

- [5] St. Kotsios, "Finite input/output representation of a class of Volterra polynomial systems," *Automatica*, vol. 33, no. 2, pp. 257–262, 1997.
- [6] St. Kotsios and D. Lappas, "A description of 2-dimensional discrete polynomial dynamics," *IMA J. Math. Control and Inform.*, vol. 13, pp. 409–428, 1996.
- [7] L. Ljung, *System Identification Theory for the User*. Englewood Cliffs, NJ: Prentice-Hall, 1987.
- [8] J. W. Rugh, *Nonlinear System Theory*. Baltimore, MD: Johns Hopkins Univ. Press, 1981.

Decentralized Model Reference Adaptive Control Without Restriction on Subsystem Relative Degrees

Changyun Wen and Yeng Chai Soh

Abstract—When the direct model reference adaptive control (MRAC) scheme with first-order local estimators is employed to design totally decentralized controllers, the stability result can only be applied to a system with all of its nominal subsystem relative degrees less than or equal to two. In this paper, this restriction is relaxed and it is achieved by employing the parameter projection together with static normalization. To implement the local controllers, no *a priori* knowledge of the subsystem unmodeled dynamics and no information exchange between subsystems are required. Global stability is established for the closed-loop system and small in the mean tracking error is ensured. With this analysis, the class of interactions and subsystem unmodeled dynamics can be enlarged to include those having infinite memory.

Index Terms—Adaptive control, decentralized control, robustness, stability.

I. INTRODUCTION

Decentralized adaptive control is an important control scheme for large scale systems, and it has continued to receive a lot of attention from control researchers over the last few decades. However, only a limited number of stability results in this area are available due to the difficulties in the analysis of ignored interactions. The first batch of results were obtained based on the direct model reference adaptive control (MRAC) approach [1]–[3]. A strong assumption for these results is that relative degrees of all the nominal subsystem models should be less than or equal to two. The stability results using the indirect pole assignment design scheme were reported later in [4]–[6] where there is no restriction on the relative degrees of the nominal subsystem. Recently, efforts on relaxing the subsystem relative degrees in the case of employing the direct adaptive control scheme have been made by using some advanced adaptive strategies. The concept of high-order tuners in [7] was applied to achieve this in [8] and [9]. In this case, a local dynamic estimator with the subsystem relative degree as its order is designed to identify the unknown parameters of each subsystem. The integrator backstepping technique of [10] was also successfully utilized to reach a similar goal in [11]–[13]. To obtain the final control for each subsystem, a number of iterative design steps should be involved to calculate

Manuscript received April 24, 1996; revised March 16, 1998. Recommended by Associate Editor, M. Krstic. This work was supported in part by NTU under the Applied Research Project Grant RP 23/92.

The authors are with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore (e-mail: ecywen@ntu.edu.sg).
Publisher Item Identifier S 0018-9286(99)04544-4.

some intermediate virtual control signals. As commented in [9], the unmodeled interactions must satisfy certain structural conditions when these advanced schemes are used.

However, for the conventional MRAC scheme, the problem of the relaxation of the subsystem relative degrees is still unsolved. Due to the simplicity of the conventional MRAC scheme, the solution to such a problem is of practical interest. In [14], Datta and Ioannou applied the normalization technique used in the single-loop robust adaptive controller design to achieve the required relaxation. But the proposed local normalizing signals require information from the other subsystems to bound the effects of interactions from these subsystems. Thus, only partially decentralized adaptive controllers can be designed. In this paper, the problem will be solved with totally decentralized controllers by employing the parameter projection together with a static normalization technique. Global stability is established for the closed-loop system and small in the mean tracking error is ensured. With our analysis, the class of interactions and subsystem unmodeled dynamics can be enlarged to include those having infinite memory.

The remaining part of the paper is organized as follows. Section II gives the class of systems to be controlled and Section III presents the decentralized controllers. The analysis of the closed-loop system and the main result are given in Section IV. Finally, the paper is concluded in Section V.

II. SYSTEM MODELS AND ASSUMPTIONS

In this paper, the class of interconnected systems considered consists of m single-input/single-output subsystems. The i th subsystem is modeled as

$$y_i(t) = H_i(D)u_i(t) + H_i(D) \sum_{j=1}^m \bar{e}_{ij} \bar{H}_{ij}(D)[u_j(t) + y_j(t)] + \sum_{j=1}^m \bar{e}_{ij} \bar{H}_{ij}(D)[u_j(t) + y_j(t)] + d_i(t) \quad (1)$$

for $i, j = 1, \dots, m$, where y_i , u_i , and d_i are, respectively, the output, input, and disturbance of the i th subsystem. In (1), $H_i(D) = \frac{B_i(D)}{A_i(D)}$ and it is the reduced-order transfer function of subsystem i with

$$A_i(D) = D^{n_i} + a_i^{n_i-1} D^{n_i-1} + \dots + a_i^0 \\ B_i(D) = b_i^{m_i} D^{m_i} + b_i^{m_i-1} D^{m_i-1} + \dots + b_i^0$$

where D denotes the differentiation operator, $m_i < n_i$, \bar{e}_{ij} , \bar{e}_{ij}^* are constants, and $\bar{H}_{ij}(D)$ and $\bar{H}_{ij}^*(D)$ denote the subsystem interactions if $i \neq j$ and unmodeled dynamics if $i = j$.

Now, a reference model given below is chosen for the i th subsystem

$$y_m^i(t) = W_m^i(D)r_i(t) \quad (2)$$

where $W_m^i(D) = k_m^i \frac{1}{D_{n_m^i}^i(D)}$ and r_i is an external reference input signal. Here, k_m^i is a constant and $D_{n_m^i}^i(D)$ is a monic Hurwitz polynomial of degree $n_m^i = n_i - m_i$, i.e., $D_{n_m^i}^i(D) = D^{n_m^i} + d_{n_m^i-1}^i D^{n_m^i-1} + \dots + d_1^i D + d_0^i$. The control problem is to design totally decentralized controllers for plant (1) such that the closed-loop system is stable in the sense that all signals in the system are bounded for arbitrary bounded r_i and initial conditions, and the output $y_i(t)$ follows the output $y_m^i(t)$ of the model (2) as closely as possible. To solve the control problem, the following assumptions are made for the plant given in (1).

