

Robust Adaptive Control of Uncertain Nonlinear Systems in the Presence of Input Saturation and External Disturbance

Changyun Wen, *Fellow, IEEE*, Jing Zhou, *Member, IEEE*,
Zhitao Liu, and Hongye Su

Abstract—In this technical note, we consider adaptive control of single input uncertain nonlinear systems in the presence of input saturation and unknown external disturbance. By using backstepping approaches, two new robust adaptive control algorithms are developed by introducing a well defined smooth function and using a Nussbaum function. The Nussbaum function is introduced to compensate for the nonlinear term arising from the input saturation. Unlike some existing control schemes for systems with input saturation, the developed controllers do not require assumptions on the uncertain parameters within a known compact set and a priori knowledge on the bound of the external disturbance. Besides showing global stability, transient performance is also established and can be adjusted by tuning certain design parameters.

Index Terms—Adaptive control, backstepping, global stability, nonlinear systems, saturation.

I. INTRODUCTION

In many practical dynamic systems, physical input saturation on hardware dictates that the magnitude of the control signal is always constrained. Saturation is a potential problem for actuators of control systems. It often severely limits system performance, giving rise to undesirable inaccuracy or leading instability [1], [2]. The development of adaptive control schemes for uncertain nonlinear systems with input saturation has been a task of major practical interest as well as theoretical significance.

However, the number of available results by taking saturation into account in the design and analysis of adaptive controllers is still limited due to the difficulty of the problem. Especially, the considered plants should satisfy certain restrictive conditions. For linear systems with input saturation, several schemes for adaptive control law design have been proposed. In [3], [4], model reference adaptive control was proposed for a linear plant in the presence of magnitude constraints on the control input, where the plant poles lie entirely in the closed left-half of the complex plane. In [5], stability was established for discrete time adaptive pole placement systems with input rate saturation constraint, where all the poles and zeros of the model are strictly inside the unit circle. In [6], a discrete time direct adaptive control was proposed for

linear systems subject to saturation constraints, where the adaptive control system is stable provided that the plant is minimum phase and is only allowed to have one pole at $z = 1$ with all remaining poles stable. Adaptive control of exponentially stable uncertain linear plants subject to input saturation was provided in [7]. In [8], the problem of controlling non-minimum phase type-1 plants in the presence of saturation constraints and disturbances was considered by using a saturated adaptive regulator based on a specific pole placement, where the plant has only also one pole at $z = 1$ and the others are within the unit circle, and uncertain parameters must be inside a known compact set. In [9], an adaptive controller is applied to linear systems in the presence of magnitude saturation of the control input, where the adaptive controller is shown to result in global stability if the plant is open loop stable and minimum phase, and local stability otherwise. The problem for compensating for saturation in controlling nonlinear systems is a topic of great importance and has received increasing attention in adaptive control, such as using neural networks (NN) control [10]–[12], model predictive control (MPC) [13], [14], reference governors [15], anti-windup technique [16] and dynamic inversion model reference control [17]. In controller design with NN approaches [10], [11], it is required that all system states are within a known compact set to handle approximation errors caused by NN approximation and the NN weights must be bounded with known bounds. In [12], it is assumed that the controlled plant should be locally stable, but only local stability of the overall closed-loop system is ensured. The result in [13] implements a certainty equivalence nominal-model MPC feedback to stabilize a parametric uncertain system subject to an input constraint. An adaptive receding horizon controller is proposed with assumption that the state and the derivative of the state are accessible for measurement. In [14], an adaptive model predictive control scheme is proposed dealing with constrained nonlinear systems, where the nonlinear function is assumed to be locally Lipschitz and the uncertain parameters lies within an initially known compact set. In [15], a direct adaptive nonlinear tracking control framework for nonlinear uncertain systems with actuator amplitude and rate saturation constraints is developed, where the control signal to a given reference (governor or supervisor) system is modified to effectively robustify the error dynamics to the saturation constraints. The governor accepts input commands and modifies their evolution so that specified constraints on control variables are satisfied. In [16], an anti-windup design is presented for single input adaptive control systems in strict feedback form with input saturation. A piecewise linear approximation network is used to estimate the unknown parts where the unknown parameters must be bounded with known bounds. In [17], a dynamic inversion based adaptive control framework is developed to a specific class of nonlinear systems in Brunovsky form, where the unknown parameters are bounded with known bounds. The proposed method, termed “positive μ -modification” in [18], protects the control law from actuator position saturation and ensures bounded tracking for initial conditions within a domain of attraction. Backstepping approach is a Lyapunov-based recursive design procedure. With this technique, transient performance can be established and improved with explicit tuning of design parameters. A great deal of attention has been paid to tackle both linear and nonlinear systems with unknown parameters. A number of results have been obtained as summarized in [19]. Some robustness issues have also been addressed, see for examples, [20], [21]. However, the effect of saturation nonlinearity has not been addressed with this approach, especially in the absence of a priori knowledge of system parameters. To solve such a problem, certain modifications of standard backstepping controllers are required. A preliminary result for a simple class of nonlinear systems with input saturation is reported in [22] by using state feedback adaptive backstepping design.

Manuscript received June 12, 2010; revised October 06, 2010; accepted February 10, 2011. Date of publication March 03, 2011; date of current version July 07, 2011. This work was supported in part by the Research Council of Norway and Statoil ASA under the MaxWells project at IRIS, the National Creative Research Groups Science Foundation of China (NCRGSFC: 60721062); National Natural Science Foundation of China (NSFC: 60736021); and by The National High Technology Research and Development Program of China (863 Program 2008AA042902). Recommended by Associate Editor A. Ferrara.

C. Wen is with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798 (e-mail: ecywen@ntu.edu.sg).

J. Zhou is with the International Research Institute of Stavanger (IRIS), Bergen N-5008, Norway (e-mail: jing.zhou@iris.no; jing.zhou@itk.ntnu.no).

Z. Liu and H. Su are with the State Key Laboratory of Industrial Control Technology, Institute of Cyber-Systems and Control, Zhejiang University, Hangzhou 310027, China (e-mail: liuzhitao100@gmail.com; ztliu@iipc.zju.edu.cn; hysu@iipc.zju.edu.cn).

Color versions of one or more of the figures in this technical note are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TAC.2011.2122730

In this technical note, we will address this problem for a class of nonlinear uncertain systems in the presence of an external disturbance, by taking saturation into consideration in controller design. Note that saturation is a nonsmooth function but the backstepping technique requires all functions differentiable. To use the technique, a smooth function is used to approximate the saturation with a bounded approximation error and the plant is then augmented to design controllers. However, the derivative of the approximate function makes the design and stability analysis a challenge problem. To solve it, a Nussbaum function is used. In this technical note, two control schemes are presented. The first is a relatively simple scheme, which follows the standard backstepping control design, in addition to using the σ -modification in the adaptation laws. The transient tracking error performance depends on an unknown 'disturbance-like' term, which is a combination of the external disturbance and the approximation error of the saturation function. To improve system performance, the second scheme is proposed to estimate the bound of the 'disturbance-like' term and compensate for it in the controller design. With the second scheme, the transient tracking error performance does not depend on the unknown 'disturbance-like' term, but on the initial estimation error of its bound. However, this is at the expense of increasing the complexity of the designed controller. With the proposed schemes, system parameters are no longer assumed to be in a known compact set. No a priori knowledge is required on the disturbance bound. Besides showing global stability of the system, the transient tracking error performances for both schemes are derived to be explicit functions of design parameters and thus our schemes allow designers to obtain the closed loop behavior by tuning design parameters in an explicit way. The proposed schemes are not only applicable to systems with input saturation, but also with other nonlinearities which can be bounded by a bounded smooth nonlinear function. As an additional contribution, this also enlarges the nonlinear systems currently studied by using backstepping approach.

II. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

We consider a class of uncertain nonlinear systems given as follows:

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}(t) + \theta^T \phi_i(\bar{x}_i(t)) \quad i = 1, \dots, n-1 \\ \dot{x}_n(t) &= \theta^T \phi_n(x(t)) + bu(v(t)) + \bar{d}(t) \end{aligned} \quad (1)$$

where $x(t) = [x_1(t), \dots, x_n(t)]^T \in R^n$ are state variables and $y(t) = x_1(t)$ is output, $\theta \in R^r$ is an unknown constant parameter vector, $\bar{x}_i(t) = [x_1(t), \dots, x_i(t)]^T$, b is the unknown control gain, $\phi_i \in R^r$, $i = 1, \dots, n$ are known nonlinear functions, $\bar{d}(t)$ denotes an external disturbance with unknown bound, v is the controller output to be designed, $u(v(t))$ denotes the plant input subject to saturation type nonlinearity. The control objective is to make the system output y track the desired trajectory $y_r(t)$.

$u(v(t))$ is described by

$$u(v(t)) = \text{sat}(v(t)) = \begin{cases} \text{sign}(v(t))u_M, & |v(t)| \geq u_M \\ v(t), & |v(t)| < u_M \end{cases} \quad (2)$$

where u_M is a known bound of $u(t)$. Clearly, the relationship between the applied control $u(t)$ and the control input $v(t)$ has a sharp corner when $|v(t)| = u_M$. Thus backstepping technique cannot be directly applied. In order to use this technique, the saturation is approximated by a smooth function defined as

$$g(v) = u_M \times \tanh\left(\frac{v}{u_M}\right) = u_M \frac{e^{v/u_M} - e^{-v/u_M}}{e^{v/u_M} + e^{-v/u_M}}. \quad (3)$$

Then $\text{sat}(v(t))$ in (2) can be expressed as

$$\text{sat}(v) = g(v) + d_1(v) = u_M \times \tanh\left(\frac{v}{u_M}\right) + d_1(v) \quad (4)$$

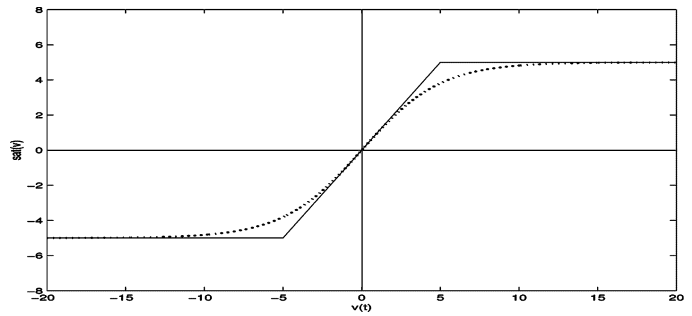


Fig. 1. Saturation (Dot-line: smooth $u_M \tanh(v/u_M)$; Solid-line: $\text{sat}(v)$).

where $d_1(v) = \text{sat}(v) - g(v)$ is a bounded function in time and its bound can be obtained as

$$|d_1(v)| = |\text{sat}(v) - g(v)| \leq u_M(1 - \tanh(1)) = D_1. \quad (5)$$

Note that in the section $0 \leq |v| \leq u_M$ the bound $d_1(v)$ increases from 0 to D_1 as $|v|$ changes from 0 to u_M , and outside this range the bound $d_1(v)$ decreases from D_1 to 0. Fig. 1 shows approximation of the saturation function. The following assumptions are made.

Assumption 1: The plant is input-to-state stable (ISS).

Assumption 2: The desired trajectory $y_r(t)$ and its n th order derivatives are known and bounded.

Assumption 3: The control gain $b > 0$.

Remark 1: Assumption 1 is reasonable since a unstable plant cannot be globally stabilized in the presence of input saturation. For example, consider the following simple system:

$$\dot{x} = Px + u(v(t))$$

where $x \in R$ is a state variable, and $u(v(t)) \in R$ denotes the plant input subject to saturation described by (2). If $P > 0$ and the initial value $x(0) > u_M/P$, there does not exist any control that satisfies the saturation constraint to stabilize the system.

III. DESIGN OF ADAPTIVE CONTROLLERS

To achieve the objective of tracking, we augment the plant to consider the saturation approximation function and the resulting approximation error as follows:

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}(t) + \theta^T \phi_i(\bar{x}_i(t)) \quad i = 1, \dots, n-1 \\ \dot{x}_n(t) &= \theta^T \phi_n(x(t)) + bg(v) + d(t) \end{aligned} \quad (6)$$

$$\dot{v} = -cv + w \quad (7)$$

where c is a positive constant and w is an auxiliary signal to be designed in the backstepping approach, $d(t) = bd_1(t) + \bar{d}(t)$. The effect of $d(t)$ is due to both external disturbances and $bd_1(t)$ and thus we call it a "disturbance-like" term for simplicity of presentation. Clearly all functions in (6) and (7) are smooth and the use of backstepping is feasible.

Remark 2: To the best knowledge of authors, the augmented system (6) and (7) does not belong to any class of systems studied by using backstepping approach so far, as \dot{x}_n is related to nonlinear function $g(v)$ instead of v directly. This results in a term $(\partial g(v)/\partial v)\dot{v}$ instead of only \dot{v} as in previous backstepping-based approaches in *Step n + 1* of the backstepping design given later. To handle $\partial g(v)/\partial v$, a Nussbaum function is employed.

Remark 3: From (6) and (7), it can be noted that the proposed schemes given later in this section are not limited for application to

only saturation functions. They can also be applied to any smooth or nonsmooth function $u(v)$ that can be approximated by a bounded smooth function $g(v)$ with a bounded approximation error and bounded derivative $\partial g(v)/\partial v$. With this, the class of systems in (1) or studied in existing literature using backstepping design is enlarged.

Remark 4: Note that system (6) involves the function $g(v)$ which plays the same role as a 'control input' in the current class of systems studied with backstepping approaches. In our case, the control signal to be designed is v , but it is hard to directly design it with the difficulty explained in Remark 2. To overcome this difficulty, (7) is artificially introduced to generate a stable control signal v by designing an auxiliary control signal w in the last step $n + 1$ of the backstepping approach. As summarized in Section III-A and Table I later, (7) is part of the controller designed.

