \chi^T\tilde{\vartheta}, \tag{80}$$

we will show (ii) in two steps. In Step 1, $\eta \in S_2(\mu)$ will be proved. Then we will establish that $\chi^T\tilde{\vartheta} \in S_2(\mu)$ in Step 2. Thus from (80), $z(t) \in S_2(\mu)$ will be obtained.

Step 1.

Computing the derivative of η gives that

$$\begin{aligned} \dot{\eta} &= \dot{z} - \dot{\chi}^T\tilde{\vartheta} - \chi^T(\dot{\vartheta} - \dot{\hat{\vartheta}}) \\ &= \bar{A}_z\eta + Q_\theta^T\dot{\hat{\theta}} + \chi^T\dot{\hat{\vartheta}} - \chi^T\dot{\vartheta}. \end{aligned} \tag{81}$$

The solution of (81) is

$$\begin{aligned} \eta(t) &= \Phi_{\bar{A}_z}(t, 0)\eta(0) \\ &\quad + \int_0^t \Phi_{\bar{A}_z}(t, \tau)Q_\theta(z(\tau), \hat{\vartheta}(\tau), \tau)^T \cdot \dot{\hat{\theta}}(\tau)\mathrm{d}\tau \\ &\quad + \int_0^t \Phi_{\bar{A}_z}(t, \tau)\chi(\tau)^T\dot{\hat{\vartheta}}(\tau)\mathrm{d}\tau \\ &\quad - \int_0^t \Phi_{\bar{A}_z}(t, \tau)\chi(\tau)^T\dot{\vartheta}(\tau)\mathrm{d}\tau. \end{aligned} \tag{82}$$

Since Q_θ and χ are bounded, we have

$$\begin{aligned} |\eta(t)| &\le \bar{\bar{k}}_1\mathrm{e}^{-r_1 t}|\eta(0)| + \bar{\bar{k}}_1\|Q_\theta\|_\infty \int_0^t \mathrm{e}^{-r_1(t-\tau)} \cdot |\dot{\hat{\theta}}(\tau)|\mathrm{d}\tau \\ &\quad + \bar{\bar{k}}_1\|\chi\|_\infty \int_0^t \mathrm{e}^{-r_1(t-\tau)}|\dot{\hat{\vartheta}}(\tau)|\mathrm{d}\tau \\ &\quad + \bar{\bar{k}}_1\|\chi\|_\infty \int_0^t \mathrm{e}^{-r_1(t-\tau)}|\dot{\vartheta}|\mathrm{d}\tau = \eta_1 + \eta_2, \end{aligned} \tag{83}$$

where η_1 and η_2 are defined respectively as

$$\begin{aligned} \eta_1 &= \bar{\bar{k}}_1\left(\|Q_\theta\|_\infty \int_0^t \mathrm{e}^{-r_1(t-\tau)}|\dot{\hat{\theta}}(\tau)|\mathrm{d}\tau \right. \\ &\quad \left. + \|\chi\|_\infty \int_0^t \mathrm{e}^{-r_1(t-\tau)}|\dot{\hat{\vartheta}}(\tau)|\mathrm{d}\tau\right) \end{aligned} \tag{84}$$

$$\eta_2 = \bar{\bar{k}}_1\left(\mathrm{e}^{-r_1 t}|\eta(0)| + \|\chi\|_\infty \int_0^t \mathrm{e}^{-r_1(t-\tau)}|\dot{\vartheta}|\mathrm{d}\tau\right). \tag{85}$$

By following similar procedures to the proof of Lemma 4(ii), it can be shown that $\eta_2 \in S_2(\mu)$. Now we show that $\eta_1 \in S_2(\mu)$. Using the Schwartz inequality, we obtain

$$\begin{aligned} \eta_1 &\le \bar{\bar{k}}_1\left[\|Q_\theta\|_\infty\left(\int_0^t \mathrm{e}^{-r_1(t-\tau)}\mathrm{d}\tau\right)^{\frac{1}{2}}\left(\int_0^t \mathrm{e}^{-r_1(t-\tau)} \right.\right. \\ &\quad \left. \times|\dot{\hat{\theta}}(\tau)|^2\mathrm{d}\tau\right)^{\frac{1}{2}} + \|\chi\|_\infty\left(\int_0^t \mathrm{e}^{-r_1(t-\tau)}\mathrm{d}\tau\right)^{\frac{1}{2}} \\ &\quad \left.\left(\int_0^t \mathrm{e}^{-r_1(t-\tau)}|\dot{\hat{\vartheta}}(\tau)|^2\mathrm{d}\tau\right)^{\frac{1}{2}}\right] \\ &\le \frac{\bar{\bar{k}}_1}{\sqrt{r_1}}\left[\|Q_\theta\|_\infty\left(\int_0^t \mathrm{e}^{-r_1(t-\tau)}|\dot{\hat{\theta}}(\tau)|^2\mathrm{d}\tau\right)^{\frac{1}{2}} \right. \\ &\quad \left. + \|\chi\|_\infty^2\left(\int_0^t \mathrm{e}^{-r_1(t-\tau)}|\dot{\hat{\vartheta}}(\tau)|^2\mathrm{d}\tau\right)^{\frac{1}{2}}\right]. \end{aligned} \tag{86}$$

By squaring both sides of (86) and integrating it over $[t, t+T]$, we have

$$\int_t^{t+T} \eta_1^2 d\tau \leq \frac{2\bar{\bar{k}}_1^2}{r_1}\left[\|Q_\theta\|_\infty^2 \int_t^{t+T}\int_0^\tau e^{-r_1(\tau-s)}|\dot{\hat{\theta}}(s)|^2 ds d\tau + \|\chi\|_\infty^2 \int_t^{t+T}\int_0^\tau e^{-r_1(\tau-s)}|\dot{\hat{\vartheta}}(s)|^2 ds d\tau\right]. \quad (87)$$