Assumption 2.1:

- A1) $B_i(D)$ is Hurwitz.
A2) An upper bound for n_i , the nominal relative degree $n_i^* = n_i - m_i$ of subsystem i , and the sign of the high frequency gain $\text{sgn}(b_i^{m_i})$ are known. Furthermore, the coefficients of $A_i(D)$ and $B_i(D)$ are inside a known compact convex region C_i .
A3) $\bar{H}_{ij}(D)$ and $\bar{\bar{H}}_{ij}(D)$ are stable, and $H_i(D)\bar{H}_{ij}(D)$ and $\bar{H}_{ij}(D)$ are strictly proper.
A4) $d_i(t)$ is bounded.

Remarks 2.1:

- 1) Note that there is no restriction on the nominal subsystem relative degrees n_i^* , $i = 1, 2, \dots, m$.
2) While modeling errors are assumed to satisfy A3) and A4), no *a priori* knowledge is required from them for the implementation of the adaptive controllers given in the later sections. Assumption A2) also implies a known lower bound for $|b_i^{m_i}|$.

III. DESIGN OF ROBUST DECENTRALIZED ADAPTIVE CONTROLLERS

For each subsystem, we define the following filtered variables:

$$\dot{\omega}_{i,1} = \Lambda_i \omega_{i,1} + q_i y_i, \quad \dot{\omega}_{i,2} = \Lambda_i \omega_{i,2} + q_i u_i \quad (3)$$

where (Λ_i, q_i) is a controllable pair satisfying

$$(DI - \Lambda_i)^{-1} q_i = \frac{1}{F_i(D)} [D^{n_i-2}, \dots, 1]^T \quad (4)$$

with $F_i(D)$ as an arbitrary Hurwitz polynomial of order $n_i - 1$. Both Λ_i and q_i are chosen by users. Let

$$\bar{\omega}_i^T = [\omega_{i,1}^T, \omega_{i,2}^T, y_i]. \quad (5)$$

Then the control is given as

$$u_i = \bar{\omega}_i^T \theta_i + c_{i,0} r_i \quad (6)$$

where $\theta_i^T(t) = [\theta_{i,1}^T(t), \theta_{i,2}^T(t), \theta_{i,3}(t)]$ is a $(2n_i - 1)$ -dimensional control parameter vector and $c_{i,0}(t)$ is a feedforward parameter scalar. From [16], it can be shown that a desired parameter vector θ_i^* of θ_i and a desired parameter $c_{i,0}^*$ of $c_{i,0}$ exist, and they can be obtained when the nominal transfer function $H_i(D)$ of the i th subsystem is known. When $H_i(D)$ is unknown, an adaptive law is required to update θ_i and $c_{i,0}$. To achieve this and to ensure the robustness of the adaptive controller in the presence of modeling errors including interactions, subsystem unmodeled dynamics, and external disturbances, we introduce parameter projection operation to the adaptive law. The adaptive law to tune θ_i and $c_{i,0}$ is divided into the following two cases.

Case 1: $b_i^{m_i} = 1$. In this case, $c_{i,0} = 1$ if k_m^i is chosen to be one and

$$\dot{\theta}_i = \mathcal{P} \left\{ \theta_i, -\frac{\Gamma_i \bar{e}_{i,1} \zeta_i}{1 + \omega_i^T \omega_i} \right\} \quad (7)$$

where $\Gamma_i = \Gamma_i^T > 0$

$$\bar{e}_{i,1} = y_i - y_{m_i} + \theta_i^T \zeta_i - v_i, \quad \zeta_i = W_m^i(D) I \bar{\omega}_i$$

$$v_i = W_m^i(D) \theta_i^T \bar{\omega}_i \quad (8)$$

$$\omega_i^T = [\bar{\omega}_i^T, \zeta_i^T, \zeta_i^{(1)T}, \dots, \zeta_i^{(n_i^*)T}] \quad (9)$$

and \mathcal{P} denotes the projection operation defined in [17] or [18].

Case 2: $b_i^{m_i}$ is unknown. In this case, $c_{i,0}(t)$ is unknown and needs to be updated. The local adaptive law in this case is a modified version of that in [15] by changing the σ -modification and the normalizing signal appropriately as in Case 1.

Remarks 3.1:

- 1) As can be noted from (6)–(9), the normalization is static. Also the implementation of local adaptive controllers does not require any information exchange between subsystems and the *a priori* knowledge on subsystem unmodeled dynamics.
2) The results for the adaptive controller in Case 2 can be obtained by following the similar analyses as in Case 1 and [15]. Thus we just focus our attention on Case 1 without any further elaboration on Case 2.

IV. STABILITY OF THE DECENTRALIZED ADAPTIVE CONTROL SYSTEMS

We need to establish the robustness of the local adaptive controllers in the presence of ignored interactions, unmodeled dynamics, and external disturbances. Before doing this, some preliminary analysis is required.

From (1)–(6), it can be shown that the i th subsystem can be expressed as

$$y_i = W_m^i(D) [\bar{\omega}_i^T \tilde{\theta}_i + r_i] + m_i(t) \quad (10)$$

where

$$\tilde{\theta}_i = \theta_i - \theta_i^* \quad (11)$$

$$m_i(t) = \eta_i(t) + [1 + W_m^i(D) \theta_{i,1}^{*T} (DI - \Lambda_i)^{-1} q_i] d_i \quad (12)$$

$$\eta_i(t) = \sum_{j=1}^m \Delta_{ij}(D) [u_j(t) + y_j(t)] \quad (13)$$

$$\Delta_{ij}(D) = W_m^i(D) \bar{e}_{ij} \bar{H}_{ij}(D) [1 - \theta_{i,2}^{*T} (DI - \Lambda_i)^{-1} q_i] \\ + \bar{e}_{ij} \bar{\bar{H}}_{ij}(D) [1 + W_m^i(D) (\theta_{i,3}^* + \theta_{i,1}^{*T} (DI - \Lambda_i)^{-1} q_i)]. \quad (14)$$

Clearly, $\Delta_{ij}(D)$ is strictly proper and stable from Assumption 2.1. From (12), we have the following result.