As in the usual tracking problem with backstepping approaches, the following change of coordinate is made:

$$\begin{aligned} z_1 &= x_1 - y_r, \\ z_i &= x_i - y_r^{(i-1)} - \alpha_{i-1}, \quad i = 2, \dots, n \\ z_{n+1} &= g(v) - \alpha_n \end{aligned} \quad (8)$$

where α_{i-1} is the virtual control at the i th step to be determined. Variable z_{n+1} is due to the new state variable v . In the following, two control schemes are proposed.

A. Control Scheme I

We first present a simpler control design approach. To illustrate the backstepping procedures, only the first and the last two steps are elaborated in details, especially on the methodology of handling $g(v)$.

- *Step i ($i = 1, \dots, n - 1$):* We choose virtual control law α_i and tuning function τ_i as

$$\alpha_1 = -c_1 z_1 - \hat{\theta}^T \phi_1(x_1(t)) \quad (10)$$

$$\begin{aligned} \alpha_i &= -c_i z_i - z_{i-1} + \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} x_{j+1} + \frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}} y_r^{(j)} \right) \\ &\quad - \left(\hat{\theta}^T - \sum_{k=2}^{i-1} z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma \right) \left(\phi_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \phi_k \right) \\ &\quad + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma l_\theta (\hat{\theta} - \theta_0) \end{aligned} \quad (11)$$

$$\tau_i = \phi_1(x_1) z_1 \quad (12)$$

$$\tau_i = \left(\phi_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \phi_k \right) z_i + \tau_{i-1} \quad (13)$$

where c_i , l_θ , and θ_0 are positive design parameters, and $\Gamma = \Gamma^T$ is the adaption gain matrix.

- *Step n :* From (6) and (8) for $i = n$, we obtain

$$\dot{z}_n = b(z_{n+1} + \alpha_n) + \theta^T \phi_n(\bar{x}_n) + d(t) - y_r^{(n)} - \dot{\alpha}_{n-1} \quad (14)$$

where $g(v) = z_{n+1} + \alpha_n$ has been used. We design the virtual control law α_n as follows:

$$\alpha_n = \hat{e} \bar{\alpha}_n \quad (15)$$

$$\begin{aligned} \bar{\alpha}_n &= -(c_n + l) z_n + \sum_{j=1}^{n-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_j} x_{j+1} + \frac{\partial \alpha_{n-1}}{\partial y_r^{(j-1)}} y_r^{(j)} \right) \\ &\quad + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \Gamma \tau_n - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \Gamma l_\theta (\hat{\theta} - \theta_0) + \dot{y}_r^{(n)} - z_{n-1} \\ &\quad - \left(\hat{\theta}^T - \sum_{k=2}^{n-1} z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma \right) \left(\phi_n - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} \phi_k \right) \end{aligned} \quad (16)$$

where c_n and l are positive constants and \hat{e} is an estimate of $e = 1/b$. The adaptive law is chosen as

$$\dot{\hat{e}} = -\gamma_e \bar{\alpha}_n z_n - \gamma_e l_e (\hat{e} - e_0) \quad (17)$$

where γ_e , l_e and e_0 are design parameters with γ_e and l_e being chosen as positive. From the above design and choice, the following useful property can be obtained:

$$\begin{aligned} l_e \hat{e} (\hat{e} - e_0) &= -\frac{1}{2} l_e \hat{e}^2 + \frac{1}{2} l_e (e - e_0)^2 - \frac{1}{2} l_e (\hat{e} - e_0)^2 \\ &\leq -\frac{1}{2} l_e \hat{e}^2 + \frac{1}{2} l_e (e - e_0)^2 \end{aligned} \quad (18)$$

where $\tilde{e} = e - \hat{e}$. We define a positive Lyapunov function V_n as

$$V_n = \sum_{k=1}^n \frac{1}{2} z_k^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{|b|}{2\gamma_e} \tilde{e}^2. \quad (19)$$

Then the derivative of V_n is

$$\begin{aligned} \dot{V}_n &\leq -\sum_{i=1}^n c_i z_i^2 - \frac{|b_m|}{2} l_e \tilde{e}^2 + \tilde{\theta}^T (\tau_n - \Gamma^{-1} \dot{\hat{\theta}}) \\ &\quad + \left(\sum_{k=2}^n z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \right) (\Gamma \tau_n - \Gamma l_\theta (\hat{\theta} - \theta_0) - \dot{\hat{\theta}}) \\ &\quad + \frac{|b|}{2} l_e (e - e_0)^2 + \frac{1}{4l} |d(t)|^2 + b z_n z_{n+1}. \end{aligned} \quad (20)$$

- *Step $n + 1$:* From (7) and (9), we obtain

$$\begin{aligned} \dot{z}_{n+1} &= \xi(-cv + w) - \frac{\partial \alpha_n}{\partial x_n} (\theta^T \phi_n + d(t) + bg(v)) \\ &\quad - \sum_{j=1}^{n-1} \frac{\partial \alpha_n}{\partial x_j} \theta^T \phi_j - \frac{\partial \alpha_n}{\partial \hat{\theta}} \dot{\hat{\theta}} - \beta \\ \beta &= \sum_{j=1}^n \left(\frac{\partial \alpha_n}{\partial x_j} x_{j+1} + \frac{\partial \alpha_n}{\partial y_r^{(j-1)}} y_r^{(j)} \right) + \frac{\partial \alpha_n}{\partial \hat{e}} \dot{\hat{e}} \\ \xi &= \frac{\partial g(v)}{\partial v} = \frac{4}{(e^{v/u_M} + e^{-v/u_M})^2} > 0. \end{aligned} \quad (21)$$

Note that ξ is varying, which makes the design and analysis difficult. To handle it, we use a Nussbaum function $N(\chi)$, similar to the approach in [23]. The control law for w is designed as follows:

$$\begin{aligned} w &= N(\chi) \bar{w} \\ \bar{w} &= -c_{n+1} z_{n+1} + \beta + cv \frac{\partial g(v)}{\partial v} + \hat{b} \left(\frac{\partial \alpha_n}{\partial x_n} g(v) - z_n \right) \\ &\quad - l \left(\frac{\partial \alpha_n}{\partial x_n} \right)^2 z_{n+1} - \frac{\partial \alpha_n}{\partial \hat{\theta}} \Gamma l_\theta (\hat{\theta} - \theta_0) + \frac{\partial \alpha_n}{\partial \hat{\theta}} \dot{\hat{\theta}} \\ &\quad + \left(\hat{\theta}^T - \sum_{k=2}^n z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma \right) \left(\sum_{k=1}^n \frac{\partial \alpha_n}{\partial x_k} \phi_k \right) \end{aligned} \quad (22)$$

where c_{n+1} is a positive constant, $N(\chi)$ is a Nussbaum type function defined as

$$N(\chi) = \chi^2 \cos(\chi), \quad \dot{\chi} = \gamma_\chi \bar{w} z_{n+1} \quad (23)$$

where γ_χ is a positive real design parameter. The Nussbaum function satisfies the two-sided properties

$$\lim_{k \rightarrow \pm\infty} \sup \frac{1}{k} \int_0^k N(s) ds = \infty \quad (24)$$

$$\lim_{k \rightarrow \pm\infty} \inf \frac{1}{k} \int_0^k N(s) ds = -\infty \quad (25)$$

where $k \rightarrow \pm\infty$ denotes both $k \rightarrow \infty$ and $k \rightarrow -\infty$.