Similar to the proof of Lemma 4, we obtain that

$$\begin{aligned}\int_t^{t+T} \eta_1^2 d\tau \leq\ & \frac{2\bar{\bar{k}}_1^2\|Q_\theta\|_\infty^2}{r_1}\left(\frac{1}{r_1}\int_0^t e^{-r_1(t-s)}|\dot{\hat{\theta}}(s)|^2 ds + \int_t^{t+T} e^{r_1 s} \times |\dot{\hat{\theta}}(s)|^2 \int_s^{t+T} e^{-r_1\tau} d\tau ds\right) + \frac{2\bar{\bar{k}}_1^2\|\chi\|_\infty^2}{r_1} \\ & \times\left(\frac{1}{r_1}\int_0^t e^{-r_1(t-s)}|\dot{\hat{\vartheta}}(s)|^2 ds + \int_t^{t+T} e^{r_1 s}|\dot{\hat{\vartheta}}(s)|^2 \int_s^{t+T} e^{-r_1\tau} d\tau ds\right) \\ \leq\ & \frac{2\bar{\bar{k}}_1^2}{r_1^2}(\|Q_\theta\|_\infty^2 + \|\chi\|_\infty^2)\frac{e^{r_1}(C_{14}\mu^2 + C_{15}\mu + C_{16})}{1 - e^{-r_1}} \\ & + \frac{2\bar{\bar{k}}_1^2\|Q_\theta\|_\infty^2}{r_1^2}\int_t^{t+T}|\dot{\hat{\theta}}(s)|^2 ds \\ & + \frac{2\bar{\bar{k}}_1^2\|\chi\|_\infty^2}{r_1^2}\int_t^{t+T}|\dot{\hat{\vartheta}}(s)|^2 ds. \end{aligned} \quad (88)$$

From Lemma 6(ii), $\dot{\hat{\vartheta}} \in S_2(\mu)$, thus $\dot{\hat{\theta}} \in S_2(\mu)$ and $\eta_1 \in S_2(\mu)$. From (83), $\eta \in S_2(\mu)$ where we have used the fact that $|\eta|^2 \leq 2(\eta_1^2 + \eta_2^2)$.

Step 2.

From (48), Lemma 4(ii) and Lemma 6(i), we have $\Omega^T\tilde{\vartheta} \in S_2(\mu)$. Thus our main task in this step is to show that $\Omega^T\tilde{\vartheta} \in S_2(\mu)$ implies $\chi^T\tilde{\vartheta} \in S_2(\mu)$.

For the simplicity of presentation, we represent the following system by an operator $T_{A_i}[\cdot]$,

$$\dot{\zeta}_i = A_i(t)\zeta_i + u, \quad (89)$$

where $A_i : \Re_+ \to \Re^{\varrho\times\varrho}$ is continuous, bounded, and exponentially stable. For example, $\zeta_1 = T_A[F^T\tilde{\vartheta}]$ if $\dot{\zeta}_1 = A\zeta_1 + F^T\tilde{\vartheta}$, where A is defined in (32).

Since the stability of the closed-loop system has been shown, F is bounded. Similarly to the proof of $\eta \in S_2(\mu)$, $\zeta_1 - \Omega^T\tilde{\vartheta} = T_A[F^T\tilde{\vartheta}] - T_A[F^T]\tilde{\vartheta} \in S_2(\mu)$ can also be shown. From $\Omega^T\tilde{\vartheta} \in S_2(\mu)$, it follows that $\zeta_1 \in S_2(\mu)$.

We now show that $\zeta_2 = T_{\bar{A}_z}[\bar{W}_\vartheta\tilde{\vartheta}] \in S_2(\mu)$, where $\bar{A}_z$ is the same as in (77).

From (24), (29) and (78), we have

$$\bar{W}_\vartheta = MF^T, \quad (90)$$

where

$$M = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -\frac{\partial\alpha_1}{\partial x_1} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ -\frac{\partial\alpha_{\varrho-1}}{\partial x_1} & \cdots & -\frac{\partial\alpha_{\varrho-1}}{\partial x_{\varrho-1}} & 1 \end{bmatrix}. \quad (91)$$

Then we obtain

$$\begin{aligned}\zeta_2 &= \Phi_{\bar{A}_z}(t,0)\zeta(0) + \int_0^t \Phi_{\bar{A}_z}(t,\tau)M(\tau)F^T(\tau)\tilde{\vartheta}(\tau)d\tau \\ &= \Phi_{\bar{A}_z}(t,0)\zeta(0) + \int_0^t \Phi_{\bar{A}_z}(t,\tau)M(\tau)[\dot{\zeta}_1 - A(\tau)\zeta_1(\tau)]d\tau \\ &= \Phi_{\bar{A}_z}(t,0)\zeta(0) + M(t)\zeta_1(t) - \Phi_{\bar{A}_z}(t,0)M(0)\zeta_1(0) \\ &\quad - \int_0^t \Phi_{\bar{A}_z}(t,\tau)[\dot{M}(\tau) + \bar{A}_z(\tau)M(\tau) + M(\tau)A(\tau)]\zeta_1(\tau)d\tau. \end{aligned} \quad (92)$$

Note that M has a similar form to matrix N on Page 253 of Krstic et al. (1995). By following similar analysis in Krstic et al. (1995), we have that M and $\dot{M}$ are bounded. Thus for the last term in (92), we get

$$\left|\int_0^t \Phi_{\bar{A}_z}(t,\tau)[\dot{M}(\tau) + \bar{A}_z(\tau)M(\tau) + M(\tau)A(\tau)]\cdot\zeta_1(\tau)d\tau\right|^2 \leq \|\dot{M} + \bar{A}_zM + MA\|_\infty^2 \int_0^t e^{-2c_0(t-\tau)}|\zeta_1(\tau)|^2 d\tau. \quad (93)$$