Lemma 4.1: The modeling error $m_i(t)$ in (12) satisfies

$$|m_i(t)| \leq \sum_{j=1}^m \epsilon_{ij} \sup_{0 \leq \tau \leq t} \|\omega_j(\tau)\| + d_0 \quad (15)$$

where ϵ_{ij} and d_0 are some nonnegative constants.

Proof: Let $V_i(D)$ be an arbitrary Hurwitz polynomial defined as

$$V_i(D) = D^{n_i-2} + v_{i,n_i-3} D^{n_i-3} + \dots + v_{i,0}$$

$$v_i^T = [1, v_{i,n_i-3}, \dots, v_{i,0}].$$

Then

$$\eta_i(t) = \sum_{j=1}^m \Delta_{ij} \frac{F_j V_j}{V_j F_j} (u_j + y_j) \\ = \sum_{j=1}^m \Delta_{ij} \frac{F_j}{V_j} [v_j^T (\omega_{j,1} + \omega_{j,2})]. \quad (16)$$

Thus from the stability and properness of $\Delta_{ij} \frac{F_j}{V_j}$, the result can be established from (16). \square

Remarks 4.1:

- 1) In the proof of Lemma 4.1, the effects of some exponentially decaying terms due to nonzero initial conditions have been absorbed by d_0 .
2) The constant ϵ_{ij} indicates the strength of the interactions between subsystems i and j when $i \neq j$, and the unmodeled dynamics of the i th subsystem are coupled to the nominal model when $i = j$.
3) In terms of the bounding signals, the bound for the modeling error in (15) allows the effects of the unmodeled dynamics and interactions to have infinite memory, thus it is looser than those given in existing literature, such as [1] and [2]. The class of modeling errors considered can also be enlarged to include any nonlinear unmodeled dynamics satisfying (15).

Now, if $\sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| = \|\omega_i(t)\|$ and $\sup_{0 \leq \tau \leq t} \|\omega_j(\tau)\| \leq \sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\|$ for all $j \neq i$ and $t > t_i^0$, then (15) becomes

$$\|m_i(t)\| \leq \epsilon \|\omega_i(t)\| + d_0, \quad \text{for all } t \geq t_i^0 \quad (17)$$

where ϵ is a nonnegative constant depending on ϵ_{ij} .

From (17), some useful properties of the local estimators can be obtained.

In the remaining part of this section, we will use m_i^k , $k = 1, \dots, 5$ to denote $M_i^k(D)m_i$ where $M_i^k(D)$ is a vector containing proper stable transfer functions. Thus, $m_i^k(t)$, $k = 1, \dots, 5$ also satisfy the same bounds as in (17) under the same conditions. Note that the constants ϵ and d_0 in (17) have been made uniform for all $i = 1, 2, \dots, m$ and $k = 1, \dots, 5$. Also in this section, all c_j , $j = 1, 2, \dots$ stand for generic constants.

We now derive an equation to describe the closed-loop system of the i th loop. It can be shown that the plant in the i th loop has the following state representation:

$$\dot{x}_i = A_i x_i + b_i u_i + m_i^1(t), \quad y_i = h_i^T x_i + m_i^2(t) \quad (18)$$

where (A_i, b_i, h_i^T) is a minimal state representation of $H_i(D)$. Note that Dm_i^2 also satisfies (17) from the strict properness of $\bar{H}_{ij}(D)$. Now the closed-loop system of the i th loop can be described as

$$\begin{aligned} \dot{\bar{x}}_i^c &= \bar{A}_i^c \bar{x}_i^c + b_i^c \bar{\omega}_i^T \bar{\theta}_i + b_i^c r_i + m_i^1(t) \\ y_i &= (h_i^c)^T \bar{x}_i^c + m_i^2(t) \end{aligned} \quad (19)$$

where \bar{A}_i^c is a stable matrix satisfying $(h_i^c)^T (DI - \bar{A}_i^c)^{-1} b_i^c = W_m^i(D)$ and $\bar{x}_i^c = [x_i^T, \omega_{i,1}^T, \omega_{i,2}^T]^T$.

Let $\bar{\zeta}_i^T = [\zeta_i^T, \zeta_i^{(1)T}, \dots, \zeta_i^{(n_i^* - 1)T}]$. Then $\zeta_i^{(k)} = D^k W_m^i(D) I \bar{\omega}_i$, $k = 0, 1, \dots, n_i^* - 1$ can have the following realization:

$$\begin{aligned} \dot{\bar{\zeta}}_i &= A_{\zeta_i} \bar{\zeta}_i + B_{\zeta_i} \bar{\omega}_i \\ &= A_{\zeta_i} \bar{\zeta}_i + B_{\zeta_i,1} \omega_{i,1} + B_{\zeta_i,2} \omega_{i,2} + B_{\zeta_i,3} h_i^T x_i + B_{\zeta_i,3} m_i^2(t) \end{aligned} \quad (20)$$

$$\zeta_i^{(k)} = C_k \bar{\zeta}_i \quad (21)$$

where $C_k = [0, 0, \dots, I, 0 \dots 0]$ with zero being $(2n_i - 1) \times (2n_i - 1)$ block matrices and I an identity matrix at the $(k+1)$ th position. A_{ζ_i} is a stable matrix satisfying $C_k^T (DI - A_{\zeta_i})^{-1} B_{\zeta_i} = D^k W_m^i(D) I$ and $B_{\zeta_i} = [B_{\zeta_i,1}, B_{\zeta_i,2}, B_{\zeta_i,3}]$.

Let $x_i^c = [\bar{x}_i^c, \bar{\zeta}_i^T]^T$. Then we have

$$\dot{x}_i^c = A_i^c x_i^c + B_i^c \bar{\omega}_i^T \bar{\theta}_i + B_i^c r + m_i^3(t) \quad (22)$$

where

$$A_i^c = \begin{bmatrix} \bar{A}_i^c & 0 \\ B_{\zeta_i,3} h_i^T & A_{\zeta_i} \end{bmatrix} \quad (23)$$

B_i^c is suitably augmented from b_i^c . Clearly, A_i^c is a stable matrix.

