Analysis and Design of Impulsive Control Systems

Z. G. Li, C. Y. Wen, and Y. C. Soh

***Abstract*—Some sufficient conditions for asymptotic stability of impulsive control systems with impulses at fixed times were recently presented. In this note, we derive some less conservative conditions for asymptotic stability of such impulsive control systems and the results are used to design impulsive control for a class of nonlinear systems. The class of nonlinear systems considered is also enlarged.**

***Index Terms*—Impulsive control, impulsive systems, stability.**

I. Introduction

There are many practical examples of impulsive control systems. Three typical examples are the population control system of a kind of insects with the number of insects and their natural enemies as state variables, a chemical reactor system with the quantities of different chemicals server as the states, and a financial system with two state variables of the amount of money in a market and the saving rates of a central bank [6]. Some other practical examples are given in [1] and [2].

Recently, impulsive systems and impulsive control have been studied by many researchers. Bainov and Simeonov [1], Lakshmikantham *et al.* [2], and Lakshmikantham and Liu [3] have considered the stability of impulsive systems by using Lyapunov functions and the Lyapunov functions are required to be nonincreasing along the whole sequence of the switchings. Li *et al.* [5] have relaxed this requirement and the Lyapunov function is only required to be nonincreasing along a subsequence of the switchings. Yang [6] has obtained some sufficient conditions for the impulsive control of a class of nonlinear systems by using the results in [2]. Yang and Chua [7] and Yang *et al.* [8] have presented some interesting applications of impulsive control in chaotic secure communication systems and chaotic spread spectrum communications. Panas *et al.* [8] have given some methods for the experimental settings to achieve the impulsive controls.

In this note, we shall also consider the impulsive control of nonlinear systems as in [6]. We first derive some less conservative conditions for the stability of impulsive systems with impulses at fixed times and then the results are used to design impulsive control laws for a class of nonlinear systems. Our method can be applied to a wider class of nonlinear systems and is helpful to improve the existing technologies used in chaotic secure communication systems and chaotic spread spectrum communications [7],[8].

The rest of this note is organized as follows. In Section II, some sufficient conditions for stability of impulsive differential systems are given. These results are used to design impulsive control law for nonlinear systems in Section III. Finally, this note is concluded in Section IV.

II. Stability of Impulsive Differential Systems

An impulsive differential system with impulses at fixed times is described by [2]

$$\begin{cases} \dot{X}(t) = f(t, X(t)); & t \neq \tau_k \\ \Delta X(t) \triangleq X(t^+) - X(t) = I_k(X); & t = \tau_k, \quad k = 1, 2 \ldots \end{cases} \tag{1}$$

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Z. G. Li is with the Center for Signal Processing, School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798.

Y. C. Soh and C. Y. Wen are with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798 (e-mail: EYCSOH@ntu.edu.sg).

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where

$f : R_+ \times R^n \rightarrow R^n (R_+ = [0, \infty])$	continuous;
$I_k : R^n \rightarrow R^n$	continuous;
$X \in R^n$	state variable;
τ_k^+	time just after τ_k and $\{\tau_k\}(1 \leq k < \infty)$ satisfy

$$0 < \tau_1 < \tau_2 < \cdots < \tau_k < \tau_{k+1} < \cdots \tau_k \rightarrow \infty \text{ as } k \rightarrow \infty.$$

To establish sufficient conditions for stability of impulsive differential systems, we shall introduce some basic definitions.

Definition 1: [1] Let $V : R_+ \times R^n \rightarrow R_+$, then V is said to belong to Class v_0 if

1) V is continuous in $(\tau_{i-1}, \tau_i] \times R^n$ and for each $X \in R^n$, $i = 1, 2, \ldots$

$$\lim_{(t,Y) \rightarrow (\tau_i^+, X)} V(t, Y) = V(\tau_i^+, X) \tag{2}$$

exists;

2) V is locally Lipschitzian in X.

Definition 2: [1] For $(t, X) \in (\tau_{i-1}, \tau_i] \times R^n$, we define

$$D^+ V(t, x) \triangleq \lim_{h \rightarrow 0} \frac{1}{h} \{V(t + h, X + hf(t, X)) - V(t, x)\}. \tag{3}$$

We also need the definition of a comparison system, which plays an important role in stability analysis of impulsive differential systems.

Definition 3: [2] Let $V \in v_0$ and assume that

$$D^+ V(t, X) \leq g(t, V(t, X)), \qquad t \neq \tau_k \tag{4}$$

$$V(t, X + \Delta X) \leq \psi_k(V(t, X)), \qquad t = \tau_k \tag{5}$$

where

$g : R_+ \times R_+ \rightarrow R$	continuous;
v_0	set defined in [2];
$\psi_k : R_+ \rightarrow R_+$	nondecreasing.