Since $\zeta_1 \in S_2(\mu)$, $\zeta_2 = T_{\bar{A}_z}[MF^T\tilde{\vartheta}] \in S_2(\mu)$ can be concluded by following similar procedures in proving $\eta_1 \in S(\mu)$ in Step 1. Moreover, $T_{\bar{A}_z}[MF^T\tilde{\vartheta}] - T_{\bar{A}_z}[MF^T]\tilde{\vartheta} \in S_2(\mu)$ can also be shown by following the similar procedures in the proof of $\eta \in S_2(\mu)$. We then obtain that $T_{\bar{A}_z}[MF^T]\tilde{\vartheta} \in S_2(\mu)$. Thus $T_{\bar{A}_z}[\bar{W}_\vartheta^T]\tilde{\vartheta} = \chi^T\tilde{\vartheta} \in S_2(\mu)$. From $z = \chi^T\tilde{\vartheta} + \eta$, $z \in S_2(\mu)$. Hence $z_1 \in S_2(\mu)$ follows.

From Lemma 1, we know that $\mu = \frac{1}{T^*}$ where T^* is the minimum time interval between two successive changes of failure/fault pattern. Clearly, μ can be very small for a large T^*.

- Proof of (iii).

For the case with a finite number of failures and faults, the result that $\dot{\hat{\vartheta}}(t) \in S_1(\mu)$ will be changed to that $\dot{\hat{\vartheta}}(t) \in L_1$. Through the similar procedures in the analysis above, $z(t) \in L_2$ will be followed instead of $z(t) \in S_2(\mu)$. From (67), $\dot{z}(t) \in L_\infty$. Together with the facts that $z(t) \in L_\infty$, from the corollary of Barbalat's Lemma as provided in Appendix A in Krstic et al. (1995), asymptotic tracking will be achieved, i.e. $\lim_{t\to\infty} z_1(t) = 0$. □

Remark 5. With our proposed scheme, all the closed-loop signals are ensured bounded even if there are infinite number of actuator failures or faults as long as the time interval between two successive changes of failure/fault pattern is bounded below by an arbitrary positive number. Such a condition is less restrictive than that conjectured in Tang et al. (2007). Moreover, from the established tracking error performance in Theorem 1(ii), we see that the frequency of changing failure/fault patterns will affect the tracking performance. In fact for a designed adaptive controller with a given set of design parameters and initial conditions, the less frequent the failure/fault pattern changes, the better the tracking performance is.

Our results can also be extended to the following situations, even though they are not the focus of the paper.

Remark 6.

- As far as the 'offline' repair situation (namely actuators may repeatedly fail, be removed from the loop and then put back into the loop after recovery) is concerned, stability result cannot be established by using the existing tuning function schemes. This is because when the actuators change only from a working mode to an 'offline' repairing mode infinitely many times, the parameter b in (8) will experience infinite number of jumps which will lead to instability if they are not carefully handled.

However, system stability can be ensured with our proposed scheme if Assumptions 1–5 are satisfied and the time intervals between two successive changes of failure/fault pattern are bounded below by an arbitrary positive number.

- The results achieved in this paper can also be applied to time varying systems. The derivatives of the unknown parameters are not required to be bounded like many other results on adaptive backstepping control of time varying systems such as Fidan, Zhang, and Ioannou (2005), Giri, Rabeh, and Ikhouane (1999) and Zhang, Fidan, and Ioannou (2003). On the other hand, the parameter μ being finite is the only condition to achieve the boundedness of all closed-loop signals in this paper. In contrast to previous results on adaptive control of systems with possible jumping parameters such as in Middleton and Goodwin (1988) and Zhang, Wen, and Soh (2000), μ is not required to satisfy that $\mu \in (0, \mu^*]$ where μ^* is a function of the bounds of unknown system parameters as well as design parameters. Thus the results here are more general than those in Middleton and Goodwin (1988) and Zhang et al. (2000).
- Similar to the comments in Tang and Tao (2009), more general failures modeled like $u_j(t) = u_{kj,h} + \sum_{i=1}^{n_j} d_{jh,i} \cdot f_{jh,i}(t)$ for $j = 1, 2, \ldots, m$, with smooth functions $f_{jh,i}(t)$ and unknown constants $u_{kj,h}, d_{jh,i}$ can also be handled with our proposed scheme. However different from Tang and Tao (2009), $f_{jh,i}(t)$ can be allowed unknown with our proposed scheme, as long as it varies in such a way that $\dot{\vartheta} \in S_1(\mu)$ is still satisfied.

5. Simulation studies

5.1. A numerical example

In this subsection, a numerical example is considered to illustrate the ability of the proposed scheme in compensating for infinite number of relatively frequent actuator failures. To carry out a comparison, the results by using a tuning function scheme in Wang and Wen (2010, Sec. 3), which can be regarded as a representative of currently available results in the area of adaptive failure/fault compensation for nonlinear systems, are also presented.