We now establish the relationship between ω_i and x_i^c . It can be shown, by taking the modeling error m_i into account and following similar steps in the proof of [16, Th. C.1], that

$$\|x_i(t)\| \leq c_1 \|\bar{\omega}_i(t)\| + \|m_i^4(t)\|. \quad (24)$$

Then

$$\begin{aligned} \|x_i^c(t)\| &= \|[x_i^T(t), \omega_{i,1}^T(t), \omega_{i,2}^T(t), \bar{\zeta}_i^T]^T\| \\ &\leq c_2 \|\omega_i(t)\| + \|m_i^4(t)\|. \end{aligned} \quad (25)$$

Also

$$\begin{aligned} \|\omega_i(t)\| &= \|[\omega_{i,1}^T(t), \omega_{i,2}^T(t), (h_i^c)^T \bar{x}_i^c(t) + m_i^2(t), \bar{\zeta}_i^T, \bar{\omega}_i^T]^T \\ &\quad - ([d_0^i I, \dots, d_{n_i^* - 1}^i I] \bar{\zeta}_i)^T]^T\| \\ &\leq c_3 \|x_i^c(t)\| + \|m_i^5(t)\|. \end{aligned} \quad (26)$$

Before establishing the stability of the system, we now explore some properties of the estimator (7)–(9).

Lemma 4.2: The estimator (7)–(9), when applied to the plant given in (1), has the following properties.

$$1) \quad \frac{|\bar{\omega}_i(t)^T \bar{\theta}_i(t)|}{(1 + \omega_i^T(t) \omega_i(t))^{1/2}} \leq c_4, \quad \text{for } t \geq 0. \quad (27)$$

2) Suppose M_0 is a positive constant s.t. $d_0/M_0 \leq \delta$. If $\|\omega_i(t)\| > M_0$, $\sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| = \|\omega_i(t)\|$ and $\sup_{0 \leq \tau \leq t} \|\omega_j(\tau)\| \leq \sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| \forall j \neq i$ and for all $t > t_{0_i}$ with some $t_{0_i} \geq 0$, then

$$\int_{t_{0_i}}^t \frac{|\bar{\omega}_i(\tau)^T \bar{\theta}_i(\tau)|}{(1 + \omega_i^T(\tau) \omega_i(\tau))^{1/2}} d\tau \leq c_5/\epsilon_0 + (\alpha_1 + \alpha_2)(t - t_{0_i}), \quad \text{for } t \geq t_{0_i} \quad (28)$$

where

$$\alpha_1 = (c_6/\epsilon_0 + \epsilon)\epsilon, \quad \alpha_2 = (c_6/\epsilon_0 + \delta)\delta + c_7\epsilon_0^2 \quad (29)$$

and $\rho > 0$, $\epsilon_0 \in (0, 1]$.

Proof:

$$1) \quad \left| \frac{\bar{\omega}_i(t)^T \bar{\theta}_i(t)}{(1 + \omega_i^T(t) \omega_i(t))^{1/2}} \right| \leq \frac{\|\bar{\omega}_i(t)\|}{(1 + \omega_i^T(t) \omega_i(t))^{1/2}} |\bar{\theta}_i(t)| \leq c_1.$$

2) From (8) and (10), we have

$$\bar{e}_{i,1} = \zeta_i^T \bar{\theta}_i + m_i(t). \quad (30)$$

Then consider the following positive definite function:

$$V_i = \frac{1}{2} \bar{\theta}_i^T \Gamma_i^{-1} \bar{\theta}_i. \quad (31)$$

Using (7) and (30) gives

$$\dot{V}_i \leq -\frac{1}{2} \frac{(\bar{\theta}_i^T \zeta_i)^2}{1 + \omega_i^T \omega_i} + \frac{1}{2} \frac{(m_i)^2}{1 + \omega_i^T \omega_i}. \quad (32)$$

From the assumption of the lemma and (17), we have

$$\frac{m_i(t)}{(1 + \omega_i^T \omega_i)^{1/2}} \leq \epsilon + \delta, \quad \text{for } t \geq t_{0_i}. \quad (33)$$

Then for $t \geq t_{0_i}$, (32) becomes

$$\dot{V}_i \leq -\frac{1}{2} \frac{(\bar{\theta}_i^T \zeta_i)^2}{(1 + \omega_i^T \omega_i)} + \frac{1}{2} (\epsilon + \delta)^2. \quad (34)$$

Then

$$\int_{t_{0_i}}^t \frac{(\zeta_i^T(\tau) \bar{\theta}_i(\tau))^2}{1 + \omega_i^T \omega_i} d\tau \leq c_8 + (\epsilon + \delta)^2 (t - t_{0_i}). \quad (35)$$

From (9), we can note that $\frac{\|\zeta_i^{(k)}\|}{(1 + \omega_i^T \omega_i)^{1/2}}$, $k = 0, 1, \dots, n_i^*$ are bounded. From (3), (6), (18), and (24), we can have

$$\left\| \frac{\dot{\bar{\omega}}_i}{(1 + \omega_i^T(t) \omega_i(t))^{1/2}} \right\| \leq c_9(\epsilon + \delta) + c_{10}. \quad (36)$$

Now

$$\begin{aligned} \zeta_i^{(n_i^* + 1)} &= \frac{D(D^{n_i^*} - D_m^i(D))}{D_m^i(D)} I \bar{\omega}_i + \dot{\bar{\omega}}_i \\ &= (-d_0^i \zeta_i^{(1)} - d_1^i \zeta_i^{(2)} - \dots - d_{n_i^* - 1}^i \zeta_i^{(n_i^*)}) + \dot{\bar{\omega}}_i. \end{aligned} \quad (37)$$