The parameter update laws are designed as

$$\dot{\hat{\theta}} = \Gamma\tau_{n+1} - \Gamma l_{\theta}(\hat{\theta} - \theta_0) \quad (26)$$

$$\dot{\hat{b}} = -\gamma_b \left(\frac{\partial \alpha_n}{\partial x_n} g(v) - z_n \right) z_{n+1} - \gamma_b l_b(\hat{b} - b_0) \quad (27)$$

where \hat{b} is an estimate of b , γ_b and b_0 are design parameters with γ_b being positive.

To analyze the designed system, we now consider a positive Lyapunov function given by

$$V = V_n + \frac{1}{2}z_{n+1}^2 + \frac{1}{2\gamma_b}\hat{b}^2. \quad (28)$$

The derivative of V is given as

$$\begin{aligned} \dot{V} &\leq (\xi N(\chi) - 1)\bar{w}z_{n+1} - \sum_{i=1}^{n+1} c_i z_i^2 + \bar{\theta}^T (\tau_{n+1} - \Gamma^{-1}\dot{\hat{\theta}}) \\ &\quad + \left(\sum_{k=2}^{n+1} (z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}) \right) (\Gamma\tau_n - \Gamma l_{\theta}(\hat{\theta} - \theta_0) - \dot{\hat{\theta}}) \\ &\quad - \frac{1}{\gamma_b}\hat{b} \left(\gamma_b \left(\frac{\partial \alpha_n}{\partial x_n} g(v) - z_n \right) z_{n+1} + \dot{\hat{b}} \right) \\ &\quad - \frac{|b_m|}{2} l_e \hat{e}^2 + \frac{|b|}{2} l_e (e - e_0)^2 + \frac{1}{2l} |d(t)|^2 \\ &\leq \frac{1}{\gamma_{\chi}} (\xi N(\chi) - 1)\dot{\chi} - \sum_{i=1}^{n+1} c_i z_i^2 - \frac{1}{2} l_{\theta} \|\hat{\theta}\|^2 - \frac{|b|}{2} l_e \hat{e}^2 \\ &\quad - \frac{1}{2} l_b \hat{b}^2 + M \end{aligned} \quad (29)$$

$$M = \bar{M} + \frac{1}{2l} D_{max}^2$$

$$\bar{M} = \frac{1}{2} l_b (b - b_0)^2 + \frac{|b|}{2} l_e (e - e_0)^2 + \frac{1}{2} l_{\theta} \|\theta - \theta_0\|^2 \quad (30)$$

where D_{max} is a constant which denotes the bound of $d(t)$ and may not be available. Notice that

$$-\sum_{i=1}^{n+1} c_i z_i^2 - \frac{|b|}{2} l_e \hat{e}^2 - \frac{1}{2} l_b \hat{b}^2 - \frac{1}{2} l_{\theta} \|\hat{\theta}\|^2 \leq -f_- \bar{V} \quad (31)$$

$$V = \sum_{i=1}^{n+1} \frac{1}{2} z_i^2 + \frac{1}{2} \bar{\theta}^T \Gamma^{-1} \hat{\theta} + \frac{|b|}{2\gamma_e} \hat{e}^2 + \frac{1}{2\gamma_b} \hat{b}^2 \leq f_+ \bar{V} \quad (32)$$

$$\bar{V} = \sum_{i=1}^{n+1} z_i^2 + \bar{\theta}^T \hat{\theta} + \hat{e}^2 + \hat{b}^2 \quad (33)$$

$$f_- = \min \left\{ c_i, \frac{|b|}{2} l_e \frac{1}{2} l_b, \frac{1}{2} l_{\theta} \right\} \quad (34)$$

$$f_+ = \max \left\{ \frac{1}{2}, \frac{|b|}{2\gamma_e}, \frac{1}{2\gamma_b}, \frac{1}{2} \lambda_{max}^{-1}(\Gamma) \right\} \quad (35)$$

where $\lambda_{max}(\Gamma)$ is the maximum eigenvalue of Γ . Therefore, from (29) we obtain

$$\dot{V} \leq -f^* V + M + \frac{1}{\gamma_{\chi}} (\xi N(\chi) - 1)\dot{\chi} \quad (36)$$

where $f^* = f^-/f^+$. By direct integrations of the differential inequality (36), we have

$$\begin{aligned} V &\leq V(0)e^{-f^*t} + \frac{M}{f^*}(1 - e^{-f^*t}) \\ &\quad + \frac{e^{-f^*t}}{\gamma_{\chi}} \int_0^t (\xi N(\chi)\dot{\chi} - \dot{\chi})e^{f^*\tau} d\tau. \end{aligned} \quad (37)$$

Theorem 1: Consider the uncertain nonlinear system (1) satisfying Assumptions 1–3. With the application of controller (10)–(13),

(15)–(16), (22)–(23), and parameter update laws (17), (26) and (27), the closed loop system is globally stable.

Proof: The boundedness of V can be established based on the Nussbaum gain properties (24) via a contradiction argument. We first define

$$V_{\xi}(t_i, t_j) = \int_{t_i}^{t_j} (\xi N(\chi) - 1)\dot{\chi} e^{-f^*(t_j - \tau)} d\tau. \quad (38)$$

For notation convenience, $V_{\xi}(\chi_i, \chi_j) = V_{\xi}(\chi(t_i), \chi(t_j)) = V_{\xi}(t_i, t_j)$, $t_i \leq t_j$. Using integral inequality $(b - a)f_{min} \leq \int_a^b f(x)dx \leq (b - a)f_{max}$ with $f_{min} = \inf_{a \leq x \leq b} f(x)$ and $f_{max} = \sup_{a \leq x \leq b} f(x)$, and noting the facts that $0 < \xi = \partial g(v)/\partial v = 4/(e^{v/u_M} + e^{-v/u_M})^2 \leq 1$, $0 < e^{-f^*(t - \tau)} \leq 1$, we have

$$\begin{aligned} |V(\chi_0, \chi)| &\leq \int_{\chi_0}^{\chi} (|N(\chi)| + 1)d\chi \\ &\leq (\chi - \chi_0)(\sup_{\chi \in [\chi_0, \chi]} |N(\chi)| + 1). \end{aligned} \quad (39)$$

For the Nussbaum function $N(\chi) = \chi^2 \cos(\pi\chi/2)$, we know that it is positive for $\chi \in (4m - 1, 4m + 1)$ and negative for $\chi \in (4m + 1, 4m + 3)$ with an integer m .

We can show that $\chi(t)$ is bounded on $[0, t_f]$ by seeking a contradiction. Suppose that $\chi(t)$ is unbounded and two cases should be considered: 1) $\chi(t)$ has no upper bound and 2) $\chi(t)$ has no lower bound.