We consider a system with dual actuators

$$\dot{\chi} = f_0(\chi) + f(\chi)\theta + \sum_{j=1}^{2} b_j g_j(\chi) u_j$$
$$y = \chi_2, \tag{94}$$

where the state $\chi \in \Re^3$,

$$f_0 = \begin{bmatrix} -\chi_1 \\ \chi_3 \\ \chi_2\chi_3 \end{bmatrix}, \qquad f = \begin{bmatrix} 0 \\ \chi_2^2 \\ \dfrac{1-e^{-\chi_3}}{1+e^{-\chi_3}} \end{bmatrix}, \tag{95}$$

and

$$g_1 = g_2 = \left[\frac{2+\chi_3^2}{1+\chi_3^2}, 0, 1\right]^T, \tag{96}$$

which is modeled similarly to Example 13.6 in Khalil (1996). As discussed in Khalil (1996), to transform (94) into the form of (1), we choose $[\xi, x_1, x_2]^T = T(\chi) = [\phi(\chi), \chi_2, \chi_3]^T$ where $\phi(\chi) = -\chi_1 + \chi_3 + \tan^{-1}\chi_3$. We have $\phi(0) = 0$ and

$$\frac{\partial \phi}{\partial \chi} g_j(\chi) = \frac{\partial \phi}{\partial \chi_1} \cdot \frac{2+\chi_3^2}{1+\chi_3^2} + \frac{\partial \phi}{\partial \chi_3} = 0. \tag{97}$$

Since the equation $T(\chi) = s$ for any $s \in \Re^3$ has a unique solution, the mapping $T(\chi)$ is a global diffeomorphism. Thus, the transformed system below

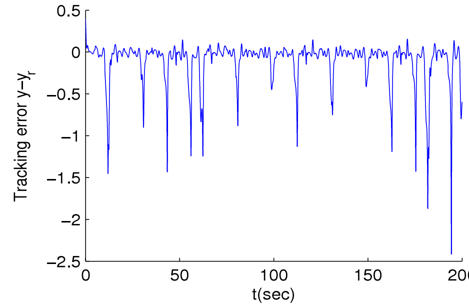


Fig. 1. Tracking error $y(t) - y_r(t)$ with the scheme in Wang and Wen (2010, Sec. 3) when $T^* = 5$ s.

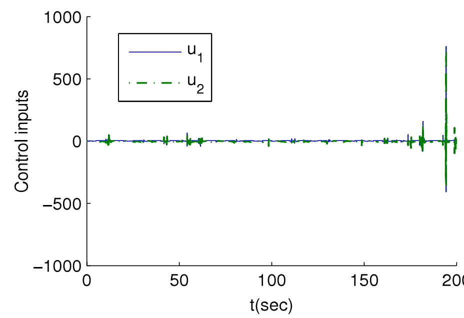


Fig. 2. Control u_1 and u_2 with scheme in Wang and Wen (2010, Sec. 3) when $T^* = 5$ s.

$$\dot{\xi} = -\xi + x_2 + \tan^{-1} x_2 + \frac{2+x_2^2}{1+x_2^2}\left(x_1x_2 + \frac{1-e^{-x_2}}{1+e^{-x_2}}\theta\right)$$
$$\dot{x}_1 = x_2 + x_1^2\theta$$
$$\dot{x}_2 = x_1x_2 + \frac{1-e^{-x_2}}{1+e^{-x_2}}\theta + \sum_{j=1}^{2} b_j u_j \tag{98}$$

is defined globally. Because of the boundedness of functions $\tan^{-1}(x_2)$, $\frac{2+x_2^2}{1+x_2^2}$ and $\frac{1-e^{-x_2}}{1+e^{-x_2}}$, it is concluded that $\dot{\xi} = -\xi + \eta(x_1, x_2, \theta)$ is ISS where $\eta = x_2 + \tan^{-1} x_2 + \frac{2+x_2^2}{1+x_2^2}\left(x_1x_2 + \frac{1-e^{-x_2}}{1+e^{-x_2}}\theta\right)$. Thus Assumption 5 is satisfied.

The considered failure case is modeled as

$$u_1(t) = u_{k1,h}, \quad t \in [hT^*, (h+1)T^*), \ h = 1, 3, \ldots, \tag{99}$$

which implies that the output of first actuator (u_1) is stuck at $u_1 = u_{k1,h}$ at every hT^* seconds and is back to normal operation at every $(h+1)T^*$ seconds until the next failure occurs.

The following information is unknown in the designs.

$$\theta = 2, \qquad b_1 = 1, \qquad b_2 = 1.1,$$
$$u_{k1,h} = 5, \qquad T^* = 5. \tag{100}$$

However, we know that $b_1, b_2 > 0$ and

$$1 \le \theta \le 3, \qquad 0.8 \le |b_1| \le 1.4, \qquad 0.6 \le |b_2| \le 2 \tag{101}$$
$$0.5 \le \rho_{jh} \le 1, \qquad |u_{kj,h}| \le 6, \quad j = 1, 2. \tag{102}$$

The reference signal $y_r = \sin(t)$.

We first design the adaptive controllers following the procedures in Wang and Wen (2010, Sec. 3). In simulation, the initial states and estimates are all set as 0 except that $\chi_2(0) = 0.4$ and $\hat{\theta}(0) = 1$. The design parameters are chosen as $c_1 = c_2 = 5$, $\Gamma = 3$, $\Gamma_\kappa = 3 \times I_3$. The performances of the tracking error and control inputs (u_1, u_2) versus time are given in Figs. 1 and 2, respectively. It can be seen that after 150 s, the magnitudes of the error

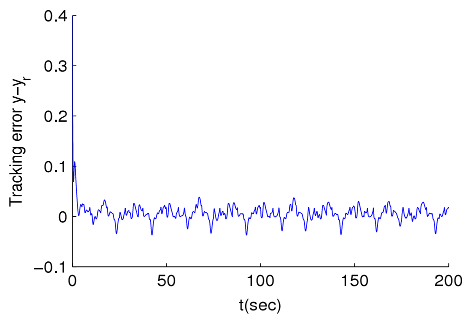


Fig. 3. Tracking error $y(t) - y_r(t)$ with proposed scheme when $T^* = 5$ s.