Therefore, similar bounds to (36) can be obtained for

$$\frac{\zeta_i^{(n_i^* + 1)}}{(1 + \omega_i^T(t) \omega_i(t))^{1/2}}, \quad \frac{\frac{d}{dt}(1 + \omega_i^T \omega_i)^{1/2}}{(1 + \omega_i^T \omega_i)^{1/2}}$$

and

$$\frac{d}{dt} \left(\frac{\tilde{\theta}_i^T \zeta_i^{(k)}}{(1 + \omega_i^T \omega_i)^{1/2}} \right)$$

$k = 0, 1, \dots, n_i^*$. Note that $\bar{\omega}_i = \sum_{k=0}^{n_i^*} d_k^i \zeta_i^{(k)}$. Thus if

$$\frac{(\zeta_i^T(t) \tilde{\theta}_i(t))^2}{1 + \omega_i^T(t) \omega_i(t)} \leq \epsilon_0 \quad \forall t \in [t_j, t_j + \Delta t_j],$$

and for some $t_j \geq t_{0_i}$ (38)

we can establish

$$\frac{|\bar{\omega}_i(t)^T \tilde{\theta}_i(t)|}{(1 + \omega_i^T(t) \omega_i(t))^{1/2}} \leq c_{11} \epsilon_0^{\rho_0} + (\epsilon + \delta) \frac{c_{12}}{\epsilon_0},$$

$\forall t \in [t_j, t_j + \Delta t_j]$. (39)

Then (28) can be established from (35) and (39). \square

Remark 4.2: α_1 can be made arbitrary small by reducing ϵ and α_2 by making M_0 a sufficiently large number. M_0 is used here for the purpose of analysis only. It is not a design parameter.

From Lemma 4.2, (25), and (26), the stability of the system can be established under a special case. This is presented in the following lemma.

Lemma 4.3: Consider the decentralized adaptive system consisting of plant (1) and local adaptive controllers (6)–(9). Suppose that $\|\omega_i(t_i^0)\| = M_0$ and for all $t > t_i^0$, $\sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| = \|\omega_i(t)\|$ and $\sup_{0 \leq \tau \leq t} \|\omega_j(\tau)\| \leq \|\omega_i(t)\|$ for all $j \neq i$. Then under Assumption 2.1, there exists a positive constant ϵ_1^* such that for all $\epsilon \leq \epsilon_1^*$ the closed-loop system ensures that

$$\sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| \leq M, \quad \forall i = 1, 2, \dots, m \quad (40)$$

where $M = c_{12}M_0 + c_{13}$ with $c_{12} \geq 1$.

Proof: The solution of (22) is

$$x_i^c(t) = e^{A_i^c(t-t_{0_i})} x_i^c(t_{0_i}) + \int_{t_{0_i}}^t e^{A_i^c(t-\tau)} [B_i^c \bar{\omega}_i^T(\tau) \tilde{\theta}_i(\tau) + B_i^c r_i(\tau) + m_i^3(\tau)] d\tau.$$

As A_i^c is a stable matrix, there exist positive constants c and σ such that

$$\|e^{A_i^c t}\| \leq ce^{-\sigma t}. \quad (41)$$

Suppose that the intermediate number M_0 is also such that $\|r_i\|_\infty \leq M_0$, $\forall i = 1, 2, \dots, m$. Then using (25), (26), (17), (27), and (41) gives

$$\begin{aligned} \|\omega_i(t)\| &\leq c_3 ce^{-\sigma(t-t_{0_i})} (c_2 \|\omega_i(t_{0_i})\| + \|m_i^4(t_{0_i})\|) \\ &+ c_{14} \int_{t_{0_i}}^t ce^{-\sigma(t-\tau)} \left[\frac{|\bar{\omega}_i^T(\tau) \tilde{\theta}_i(\tau)|}{(1 + \|\omega_i(\tau)\|^2)^{1/2}} \|\omega_i(\tau)\| \right. \\ &\quad \left. + \frac{|\omega_i^T(\tau) \tilde{\theta}_i(\tau)|}{(1 + \|\omega_i(\tau)\|^2)^{1/2}} \right. \\ &\quad \left. + M_0 + \|m_i^3(\tau)\| \right] d\tau + \|m_i^5(t)\| \\ &\leq c_{15} M_0 + c_{16} \epsilon \sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| + c_{17} \int_{t_{0_i}}^t ce^{-\sigma(t-\tau)} \\ &\quad \times \frac{|\omega_i^T(\tau) \tilde{\theta}_i(\tau)|}{(1 + \|\omega_i(\tau)\|^2)^{1/2}} \|\omega_i(\tau)\| d\tau + c_{18}. \end{aligned} \quad (42)$$

After some rearrangement of (42), the Bellman–Grownwall lemma can be applied. Then from Lemma 4.2, we can obtain that

$$\|\omega_i(t)\| \leq c_{19} M_0 + c_{20} + c_{21} \epsilon \sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| \quad (43)$$

for $\epsilon \leq \bar{\epsilon}_1^*$ and $\delta \leq \bar{\delta}^*$ where $\bar{\epsilon}_1^*$ and $\bar{\delta}^*$ are sufficiently small constants satisfying

$$c_{22}(\alpha_1^* + \alpha_2^*) < \sigma \quad (44)$$

with α_1^* depending on $\bar{\epsilon}_1^*$ and α_2^* on $\bar{\delta}^*$.

Note that the right side of (43) is nondecreasing. Thus it can be rewritten as

$$\sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| \leq c_{19} M_0 + c_{20} + c_{21} \epsilon \sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\|. \quad (45)$$

Then from (45), we get

$$\sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| \leq \frac{c_{19}}{1 - c_{21}\epsilon} M_0 + \frac{c_{20}}{1 - c_{21}\epsilon} \quad (46)$$

if $\epsilon \leq \bar{\epsilon}_1^*$ with the positive constant $\bar{\epsilon}_1^*$ satisfying $c_{21}\bar{\epsilon}_1^* < 1$.

Finally, the result is proved by letting $\epsilon_1^* = \min\{\bar{\epsilon}_1^*, \bar{\epsilon}_1^*\}$ and $c_{12} = \max\{\frac{c_{19}}{1 - c_{21}\epsilon_1^*}, 1\}$, $c_{13} = \frac{c_{20}}{1 - c_{21}\epsilon_1^*}$. \square

To establish the stability result for the general case, we explore the parameter estimator further, and this gives Lemma 4.4 as follows.