Case 1): $\chi(t)$ has no upper bound on $[0, t_f]$. In this case, there must exist a monotone increasing variable $\chi_i = \chi(t_i)$ with $\chi_0 = |\chi(t_0)| > 0$, $\lim_{i \rightarrow \infty} t_i = t_f$, and $\lim_{i \rightarrow \infty} \chi_i = \infty$. Note that $\xi > 0$. From (37), we know that, for $[\chi_0, \chi_1] = [\chi_0, 4m + 1]$

$$\begin{aligned} |V_{\xi}(\chi_0, \chi_1)| &= \left| \int_{t_0}^{t_1} (\xi N(\chi) - 1)\dot{\chi} e^{-f^*(t_1 - \tau)} d\tau \right| \\ &\leq (\chi_1 - \chi_0)(\sup_{\chi \in [\chi_0, \chi_1]} |N(\chi)| + 1) \\ &= l_{m1}(4m + 1)^2 + l_{m1} \end{aligned} \quad (40)$$

where $l_{m1} = 4m + 1 - \chi_0$. By noting that $N(\chi) \leq 0, \forall \chi \in [\chi_1, \chi_2] = [4m + 1, 4m + 3]$, we have

$$V_{\xi}(\chi_1, \chi_2) \leq \int_{4m+2-c_{m1}}^{4m+2+c_{m1}} (\xi N(\chi) - 1)\dot{\chi} e^{-f^*(t_2 - \tau)} d\tau \quad (41)$$

where $c_{m1} \in (0, 1)$. Using Hermite–Hadamard integral inequality and noting that $\xi \geq \xi_{min} > 0$, $e^{-f^*(t_2 - \tau)} \geq e^{-f^*(t_2 - t_1)} > 0$ for $\tau \in [t_2, t_1]$, we have

$$\begin{aligned} V_{\xi} &\leq 2c_{m1}(\xi_{min} \inf_{\chi \in [\chi_1, \chi_2]} N(\chi) + 1)e^{-f^*(t_2 - t_1)} \\ &= -g_0(4m + 2 - c_{m1})^2 + g_1 \end{aligned} \quad (42)$$

where $g_0 = 2c_{m1}e^{-f^*(t_2 - t_1)} \cos(\pi c_{m1}/2) > 0$, and $g_1 = 2c_{m1}e^{-f^*(t_2 - t_1)} > 0$. Thus, from (40) and (42)

$$\begin{aligned} V_{\xi}(\chi_0, \chi_2) &= V_{\xi}(\chi_0, \chi_1) + V_{\xi}(\chi_1, \chi_2) \\ &\leq (4m + 1)^2 \\ &\quad \times \left(-g_0(2(4m + 1)(1 - c_{m1}) + (1 - c_{m1})^2) \right. \\ &\quad \left. + (4m + 1 - \chi_0) + \frac{l_{m1} + g_1}{(4m + 1)^2} \right). \end{aligned} \quad (43)$$

From (43), we know that $V_{\xi}(\chi_0, \chi_2) = V_{\xi}(\chi_0, 4m + 3) \rightarrow -\infty$ as $m \rightarrow \infty$. On the other hand, $V(t) > 0$ for all t . Thus we can always find a subsequence that leads to a contradiction. So $\chi(t)$ has an upper bound.

Case 2): χ has no lower bound on $[0, t_f]$. Define $\chi = -\epsilon$. Accordingly, ϵ has no upper bound. Further noting that $N(\cdot)$ is an even function, (37) becomes

$$\begin{aligned} V(t) &\leq M - \int_0^t (\xi N(\epsilon) - 1) \dot{\epsilon} e^{-f^*(T-\tau)} d\tau \\ &= M - V_\xi(\epsilon(0), \epsilon(t)), \quad \forall t \in [0, t_f]. \end{aligned} \quad (44)$$

Thus, there must exist a monotone increasing variable $\{\epsilon_i = \epsilon(t_i)\}$ with $\epsilon_0 = |\epsilon(t_0)| > 0$, $\lim_{i \rightarrow \infty} t_i = t_f$, and $\lim_{i \rightarrow \infty} \epsilon_i = \infty$. Following the same procedure as in Case 1, we can also construct a subsequence that leads to a contradiction. Accordingly, we can claim that ϵ has an upper bound on $[0, t_f]$. Since $\chi = -\epsilon$, we know that χ has a lower bound on $[0, t_f]$.

The above argument is true for all $t_f > 0$. Therefore, χ must be bounded. And also $\int_0^t [\xi N(\chi) - 1] \dot{\chi} d\tau$ is bounded. So $V(t)$ is bounded from (37), which implies $z_i, i = 1, \dots, n+1, \hat{\theta}, \hat{b}, \hat{e}$ are bounded. From Assumption 1, the plant is ISS. Then there exists a class of \mathcal{K} function σ for continuous input u satisfying

$$\sup_{0 \leq \tau \leq t} \|x(\tau)\| \leq K + \sigma \left(\sup_{t_0 \leq \tau \leq t} \|u(\tau)\| \right) \leq K + \sigma(u_M)$$

where K is a constant. Thus the global boundedness of x_1, x_2, \dots, x_n is established. Note that

$$|g(v)| = u_M \left| \tanh \left(\frac{v}{u_M} \right) \right| \leq u_M \quad (45)$$

$$\left| \frac{\partial g(v)}{\partial v} \right| = \left| \frac{4}{(e^{v/u_M} + e^{-v/u_M})^2} \right| \leq 1 \quad (46)$$

$$\left| \frac{\partial g(v)}{\partial v} v \right| = \left| \frac{4v}{(e^{v/u_M} + e^{-v/u_M})^2} \right| \leq \frac{u_M}{2}. \quad (47)$$

From (45)–(47) and (22), \bar{w} is bounded because of the boundedness of $z_i, i = 1, \dots, n+1, \hat{\theta}, \hat{b}, \hat{e}$. This further implies that w in (22) is bounded because χ is bounded. Then v is bounded from (7). $\triangle\triangle\triangle$

We now derive a bound for the vector $z(t) = [z_1, z_2, \dots, z_{n+1}]^T$. First, the following definitions are made:

$$c_0 = \min_{1 \leq i \leq n+1} c_i, \quad \|z\|_{[0, T]} = \sqrt{\frac{1}{T} \int_0^T (z(t))^2 dt}. \quad (48)$$

Integrating both sides of (29), we obtain

$$\begin{aligned} \|z\|_{[0, T]} &\leq \frac{1}{c_0} \left(\frac{|V(0) - V(T)|}{T} + \bar{M} + \frac{1}{2lT} \int_0^T d(t)^2 dt \right. \\ &\quad \left. + \frac{1}{\gamma_\chi T} \int_0^T (\xi N(\chi) \dot{\chi} - \dot{\chi}) d\tau \right). \end{aligned} \quad (49)$$

On the other hand, from (36), we have

$$\begin{aligned} &\frac{|V(0) - V(T)|}{T} \\ &\leq \bar{M} + f^* V(0) + \frac{1}{T} \int_0^T e^{-f^*(T-t)} \\ &\quad \times \left(\frac{d(t)^2}{2l} + \frac{\xi N(\chi) \dot{\chi} - \dot{\chi}}{\gamma_\chi} \right) dt, \quad \forall T \geq 0 \end{aligned} \quad (50)$$

where we have used the fact that $e^{-f^*(T-t)} \leq 1$ and $1 - e^{-f^*T}/T \leq f^*$.