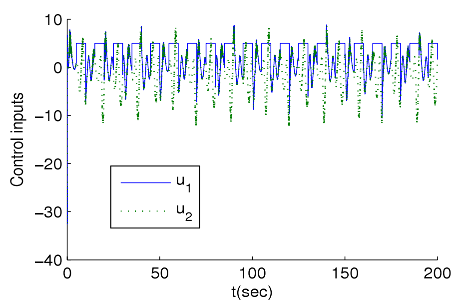


Fig. 4. Control u_1 and u_2 with proposed scheme when $T^* = 5$ s.

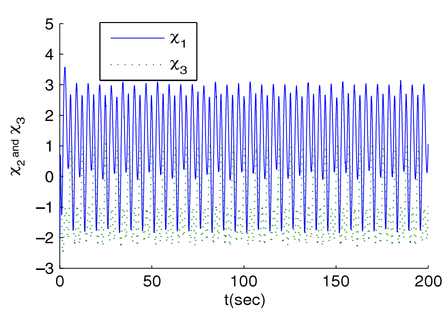


Fig. 5. χ_1 and χ_3 with proposed scheme when $T^* = 5$ s.

signal grows larger and larger. Growing phenomenon can also be observed from the control signal even more rapidly. It seems that the boundedness of the signals cannot be guaranteed in this case.

We then adopt the proposed modular scheme to redesign the adaptive controllers. $\hat{b}(0) = 1.5$, the rest of the initial states and estimates are kept the same as in the tuning function design. The design parameters c_1, c_2 are fixed at $c_1 = c_2 = 5$, while other design parameters are chosen as

$$\zeta = 0.3, \quad \kappa_1 = \kappa_2 = 3, \quad g_2 = 3, \quad \Gamma = 40 \times I_4, \tag{103}$$

$$\nu_1 = \frac{0.3 + 3.4}{2}, \quad \nu_3 = \nu_4 = 0, \tag{104}$$

$$\sigma_1 = 3.4 - 2 = 1.4, \quad \sigma_3 = \sigma_4 = 12, \tag{105}$$

$$\theta_0 = \frac{1+3}{2} = 2, \quad \bar{\theta} = 2, \quad q = 40, \quad \varsigma = 0.01. \tag{106}$$

The performances of tracking error and control signals in this case are given in Figs. 3–4. Apart from these, the states χ_1 and χ_3, parameter estimates are also plotted in Figs. 5–6. Obviously, the boundedness of all the signals is now ensured.

To show how T^* affects the tracking performance when the proposed design scheme is utilized, we set $T^* = 25$ s. The performance of tracking error is now shown in Fig. 7. Comparing

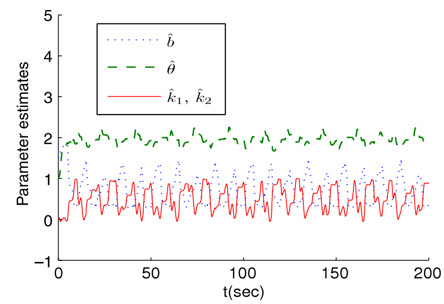


Fig. 6. Parameter estimates with proposed scheme when $T^* = 5$ s.

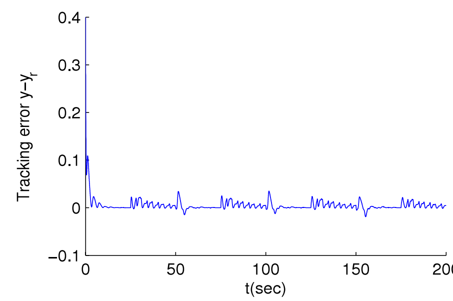


Fig. 7. Tracking error $y(t) - y_r(t)$ with proposed scheme when $T^* = 25$ s.

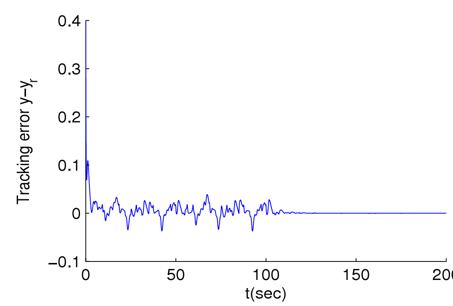


Fig. 8. Tracking error $y(t) - y_r(t)$ with finite number of failures when the proposed scheme is applied.

Figs. 7 and 3, better tracking error performance in the mean square sense is observed.

Now we consider the case that there are finite number of failures by setting $T^* = 5$ s and there will be no failure for $t > 100$ s. The performance of tracking error with our proposed scheme is given in Fig. 8, which shows that the tracking error will converge to zero asymptotically in this case.

5.2. An aircraft application

In this subsection, we apply the proposed scheme to accommodate infinite number of complete failures or PLOE faults for the twin otter aircraft longitudinal nonlinear dynamics model as described in Tang et al. (2003), i.e.

$$\begin{aligned}
\dot{V} &= \frac{F_x \cos(\alpha) + F_z \sin(\alpha)}{m} \\
\dot{\alpha} &= \bar{q} + \frac{-F_x \sin(\alpha) + F_z \cos(\alpha)}{mV} \\
\dot{\theta} &= \bar{q} \\
\dot{\bar{q}} &= \frac{M}{I_y},
\end{aligned} \tag{107}$$

Table 1
Notations through the model (107)–(109).

V	The velocity
α	The attack angle
θ	The pitch angle
$\bar{q}$	The pitch rate
δ_{e1}, δ_{e2}	The elevator angles of an augmented two-piece elevator
m	The mass
I_y	The moment of inertia
ρ	The air density
S	The wing area
c	The mean chord
T_x	The components of the thrust along the body x
T_z	The components of the thrust along the body z

Table 2
Aircraft parameters in simulation.