Lemma 4.4: If $\|\omega_i(t)\| > M_0$ for all $t \geq t_{0_i}$, and $\sup_{0 \leq \tau \leq t} \|\omega_j(\tau)\| \leq c_{12}M_0 + c_{13}$ for all $t \in [0, t_1]$ and $j = 1, 2, \dots, m$, and $\sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| = \|\omega_i(t)\|$, $\sup_{0 \leq \tau \leq t} \|\omega_j(\tau)\| \leq \sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\|$, $\forall j \neq i$ for all $t \geq t_1$ with some $t_1 \geq t_{0_i}$, then

$$\int_{t_{0_i}}^t \frac{|\omega_i(\tau)^T \tilde{\theta}_i(\tau)|}{(1 + \omega_i^T(\tau) \omega_i(\tau))^{1/2}} d\tau \leq c_5/\epsilon_0 + (\bar{\alpha}_1 + \alpha_2)(t - t_{0_i}),$$

for $t \geq t_{0_i}$ (47)

where

$$\bar{\alpha}_1 = [c_6/\epsilon_0 + (c_{12} + c_{13})\epsilon](c_{12} + c_{13})\epsilon. \quad (48)$$

Proof: By noting the condition of the lemma and using (17), we have

$$\frac{|m_i(t)|}{(1 + \|\omega_i(t)\|^2)^{1/2}} \leq \epsilon(c_{12} + c_{13}) + \delta, \quad \forall t \in [t_{0_i}, t_1]. \quad (49)$$

For $t \geq t_1$, (33) becomes valid. Noting that $c_{12} \geq 1$, we can have (49) for all $t \geq t_{0_i}$. Then replacing (33) by (49) and ϵ by $\epsilon(c_{12} + c_{13})$ in the proof of (28), we can establish (47). \square

Remark 4.3: Note that the property in the above lemma is quite similar to (28) in Lemma 4.2 except that ϵ is changed to $\epsilon(c_{12} + c_{13})$.

From Lemmas 4.2–4.4, we can establish our main stability result as follows.

Theorem 4.1: Consider the decentralized adaptive system consisting of plant (1) and local adaptive controllers (6)–(9). Under Assumption 2.1, there exists a constant ϵ^* such that for all $\epsilon \leq \epsilon^*$, we have the following.

- 1) The closed-loop system is globally stable in the sense that all signals remain bounded $\forall t$ and for all finite initial states, any bounded r_i , and arbitrarily bounded external disturbances.
- 2) The tracking error $e_{i,1}(t) = y_i - y_{m_i}$ satisfies

$$\int_{t_i^0}^t e_{i,1}^2(\tau) d\tau \leq \beta_1 + \beta_2(\epsilon + d_0 + \epsilon_0)(t - t_i^0), \quad \text{for all } t_i^0 \geq 0 \quad (50)$$

where β_1, β_2 are positive constants.

Proof:

- 1) To show the boundedness of all the trajectories $\|\omega_i\|$, $i = 1, 2, \dots, m$, we consider a function $\|\omega(t)\|$ defined as

$$\|\omega(t)\| = \max\{\|\omega_1(t)\|, \|\omega_2(t)\|, \dots, \|\omega_m(t)\|\}. \quad (51)$$

Clearly, the result is proved if $\|\omega(t)\|$ is bounded. It can be noted that $\|\omega(t)\|$ is continuous and thus, starting with $\tau_0 = 0$

and $k = 1, 2, \dots$, we can divide the time axis $[0, \infty)$ into the following two subsequences:

$$\mathfrak{R}_k^- = [\tau_{k-1}, s_k] \quad \mathfrak{R}_k^+ = (s_k, \tau_k)$$

where

$$\mathfrak{R}_k^- = \{t \mid \|\omega(t)\| \leq M_0\} \quad \text{and} \quad \mathfrak{R}_k^+ = \{t \mid \|\omega(t)\| > M_0\} \quad (52)$$

i.e.,

$$[0, \infty) = \left(\bigcup_{k=1}^{\infty} \mathfrak{R}_k^- \right) \cup \left(\bigcup_{i=1}^{\infty} \mathfrak{R}_i^+ \right). \quad (53)$$

Obviously, it suffices to prove that $\|\omega(t)\|$ is bounded in \mathfrak{R}_k^+ , $\forall k \geq 1$. This can be shown through induction. Thus we consider $t \in \mathfrak{R}_1^+$ first. From the continuity of $\|\omega(t)\|$, $\exists t_1 \in \mathfrak{R}_1^+$ and an $i \in \{1, 2, \dots, m\}$ such that $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| = \|\omega(t)\|$ and $\|\omega(t)\| = \|\omega_i(t)\|$ for all $t \leq t_1$ and $t \in \mathfrak{R}_1^+$. Thus $\sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| = \|\omega_i(t)\|$ and $\sup_{0 \leq \tau \leq t} \|\omega_j(\tau)\| \leq \sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| \forall j \neq i$, for all $t \leq t_1$ and $t \in \mathfrak{R}_1^+$. Therefore, the conditions of Lemma 4.3 are satisfied for all $t \leq t_1$ and $t \in \mathfrak{R}_1^+$. Then from this lemma and noting that $\|\omega_i(t_{0_i})\| = \|\omega(t_{0_i})\| = M_0$, we can have, for all $t \leq t_1$ and all $\epsilon \leq \epsilon_1^*$, that

$$\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| \leq M \quad (54)$$

i.e.,

$$\sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| \leq M, \quad \forall i = 1, 2, \dots, m. \quad (55)$$

If the conditions of Lemma 4.2.2 or 4.3 are violated for $t \geq t_1$ and $t \in \mathfrak{R}_1^+$, the following two possibilities may occur to $\|\omega(t)\|$.

- Case 1: $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| = \|\omega(t)\|$ but $\|\omega(t)\| = \|\omega_j(t)\|$, $j \in \{1, 2, \dots, m\} \setminus i$ for all $t > t_1$.