Note that $\|y - y_r\|_{[0, T]} \leq \|z\|_{[0, T]}$. By setting $z_i(0) = 0$ in the initial value of the Lyapunov function, and using (35), (49)–(50), and the fact that $f^*/c_0 \leq 2$, a bound on the tracking error $\|y - y_r\|_{[0, T]}$ is established and stated in the following Theorem.

Theorem 2: The tracking error $\|y - y_r\|_{[0, T]}$ satisfies

$$\begin{aligned} &\|y - y_r\|_{[0, T]} \\ &\leq \|\tilde{\theta}(0)\|_{\Gamma^{-1}}^2 + \frac{|b|}{\gamma_e} |\tilde{e}(0)|^2 \\ &\quad + \frac{1}{c_0} l_\theta \|\theta - \theta_0\|^2 \\ &\quad + \frac{1}{\gamma_b} |\tilde{b}(0)|^2 + \frac{1}{c_0} (|b| l_e (e - e_0)^2 + l_b (b - b_0)^2) \\ &\quad + \frac{2}{T c_0} \int_0^T e^{-f^*(T-t)} \left(\frac{d^2(t)}{2l} + \frac{\xi N(\chi) \dot{\chi} - \dot{\chi}}{\gamma_\chi} \right) dt. \end{aligned} \quad (51)$$

Remark 5: Regarding the above bound, the following conclusions can be drawn:

- Theorem 2 gives a quantification of the transient performance in the sense of the norm defined in (48) for the tracking error. This bound depends on the initial estimate errors $\tilde{\theta}(0)$, $\tilde{e}(0)$ and $\tilde{b}(0)$. The closer the initial estimates to the true values, the better the transient performance.
- This bound can also be systematically reduced by increasing Γ , γ_b , γ_e , c_0 and decreasing l_θ , l_b , l_e .
- The last term of (51) shows that this bound depends on the 'disturbance-like' term $d(t)$.

B. Control Scheme II

In order to improve tracking performance, we develop another scheme to handle the term $d(t)$ by estimating its bound. To do this, we first define functions $sg_i(z_i)$ and $f_i(z_i)$ as follows:

$$sg_i(z_i) = \begin{cases} \frac{z_i}{|z_i|} & |z_i| \geq \delta_i \\ \frac{z_i}{(\delta_i^2 - z_i^2)^{n-i+3} + |z_i|} & |z_i| < \delta_i \end{cases} \quad (52)$$

$$f_i(z_i) = \begin{cases} 1 & |z_i| \geq \delta_i \\ 0 & |z_i| < \delta_i \end{cases} \quad (53)$$

where $\delta_i (i = 1, \dots, n+1)$ is a positive design parameter. It can be shown that $sg_i(z_i)$ is $(n-i+3)$ th order differentiable. To compensate for the effect of $d(t)$ and also ensure the resultant functions differentiable, we use $(|z_i| - \delta_i)^{n-i+2} sg_i(z_i)$ to replace z_i in the virtual controls α_i and \bar{w} , and use $(|z_i| - \delta_i)^{n-i+3} f_i$ to replace z_i^2 in the Lyapunov functions used in the design procedure. The resulting adaptive controller with this scheme is summarized in Table I.

Theorem 3: Consider the uncertain nonlinear system (1) satisfying Assumptions 1–3. With the application of controller (T.1)–(T.10) and the parameter update laws (T.11)–(T.15), the following statements hold:

- The resulting closed loop system is globally stable.
- The transient tracking error performance is given by

$$\begin{aligned} &\|y(t) - y_r(t)\|_2 \\ &\leq \delta_1 + c_1^{-1/2(n+1)} \\ &\quad \times \left(\frac{1}{2} \tilde{\theta}(0)^T \Gamma^{-1} \tilde{\theta}(0) + \frac{1}{2\gamma_b} \tilde{b}(0)^2 \right. \\ &\quad \left. + \frac{|b|}{2\gamma_e} \tilde{e}(0)^2 + \sum_{i=1}^2 \frac{1}{2\gamma_i} \tilde{D}_i(0)^2 \right)^{1/2(n+1)} \\ &\quad + \left(\frac{1}{c_1 \gamma_\chi} \int_0^\infty (\xi N(\chi) - 1) \dot{\chi} d\tau \right)^{1/2(n+1)} \end{aligned} \quad (54)$$

with $z_i(0) = \delta_i, i = 1, \dots, n+1$.

Proof: We define a positive Lyapunov function as

$$\begin{aligned} V &= \sum_{i=1}^{n+1} \frac{1}{n-i+3} (|z_i| - \delta_i)^{n-i+3} f_i + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \\ &\quad + \frac{1}{2\gamma_b} \tilde{b}^2 + \frac{|b|}{2\gamma_e} \tilde{e}^2 + \frac{1}{2\gamma_1} \tilde{D}_1^2 + \frac{1}{2\gamma_2} \tilde{D}_2^2. \end{aligned} \quad (55)$$