$m = 4600$ kg	$I_y = 31027$ kg m^2	$S = 39.02$ m^2
$c = 1.98$ m	$\rho = 0.7377$ kg/m^3	$T_x = 4864$ N
$T_z = 212$ N	$C_{x1} = 0.39$	$C_{x2} = 2.9099$
$C_{x3} = -0.0758$	$C_{x4} = 0.0961$	$C_{z1} = -7.0186$
$C_{z2} = 4.1109$	$C_{z3} = -0.3112$	$C_{z4} = -0.2340$
$C_{z5} = -0.1023$	$C_{m1} = -0.8789$	$C_{m2} = -3.852$
$C_{m3} = -0.0108$	$C_{m4} = -1.8987$	$C_{m5} = -0.6266$

where

$$F_x = \bar{\bar{q}}SC_x + T_x - mg\sin(\theta)$$
$$F_z = \bar{\bar{q}}SC_z + T_z + mg\cos(\theta)$$
$$M = \bar{\bar{q}}cSC_m, \tag{108}$$

and $\bar{\bar{q}} = \frac{1}{2}\rho V^2$, C_x, C_z and C_m are polynomial functions that

$$C_x = C_{x1}\alpha + C_{x2}\alpha^2 + C_{x3} + C_{x4}(d_1\delta_{e1} + d_2\delta_{e2})$$
$$C_z = C_{z1}\alpha + C_{z2}\alpha^2 + C_{z3} + C_{z4}(d_1\delta_{e1} + d_2\delta_{e2}) + C_{z5}\bar{q}$$
$$C_m = C_{m1}\alpha + C_{m2}\alpha^2 + C_{m3} + C_{m4}(d_1\delta_{e1} + d_2\delta_{e2}) + C_{m5}\bar{q}. \tag{109}$$

The notations through the model (107)–(109) are illustrated in Table 1. We choose V, α, θ, q as the states $\chi_1, \chi_2, \chi_3, \chi_4$, respectively, and δ_{e1}, δ_{e2} as the outputs of the two actuators redundant for each other. As explained in Tang et al. (2003), there exists a diffeomorphism $[\xi, x]^T = T(\chi) = [T_1(\chi), T_2(\chi), \chi_3, \chi_4]$ that (107) can be transformed into the form of (1), i.e.

$$\dot{\chi}_3 = \chi_4$$
$$\dot{\chi}_4 = \varphi(\chi)^T\bar{\bar{\theta}} + \sum_{i=1}^{2} b_i\chi_1^2 u_j$$
$$\dot{\xi} = \Psi(\xi, x) + \Phi(\xi, x)\bar{\bar{\theta}} \tag{110}$$

where $\bar{\bar{\theta}} \in R^4$ is an unknown constant vector, $\varphi(\chi) = [\chi_1^2\chi_2, \chi_1^2\chi_2^2, \chi_1^2, \chi_1^2\chi_4]^T, x = [\chi_3, \chi_4]^T, u_1 = \delta_{e1}$ and $u_2 = \delta_{e2}$. Input-to-state stability of the subsystem $\dot{\xi} = \Psi(\xi, x) + \Phi(\xi, x)\bar{\bar{\theta}}$ with x as its input was shown in Tang et al. (2003).

In simulation, the aircraft parameters in use are set based on the data sheet in Miller and William (1999), which is listed in Table 2. In addition, we choose $d_1 = 6, d_2 = 4$. All these parameters are unknown in the designs.

The faulty case considered in this example is modeled as

$$\begin{matrix} u_1(t) = u_{k1,h} \\ u_2 = \rho_{2h}u_{c2} \end{matrix}, \quad t \in [hT^*, (h+1)T^*),\ h = 1, 3, \ldots, \tag{111}$$

which implies that at every hT^* seconds, the output of the 1st actuator (u_1) is stuck at $u_1 = u_{k1,h}$ and the 2nd actuator loses $(1 - \rho_{2h})$ percent of its effectiveness. While at every $(h+1)T^*$ seconds, both actuators are back to normal operation until the next failure or fault occurs. In simulation, we choose that $u_{k1,h} = 0.4$, $\rho_{2h} = 30\%$ and $T^* = 10$ s, which are also unknown in the designs. However, we know that b_1, b_2 in (110) are both negative and

$$|\bar{\bar{\theta}}| \le 0.02, \qquad 0.01 \le |b_1| \le 0.02,$$
$$0.005 \le |b_2| \le 0.01, \tag{112}$$
$$0.2 \le \rho_{jh} \le 1, \qquad |u_{kj,h}| \le 1. \tag{113}$$

The reference signal $y_r = 0.1\sin(0.05t)$. The initial states and estimates are all set as 0 except that $\chi(0) = [85, 0, 0.03, 0]^T$, $\hat{b}(0) = 0.01$. The design parameters are chosen as

$$\xi = 0.001, \qquad c_1 = c_2 = 1,$$
$$\kappa_2 = 10^{-6}, \qquad \Gamma = 0.1 \times I_7, \tag{114}$$
$$\nu_1 = \frac{0.03 + 0.001}{2}, \qquad \sigma_1 = 0.03 - 0.001, \tag{115}$$
$$\theta_0 = [0, 0, 0, 0]^T, \qquad \bar{\theta} = 0.02, \tag{116}$$
$$\nu_6 = \nu_7 = 0, \qquad \sigma_6 = 0.02, \qquad \sigma_7 = 0.01, \tag{117}$$
$$q = 20, \qquad \varsigma = 0.01. \tag{118}$$

The performances of tracking error, velocity, attack angle, pitch rate and control u_1, u_2 are given in Figs. 9–13, respectively. It can be seen that all the signals are bounded and the tracking error is small in the mean square sense.

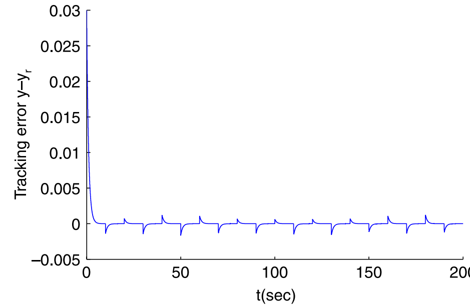


Fig. 9. Tracking error $y(t) - y_r(t)$.