In this case, the condition that $\sup_{0 \leq \tau \leq t} \|\omega_j(\tau)\| \leq \sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\|$, $\forall j \neq i$ cannot be satisfied. Thus Lemma 4.3 cannot be applied for $t > t_1$. However, we now consider $\|\omega_j(t)\|$. Clearly, there exists a t_j^1 such that $\|\omega_j(t_j^1)\| = M_0$ and $\|\omega_j(t)\| > M_0$ for all $t \in [t_j^1, t_1] \subset \mathfrak{R}_1^+$. Also in this case, we have, for $t > t_1$

$$\sup_{0 \leq \tau \leq t} \|\omega_j(\tau)\| = \|\omega_j(t)\|$$

and

$$\sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| \leq \sup_{0 \leq \tau \leq t} \|\omega_j(\tau)\| \quad i \in \{1, 2, \dots, m\} \setminus j. \quad (56)$$

Thus from (55) and (56), Lemma 4.4 can be applied to $\|\omega_j(t)\|$ for $t \geq t_j^1$. Then following the same steps as in the proof of Lemma 4.3 and applying Lemma 4.4 with "initial condition" $\|\omega_j(t_j^1)\| = M_0$, we shall obtain (54) or (55) for all $t \geq t_1$ and all $\epsilon \leq \epsilon^*$ where

$$\epsilon^* = \frac{\epsilon_1^*}{c_{12} + c_{13}}. \quad (57)$$

- Case 2: $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| \neq \|\omega(t)\|$ for $t \in [t_1, t_2] \subset \mathfrak{R}_1^+$ and $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| = \|\omega(t)\|$ for $t > t_2$.

In this case, the condition that $\sup_{0 \leq \tau \leq t} \|\omega_i(\tau)\| = \|\omega_i(t)\|$ cannot be satisfied for $t \geq t_1$. However, (54) or (55) automatically holds for $t \in [t_1, t_2]$. If t_2 is infinite, the result is proved. For a finite t_2 and when $t > t_2$, (54) or (55) can be shown under the condition (57) by following the same argument as in Case 1.

In this way, the boundedness of $\|\omega(t)\|$ is established over \mathfrak{R}_1^+ .

Now assuming (54) or (55) holds $\forall t \in \mathfrak{R}_k^+$, it can be shown that, by following the proof of Lemma 4.3 and the above argument, (54) is also true $\forall t \in \mathfrak{R}_{k+1}^+$ from Lemma 4.4 with the "initial condition" $\|\omega_i(t_i^{k+1})\| = M_0$ for $i \in \{1, 2, \dots, m\}$ and $t_i^{k+1} \in \mathfrak{R}_{k+1}^+$.

After establishing the boundedness of $\|\omega_i(t)\|$, $\forall i = 1, 2, \dots, m$, we can have $y_i(t)$ and $u_i(t)$ bounded.

- 2) Once the boundedness of all the signals is established, then the tracking properties can be obtained by following similar analyses used in [15]. \square

Remarks 4.4:

- 1) Note that trajectory $\|\omega(t)\|$ only has three possibilities. That is, it satisfies the condition of Lemmas 4.2.2 and 4.3, or Case 1, or Case 2.
- 2) In the stability analysis, we only need to take care of the subsystem in which the static normalizing signal has maximum magnitude among all the subsystems over a certain time interval and consider the situation that the magnitude exceeds a certain level, i.e., $\|\omega(t)\| \geq M_0$. In this case, the locally normalized parameter estimation prediction error in the subsystem concerned becomes small and satisfies certain conditions specified in Lemmas 4.2 and 4.4. Also the inductive technique used and the division of time interval into two subsequences are crucial in the establishment of a uniform bound for $\|\omega(t)\|$ over all subintervals.

V. CONCLUSION

In this paper, we have relaxed the subsystem relative degrees requirement imposed in model reference decentralized adaptive control using first-order local estimators. These local estimators are designed using parameter projection together with static normalization. In the implementation of the local controllers, no *a priori* knowledge of the subsystem unmodeled dynamics and no information exchanges between subsystems are required. It has been shown that global stability of the overall adaptive feedback system can be ensured, provided the strength of the interactions and subsystem unmodeled dynamics is sufficiently weak. For each subsystem, the effect of the modeling error, including interactions from other subsystems, can be allowed to have infinite memory. Despite the modeling error, we have shown that small in the mean tracking error can be achieved.

REFERENCES

- [1] P. Ioannou and P. Kokotovic, "Decentralized adaptive control of interconnected systems with reduced-order models," *Automatica*, vol. 21, pp. 401–412, 1985.
- [2] P. Ioannou, "Decentralized adaptive control of interconnected systems," *IEEE Trans. Automat. Contr.*, vol. 31, pp. 291–298, 1986.
- [3] D. T. Gavel and D. D. Siljak, "Decentralized adaptive control: Structural conditions for stability," *IEEE Trans. Automat. Contr.*, vol. 34, pp. 413–426, 1989.
- [4] C. Wen and D. J. Hill, "Decentralized adaptive control of linear time varying systems," in *Proc. 11th World Congr. Automatic Control*, Tallinn, 1990, vol. 4.
- [5] —, "Global boundedness of discrete-time adaptive control just using estimator projection," *Automatica*, vol. 28, pp. 1143–1157, 1992.
- [6] C. Wen, "Indirect robust totally decentralized adaptive control of continuous-time interconnected systems," *IEEE Trans. Automat. Contr.*, vol. 40, pp. 1122–1126, 1995.