TABLE I
ADAPTIVE BACKSTEPPING CONTROL—SCHEME II

Adaptive Control Laws:	
$\dot{v} = -cv + w$	(T.1)
$w = N(\chi)\bar{w}, N(\chi) = \chi^2 \cos(\chi)$	(T.2)
$\dot{\chi} = \gamma_\chi \bar{w} (z_{n+1} - \delta_{n+1}) f_{n+1} s_{g_{n+1}}(z_{n+1})$	(T.3)
$\bar{w} = -(c_{n+1} + 1)(z_{n+1} - \delta_{n+1}) s_{g_{n+1}}(z_{n+1}) + cv \frac{\partial g(v)}{\partial v}$ $- \hat{D}_2 \sqrt{\left\ \frac{\partial \alpha_n}{\partial x_n} \right\ ^2 + \delta_0 s_{g_{n+1}}(z_{n+1}) + \beta + \frac{\partial \alpha_n}{\partial \hat{\theta}} \dot{\hat{\theta}}}$ $+ \hat{b} \left(\frac{\partial \alpha_n}{\partial x_n} g(v) - (z_n - \delta_n)^2 f_n s_{g_{n+1}}(z_{n+1}) \right)$ $+ \left(\hat{\theta}^T - \sum_{k=2}^n (z_k - \delta_k)^{n-k+2} s_{g_k}(z_k) f_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \right) \cdot$ $\left(\sum_{k=1}^n \frac{\partial \alpha_n}{\partial x_k} \phi_k \right)$	(T.4)
$\alpha_n = \hat{e} \bar{\alpha}_n - \delta_{n+1} s_{g_n}$	(T.5)
$\bar{\alpha}_n = -c_n (z_n - \delta_n)^2 s_{g_n}(z_n) - s_{g_n} \hat{D}_1 + y_r^{(n)}$ $- (\delta_{n+1} + 1) s_{g_n} + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \Gamma \tau_n + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} x_{j+1}$ $- \left(\hat{\theta}^T - \sum_{k=2}^{n-1} (z_i - \delta_i)^{n+2-i} f_i s_{g_i}(z_i) \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \right) \cdot$ $\left(\phi_n - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} \phi_k \right) + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j-1)}} y_r^{(j)}$	(T.6)
$\alpha_i = -(c_i + \frac{5}{4})(z_i - \delta_i)^{n-i+2} s_{g_i}(z_i) - (\delta_{i+1} + 1) s_{g_i}(z_i)$ $+ \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} x_{j+1}$ $- \left(\hat{\theta}^T - \sum_{k=2}^{i-1} (z_k - \delta_k)^{n-k+2} f_k s_{g_k}(z_k) \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \right) \cdot$ $\left(\phi_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \phi_k \right) + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}} y_r^{(j)}$	(T.7)
$\alpha_1 = -(c_1 + \frac{1}{4})(z_1 - \delta_1)^{n+1} s_{g_1}(z_1) - \hat{\theta}^T \phi_1(x_1(t))$ $- (\delta_2 + 1) s_{g_1}(z_1)$	(T.8)
$\tau_1 = \phi_1 (z_1 - \delta_1)^n f_1 s_{g_1}(z_1)$	(T.9)
$\tau_i = \left(\phi_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \phi_k \right) (z_i - \delta_i)^{n-i+2} f_i s_{g_i}(z_i) + \tau_{i-1}$	(T.10)
Parameter Update Laws:	
$\dot{\hat{\theta}} = \Gamma \tau_{n+1}$	(T.11)
$\dot{\hat{b}} = -\gamma_b (z_{n+1} - \delta_{n+1}) \left(\frac{\partial \alpha_n}{\partial x_n} g(v) \right)$ $- (z_n - \delta_n)^2 f_n s_{g_{n+1}}(z_{n+1}) f_{n+1} s_{g_{n+1}}(z_{n+1})$	(T.12)
$\dot{\hat{e}} = -\gamma_e \bar{\alpha}_n (z_n - \delta_n)^2 f_n s_{g_n}(z_n)$	(T.13)
$\dot{\hat{D}}_1 = \gamma_1 (z_n - \delta_n)^2 f_n$	(T.14)
$\dot{\hat{D}}_2 = \gamma_2 \sqrt{\left\ \frac{\partial \alpha_n}{\partial x_n} \right\ ^2 + \delta_0 (z_{n+1} - \delta_{n+1}) f_{n+1}}$	(T.15)

Then from (T.1)–(T.15), the derivative of V is given as

$$\dot{V} \leq \frac{1}{\gamma_\chi} (\xi N(\chi) - 1) \dot{\chi} - \sum_{i=1}^{n+1} c_i (|z_i| - \delta_i)^{2(n-i+2)} f_i. \quad (56)$$

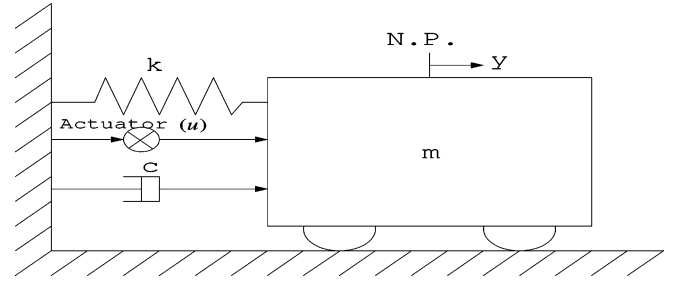


Fig. 2. Spring, mass and damper system.

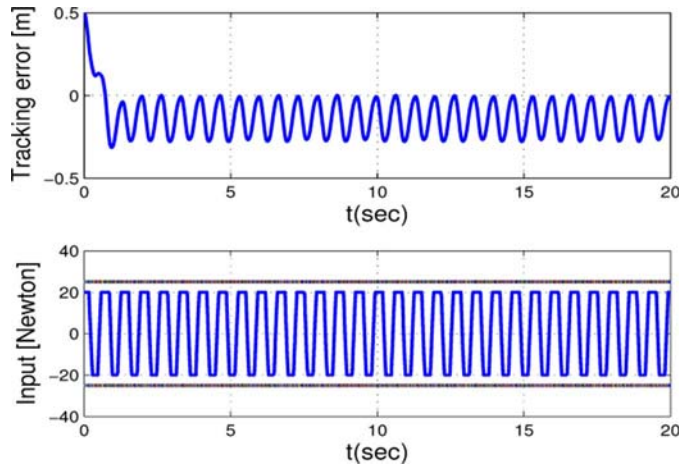


Fig. 3. Tracking error and control signal without considering saturation.

By following similar argument to the proof of Scheme I, Theorem 3 can be proved. $\triangle\triangle\triangle$

Note that the effect of $d(t)$ has been eliminated in (54), so $\|y(t) - y_r(t)\|_2$ does not depend on $d(t)$, but on the initial estimation error $\hat{D}(0)$. In addition, from (54), the tracking error will converge to δ_1 asymptotically where δ_1 can be pre-specified as an arbitrarily small constant by designers. Comments similar to the first two points in Remark 5 are also valid here.

IV. SIMULATION STUDIES

In this section, we illustrate the above methodologies on a second-order system in Fig. 2

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{k}{m} x_1 - \frac{c}{m} x_2 + \frac{1}{m} sat(v) + \bar{d}(t) \end{aligned} \quad (57)$$

where $y = x_1$, the external disturbance $\bar{d}(t) = 0.1 \sin(2\pi t)$, x_1 and x_2 are the position and velocity, m is the mass of the object, k is the stiffness constant of the spring and c is the damping. The input saturation limit is 20N. The true parameters are set as $m = 1$ kg, $c = 2N \cdot s/m$, $k = 8N/m$, which are not needed to be known in our controller design. The desired trajectory is given as $y_r = -0.2 \cos(3\pi t) + 0.2$ [m] and the initial conditions are $x_1(0) = 0.5$ m, $x_2(0) = 0$ m/s. For comparisons, three controllers are applied to system (57) by using the normal backstepping approach without considering saturation, Schemes I and II, respectively. Simulation results on system tracking error and control signal are presented in Figs. 3–5. Clearly significantly improved performances are observed with the proposed schemes. These results verify

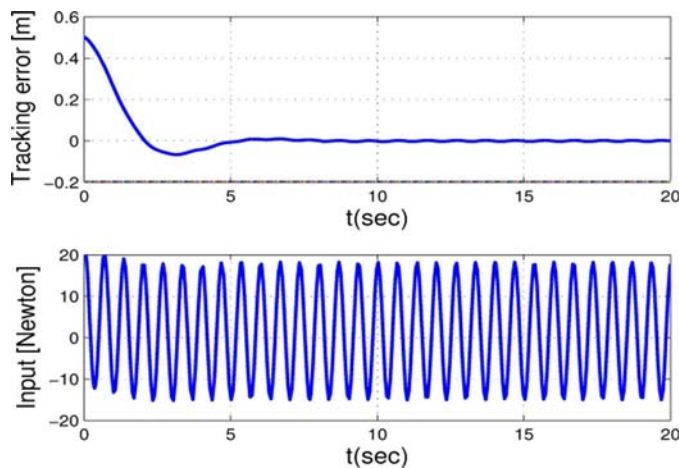


Fig. 4. Tracking error and control signal with scheme I.