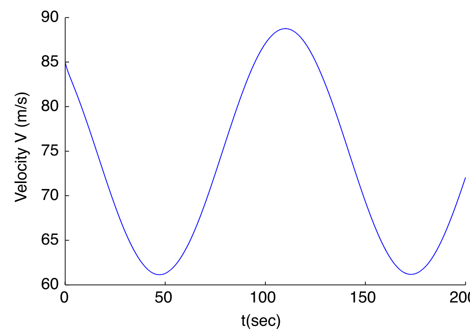


Fig. 10. Velocity V.

6. Conclusion

In this paper, the problem of adaptive control of uncertain nonlinear systems in the presence of infinite number of actuator failures or faults is addressed. It has been proved that the boundedness of all closed-loop signals can be ensured by adopting the proposed scheme, provided that the time interval between two successive changes of failure/fault pattern is bounded below by an arbitrary positive number. From the established performance of tracking error in the mean square sense, it is shown that the less frequent the failure/fault pattern changes, the better the tracking

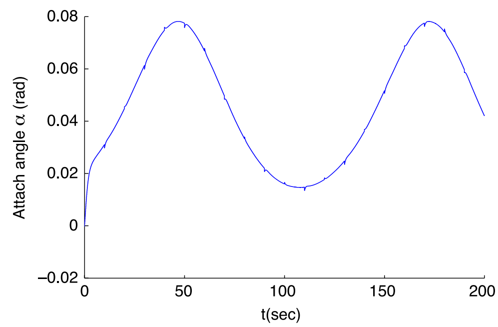


Fig. 11. Attach angle α.

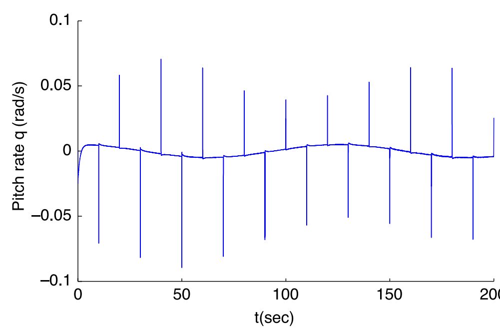


Fig. 12. Pitch rate $\bar{q}$.

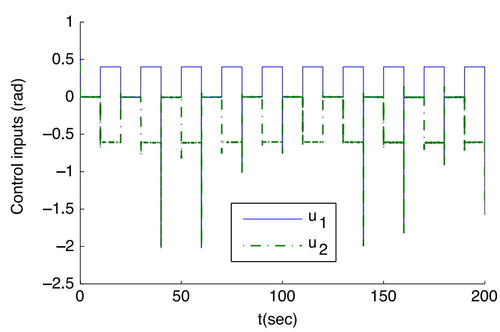


Fig. 13. Control inputs (elevator angle (rad)) u_1 and u_2.

performance is. Moreover, the tracking error can converge to zero asymptotically in the case with finite number of failures and faults. In simulation studies, the ability of the proposed scheme to compensate for infinite number of relatively frequent failures is compared with a tuning function design scheme through a numerical example. The effectiveness of the proposed scheme is also shown on an aircraft system through simulation.

We feel that it is worthy to investigate the transient performance of the system with the proposed scheme in the presence of failures. Moreover, it is an interesting topic to extend the results for more general class of redundant actuators, for example with the condition $g_j \in \text{span}\{g_0\}$ in Remark 1 relaxed. Further research is also needed to explore rigorous analysis in accommodating infinite number of actuator failures with tuning function designs.

References

Ahmed-Zaid, F., Ioannou, P., Gousman, K., & Rooney, R. (1991). Accommodation of failures in the F-16 aircraft using adaptive control. *IEEE Control Systems Magazine*, *11*(1), 73–78.

Ashari, A. E., Sedigh, A. K., & Yazdanpanah, M. J. (2005). Reconfigurable control system design using eigenstructure assignment: static, dynamic and robust approaches. *International Journal of Control*, *78*(13), 1005–1016.

Bodson, M., & Groszkiewicz, J. E. (1997). Multivariable adaptive algorithms for reconfigurable flight control. *IEEE Transactions on Control Systems Technology*, *5*(2), 217–229.

Boskovic, J. D., Jackson, J. A., Mehra, R. K., & Nguyen, N. T. (2009). Multiple-model adaptive fault-tolerant control of a planetary lander. *Journal of Guidance, Control, and Dynamics*, *32*(6), 1812–1826.

Boskovic, J. D., & Mehra, R. K. (1999). Stable multiple model adaptive flight control for accommodation of a large class of control effector failures. In *Proceedings of the 1999 American control conference* (pp. 1920–1924).

Boskovic, J. D., & Mehra, R. K. (2002a). A decentralized scheme for accommodation of multiple simultaneous actuator failures. In *Proceedings of the 2002 American control conference* (pp. 5098–5103).

Boskovic, J. D., & Mehra, R. K. (2002b). Multiple-model adaptive flight control scheme for accommodation of actuator failures. *Journal of Guidance, Control, and Dynamics*, *25*(4), 712–724.

Boskovic, J. D., Yu, S. -H., & Mehra, R. K. (1998). Stable adaptive fault-tolerant control of overactuated aircraft using multiple models, switching and tuning. In *Proceedings of the 1998 AIAA guidance, navigation and control conference. Boston, MA* (pp. 739–749).

Corradini, M. L., & Orlando, G. (2007). Actuator failure identification and compensation through sliding modes. *IEEE Transactions on Control Systems Technology*, *15*(1), 184–190.