- [7] A. S. Morse, "A comparative study of normalized and unnormalized tuning errors in parameter-adaptive control," in *Proc. 30th IEEE Conf. Decision and Control*, 1991.
- [8] R. Ortega and A. Herrera, "A solution to the decentralized stabilization problem," *Syst. Contr. Lett.*, vol. 20, pp. 299–306, 1993.
- [9] R. Ortega, "An energy amplification condition for decentralized adaptive stabilization," *IEEE Trans. Automat. Contr.*, vol. 41, pp. 285–288, 1996.
- [10] M. Krstic, I. Kanellakopoulos, and P. Kokotovic, "A new generation of adaptive controllers for linear systems," *IEEE Trans. Automat. Contr.*, vol. 39, pp. 738–752, 1994.
- [11] C. Wen, "Decentralized adaptive regulation," *IEEE Trans. Automat. Contr.*, vol. 39, pp. 2163–2166, 1994.
- [12] C. Wen and Y. C. Soh, "Decentralized adaptive control using integrator backstepping," *Automatica*, vol. 33, pp. 1719–1724, 1997.
- [13] S. Jain and F. Khorrami, "Global decentralized adaptive control of large scale nonlinear systems without strict matching," in *Proc. American Control Conf.*, 1995, pp. 2938–2942.
- [14] A. Datta and P. Ioannou, "Decentralized adaptive control," in *Advances in Control and Dynamic Systems*, C. T. Leondes, Ed. New York: Academic, 1992.
- [15] P. A. Ioannou and K. S. Tsakalis, "A robust direct adaptive controller," *IEEE Trans. Automat. Contr.*, vol. 31, pp. 1033–1043, 1986.
- [16] K. S. Narendra and A. M. Annaswamy, *Stable Adaptive Systems*. Englewood Cliffs, NJ: Prentice-Hall, 1989.
- [17] S. M. Naik, P. R. Kumar, and B. E. Ydstie, "Robust continuous-time adaptive control by parameter projection," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 182–197, 1992.
- [18] J.-B. Pomet and L. Praly, "Adaptive nonlinear regulation: Estimation from the Lyapunov equation," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 729–740, 1992.
- [19] R. H. Middleton, G. C. Goodwin, D. J. Hill, and D. Q. Mayne, "Design issues in adaptive control," *IEEE Trans. Automat. Contr.*, vol. 33, pp. 50–58, 1988.

A Diagonal Recurrent Neural Network-Based Hybrid Direct Adaptive SPSA Control System

Xiao D. Ji and Babajide O. Familoni

Abstract—A direct adaptive simultaneous perturbation stochastic approximation (DA SPSA) control system with a diagonal recurrent neural network (DRNN) controller is proposed. The DA SPSA control system with DRNN has simpler architecture and parameter vector size that is smaller than a feedforward neural network (FNN) controller. The simulation results show that it has a faster convergence rate than FNN controller. It results in a steady-state error and is sensitive to SPSA coefficients and termination condition. For trajectory control purpose, a hybrid control system scheme with a conventional PID controller is proposed.

Index Terms—Diagonal recurrent neural network (DRNN), neural network controller (NNC), simultaneous perturbation stochastic approximation (SPSA).

I. INTRODUCTION

Nonlinear adaptive control system design is a challenge in nonlinear control system theory. In general, one may use neural networks (NN) to identify and/or control unknown and/or uncertain nonlinear

Manuscript received May 9, 1996; revised March 6, 1998. Recommended by Associate Editor, J. C. Spall.

The authors are with the Department of Electrical Engineering, The University of Memphis, Memphis, TN 38152 USA (e-mail: familoni@memphis.edu).
Publisher Item Identifier S 0018-9286(99)04545-6.

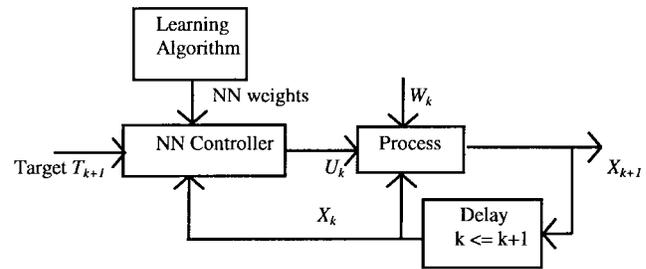


Fig. 1. Block diagram of the DA SPSA neural network-based control system.

systems. To accomplish this, one needs to train (usually, offline) an inverse neural network as a controller. This is generally difficult since the system is unknown. An ideal scheme is a direct adaptive (DA) neural network control system. Spall described a generalized NN based on the simultaneous perturbation stochastic approximation (SPSA) approach to estimate the gradient of the performance function of an unknown nonlinear system [1]. Such a direct adaptive SPSA approach does not require any prior knowledge of the unknown system and does not need a separate training phase. An SPSA direct adaptive control system will converge to an optimal neural network parameter set, if it exists [3]. The NN-based SPSA approach as discussed in [2] and [3] uses a forward neural network (FNN) as the controller. The parameter vector size, in general, is large. For example, a network which contains four layers, two inputs, one output, with two hidden layers containing 20 and 10 nodes, respectively (denoted as $N_{2,20,10,1}^4$), has 280 elements in the parameter vector. Other things being equal, its increased computational cost results in a slow performance measurement period (i.e., sampling period), and the performance measurement period is very important for a real-time control application.

As is well known, a recurrent neural network (RNN) has some advantages over FNN such as faster convergence, more accurate mapping ability, etc., but it is difficult to apply the gradient-descent method to update the neural network weights in RNN [4]. Ku *et al.* [5], [6] proposed the DRNN scheme that captures the dynamic behavior of a system and, since it is not fully connected, training is expected to be much faster than RNN. DRNN with time delay has RNN behavior but simple connections and is easy to use when applying the gradient-descent method. Therefore, in this paper, a diagonal recurrent neural network (DRNN) is employed in a DA SPSA control system. Simulation results are compared with those of the FNN SPSA scheme. These results also show that in general, after the SPSA process, the fixed DA SPSA neural network-based control results in a steady-state error because of the finite sample constraint of the SPSA approach. To improve the control performance, a conventional PID controller was employed to form a hybrid DA SPSA scheme. The proposed hybrid DA SPSA control system was examined by simulation and showed good performance.

II. SPSA BACKGROUND

Consider the problem of finding a root θ^* of the gradient equation

$$g(\theta) \equiv \frac{\partial L(\theta)}{\partial \theta} = 0 \quad (1)$$

for some differentiable cost function $L : R^p \rightarrow R^1$. There are many methods for finding θ^* . In the case where L is observed in the presence of noise, an SA algorithm of the generic