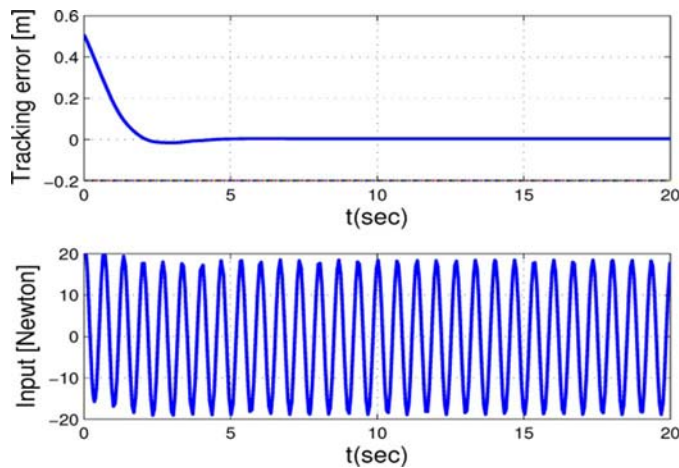


Fig. 5. Tracking error and control signal with scheme II.

the effectiveness of the proposed backstepping adaptive controllers in handling input saturation.

V. CONCLUSION

In this technical note, we consider controlling a class of uncertain nonlinear systems in the presence of input saturation and external disturbances based on adaptive backstepping approaches. Two new schemes are developed to design adaptive controllers to compensate for the effects of the saturation nonlinearity and disturbances. The controllers do not require the model parameters within known intervals. Also no knowledge is assumed on the 'disturbance-like' term. Besides showing global stability, we also give an explicit bound on the performance of the tracking error in terms of design parameters.

ACKNOWLEDGMENT

The authors wish to thank R. H. Middleton for pointing out the simple system that cannot be globally stabilized in the presence of input saturation in Remark 1.

REFERENCES

- [1] N. O. Perez-Arancibia, T.-C. Tsao, and J. S. Gibson, "Saturation-induced instability and its avoidance in adaptive control of hard disk drives," *IEEE Trans. Control Syst. Technol.*, vol. 18, no. 2, pp. 368–382, Mar. 2010.
- [2] V. Kapila and K. Grigoriadis, Eds., *Actuator Saturation Control (Automation and Control Engineering)*. Orlando, FL: CRC Press, 2002.
- [3] S. P. Karason and A. M. Annaswamy, "Adaptive control in the presence of input constraints," *IEEE Trans. Autom. Control*, vol. 39, no. 11, pp. 2325–2330, Nov. 1994.
- [4] A. M. Annaswamy, S. Evesque, S. I. Niculescu, and A. P. Dowling, "Adaptive control of a class of time-delay systems in the presence of saturation," *Adaptive Control Nonsmooth Dyn. Syst.*, pp. 289–310, 2001.
- [5] G. Feng, "Robust adaptive control of input rate constrained discrete time systems," *Adaptive Control Nonsmooth Dyn. Syst.*, pp. 333–348, 2001.
- [6] C. Zhang, "Adaptive control with input saturation constraints," *Adaptive Control Nonsmooth Dyn. Syst.*, pp. 361–381, 2001.
- [7] F. Z. Chaoui, F. Giri, J. M. Dion, and M. M'Saad, "Adaptive tracking with saturating input and controller integration action," *IEEE Trans. Autom. Control*, vol. 43, no. 11, pp. 1638–1643, Nov. 1998.
- [8] F. Z. Chaoui, F. Giri, and M. M'Saad, "Adaptive control of input-constrained tyoe-1 plants stabilization and tracking," *Automatica*, vol. 37, pp. 197–203, 2001.
- [9] A. M. Annaswamy and J.-E. Wong, "Adaptive control in the presence of saturation non-linearity," *Int. J. Adaptive Control Signal Processing*, vol. 11, pp. 3–19, 1997.
- [10] Y. H. Kim and F. L. Lewis, "Reinforcement adaptive control of a class of neural-net-based friction compensation control for high speed and precision," *IEEE Trans. Control Syst. Technol.*, vol. 8, no. 1, pp. 118–126, Jan. 2000.
- [11] M. Polycarpou, J. Farrell, and M. Sharma, "On-line approximation control of uncertain nonlinear systems: Issues with control input saturation," in *Proc. Amer. Control Conf.*, Denver, CO, 2003, pp. 543–548.
- [12] E. N. Johnson and A. J. Calise, "Neural network adaptive control of systems with input saturation," in *Proc. Amer. Control Conf.*, 2001, pp. 3527–3532.
- [13] D. Mayne and H. Michalska, "Adaptive receding horizon control for constrained nonlinear systems," in *Proc. 32nd IEEE Conf. Decision Control*, Dec. 1993, pp. 1286–1291.
- [14] V. Adetola, D. D. Adn, and M. Guay, "Adaptive model predictive control for constrained nonlinear systems," *Syst. Control Lett.*, vol. 58, pp. 320–326, 2009.
- [15] A. Leonessa, W. M. Haddad, T. Hayakawa, and Y. Morel, "Adaptive control for nonlinear uncertain systems with actuator amplitude and rate saturation constraints," *Int. J. Adaptive Control Signal Processing*, vol. 23, no. 1, pp. 73–96, 2009.
- [16] H. M. Do, T. Basar, and J. Y. Choi, "An anti-windup design for single input adaptive control systems in strict feedback form," in *Proc. Amer. Control Conf.*, 2004, pp. 2551–2556.
- [17] E. Lavretsky and N. Hovakimyan, "Positive μ -modification for stable adaptation in dynamic inversion based adaptive control with input saturation," in *Proc. Amer. Control Conf.*, 2005, pp. 3373–3378.
- [18] E. Lavretsky and N. Hovakimyan, "Positive μ -modification for stable adaptation in the presence of input constraints," in *Proc. Amer. Control Conf.*, 2004, pp. 2545–2550.
- [19] M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*. New York: Wiley, 1995.
- [20] Y. Zhang, C. Wen, and Y. C. Soh, "Adaptive backstepping control design for systems with unknown high-frequency gain," *IEEE Trans. Autom. Control*, vol. 45, no. 12, pp. 2350–2354, Dec. 2000.
- [21] J. Zhou, C. Wen, and Y. Zhang, "Adaptive backstepping control of a class of uncertain nonlinear systems with unknown backlash-like hysteresis," *IEEE Trans. Autom. Control*, vol. 49, no. 10, pp. 1751–1759, Oct. 2004.
- [22] J. Zhou and C. Wen, "Robust adaptive control of uncertain nonlinear systems in the presence of input saturation," in *Proc. 14th IFAC Symp. Syst. Ident.*, Newcastle, Australia, 2006, pp. 149–154.
- [23] J. Zhou, C. Wen, and Y. Zhang, "Adaptive output control of a class of time-varying uncertain nonlinear systems," *J. Nonlin. Dyn. Syst. Theory*, pp. 285–298, 2005.