Diao, Y., & Passino, K. M. (2001). Stable fault tolerant adaptive/fuzzy/neural control for a turbine engine. *IEEE Transactions on Control Systems Technology*, *9*(3), 494–509.

Fidan, B., Zhang, Y., & Ioannou, P. A. (2005). Adaptive control of a class of slowly time varying systems with modeling uncertainties. *IEEE Transactions on Automatic Control*, *50*(6), 915–920.

Gao, Z., & Antsaklis, P. (1991). Stability of the pseudo-inverse method for reconfigurable control systems. *International Journal of Control*, *53*(3), 717–729.

Giri, F., Rabeh, A., & Ikhouane, F. (1999). Backstepping adaptive control of time-varying plants. *System & Control Letters*, *36*(4), 245–252.

Ioannou, P. A., & Sun, J. (1996). *Robust adaptive control*. New Jersey: Prentice Hall.

Kale, M. M., & Chipperfield, A. J. (2005). Stabilized MPC formulations for robust reconfigurable flight control. *Control Engineering Practice*, *13*(6), 771–788.

Khalil, H. K. (1996). *Nonlinear systems*. New Jersey: Prentice Hall.

Krstic, M., Kanellakopoulos, I., & Kokotovic, P. V. (1995). *Nonlinear and adaptive control design*. New York: Wiley.

Maybeck, P. S., & Stevens, R. D. (1991). Reconfigurable flight control via multiple model adaptive control methods. *IEEE Transactions on Aerospace and Electronic Systems*, *27*(3), 470–480.

Middleton, R. H., & Goodwin, G. C. (1988). Adaptive control of time-varying linear systems. *IEEE Transactions on Automatic Control*, *33*(2), 150–155.

Miller, R. H., & William, B. R. (1999). The effects of icing on the longitudinal dynamics of an icing research aircraft. In *37th aerospace sciences. AIAA no. 99-0637*.

Niemann, H., & Stoustrup, J. (2005). Passive fault tolerant control of a double inverted pendulum—a case study. *Control Engineering Practice*, *13*(8), 1047–1059.

Pomet, J.-B., & Praly, L. (1992). Adaptive nonlinear regulation: estimation from the Lyapunov equation. *IEEE Transactions on Automatic Control*, *37*(6), 729–740.

Richter, J. H., Schlage, T., & Lunze, J. (2007). Control reconfiguration of a thermofluid process by means of a virtual actuator. *IET Control Theory & Applications*, *1*(6), 1606–1620.

Richter, J. H., Schlage, T., & Lunze, J. (2008). Control reconfiguration after actuator failures by Markov parameter matching. *International Journal of Control*, *81*(9), 1382–1398.

Tang, X. D., & Tao, G. (2009). An adaptive nonlinear output feedback controller using dynamic bounding with an aircraft control application. *International Journal of Adaptive Control and Signal Processing*, *23*(7), 609–639.

Tang, X. D., Tao, G., & Joshi, S. M. (2003). Adaptive actuator failure compensation for parametric strict feedback systems and an aircraft application. *Automatica*, *39*(11), 1975–1982.

Tang, X. D., Tao, G., & Joshi, S. M. (2007). Adaptive actuator failure compensation for nonlinear MIMO systems with an aircraft control application. *Automatica*, *43*(11), 1869–1883.

Tao, G., Chen, S. H., & Joshi, S. M. (2002). An adaptive failure compensation controller using output feedback. *IEEE Transactions on Automatic Control*, *47*(3), 506–511.

Tao, G., Joshi, S. M., & Ma, X. L. (2001). Adaptive state feedback control and tracking control of systems with actuator failures. *IEEE Transactions on Automatic Control*, *46*(1), 78–95.

Veillette, R. J., Medanic, J. V., & Perkins, W. R. (1992). Design of reliable control systems. *IEEE Transactions on Automatic Control*, *37*(3), 290–304.

Wang, W., & Wen, C. (2010). Adaptive actuator failure compensation control of uncertain nonlinear systems with guaranteed transient performance. *Automatica*, *46*(12), 2082–2091.

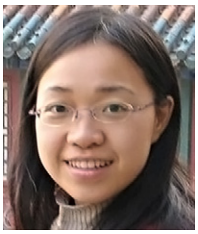
Yang, G.-H., Wang, J. L., & Soh, Y. C. (2001). Reliable H_∞ controller design for linear systems. *Automatica*, *37*(5), 717–725.

Zhang, Z., & Chen, W. (2009). Adaptive output feedback control of nonlinear systems with actuator failures. *Information Sciences*, *179*(24), 4249–4260.

Zhang, Y., Fidan, B., & Ioannou, P. A. (2003). Backstepping control of linear time-varying systems with known and unknown parameters. *IEEE Transactions on Automatic Control*, *48*(11), 1908–1925.

Zhang, X. D., Parisini, T., & Polycarpou, M. M. (2004). Adaptive fault-tolerant control of nonlinear uncertain systems: an information-based diagnostic approach. *IEEE Transactions on Automatic Control*, *49*(8), 1259–1274.

Zhang, Y., & Qin, S. J. (2008). Adaptive actuator/component fault compensation for nonlinear systems. *AIChE Journal*, *54*(9), 2404–2412.
Zhang, Y., Wen, C., & Soh, Y. C. (2000). Discrete-time robust backstepping adaptive control nonlinear time-varying systems. *IEEE Transactions on Automatic Control*, *45*(9), 1749–1755.
Zhang, Z., Xu, S., Guo, Y., & Chu, Y. (2010). Robust adaptive output-feedback control for a class of nonlinear systems with time-varying actuator faults. *International Journal of Adaptive Control and Signal Processing*, *24*(9), 743–759.
Zhao, Q., & Jiang, J. (1998). Reliable state feedback control system design against actuator failures. *Automatica*, *34*(10), 1267–1272.



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