Then, the following system:

$$\begin{cases} \dot{w} = g(t, w), & t \neq \tau_k \\ w(\tau_k^+) = \psi_k(w(\tau_k)) \\ w(t_0^+) = w_0 \geq 0 \end{cases} \tag{6}$$

is the comparison system of (1). Let $S(\rho) = \{X \in R^n | \|X\| < \rho\}$. We can then obtain a comparing theorem as follows.

Theorem 1: [2] Assume that

1) $V : R_+ \times S(\rho) \rightarrow R_+, V \in v_0$

$$D^+ V(t, X) \leq g(t, V(t, x)) \quad t \neq \tau_k$$

where $g : R_+ \times R_- \rightarrow R \cdot g(t, 0) = 0$, and g is continuous in $(\tau_{k-1}, \tau_k] \times R_+$ and for each $x \in R_+, k = 1, 2, \ldots$. $\lim_{(t,y) \rightarrow (\tau_k^+, x)} g(t, y) = g(\tau_k^+, x)$ exists;

2) there exists a $\rho_0 > 0$ such that $X \in S(\rho_0)$ implies that $X + I_k(X) \in S(\rho)$ for all k and $V(t, X + I_k(X)) \leq$

$\psi_k(V(t,x)), t = \tau_k, X \in S(\rho_0)$, where $\psi : R_+ \to R_+$ is nondecreasing;

3) $\beta(\|X\|) \le V(t,X) \le \alpha(\|X\|)$ on $R_+ \times S(\rho)$ where $\alpha, \beta \in K$ (class of continuous functions $a : R_+ \to R_+$ such that $a(0) = 0$) [2].

Then, the stability properties of the trivial solution $w = 0$ of (6) imply the corresponding stability properties of the trivial solution $X = 0$ of (1).

Let $g(t,w) = \dot{\lambda}(t)w$, $\lambda \in C^1[R_+, R_+]$, $\psi_k(w) = d_k w$, $d_k \ge 0$ for all $k \ge 1$. Then, we have the following stability result.

Theorem 2: The origin of system (1) is asymptotically stable if the following conditions hold:

1) $\sup_i\{d_i \exp(\lambda(\tau_{i+1}) - \lambda(\tau_i))\} = \epsilon_0 < \infty$;
2) there exists an $r > 1$ such that

$$\lambda(\tau_{2k+3}) + \ln(r d_{2k+2} d_{2k+1}) \le \lambda(\tau_{2k+1}) \text{ holds for all } d_{2k+2}d_{2k+1} \ne 0;\quad k = 0, 1, \ldots; \tag{7}$$

3) $\lambda(t)$ satisfies that

$$\dot{\lambda}(t) \ge 0; \tag{8}$$

4) there exist $\alpha(\cdot)$ and $\beta(\cdot)$ in Class K such that

$$\beta(\|X\|) \le V(t,X) \le \alpha(\|X\|). \tag{9}$$

Proof: It can be seen that the solution $w(t,t_0,w_0)$ of the comparison system

$$\begin{cases} \dot{w}(t) = \dot{\lambda}(t)w(t) \\ w(\tau_k^+) = d_k w(\tau_k) \\ w(t_0^+) = w_0 \ge 0 \end{cases} \tag{10}$$

is given by

$$w(t,t_0,w_0) = w_0 \prod_{t_0<\tau_k<t} d_k \exp(\lambda(t) - \lambda(t_0)). \tag{11}$$

We shall show that

$$w(t,t_0,w_0) \le \max\{1,\epsilon_0\} w_0 \exp(\lambda(\tau_1) - \lambda(t_0)); \quad t \ge t_0;\quad 0 \le t_0 < \tau_1 \tag{12}$$

To do this, we shall first consider the case where $d_k \ne 0$ holds for all k. In detail, the following three situations are considered.

1) $t_0 < t < \tau_1$. From (11), we have

$$\begin{aligned} w(t,t_0,w_0) &= w_0 \exp(\lambda(t) - \lambda(t_0)) \\ &\le w_0 \exp(\lambda(\tau_1) - \lambda(t_0)). \end{aligned}$$

2) $\tau_{2k-1} < t < \tau_{2k}$ for all $k \ge 1$. From (11), we have the equation shown at the bottom of the page.

3) $\tau_{2k} < t < \tau_{2k+1}$ for all $k \ge 1$. From (11), we have

$$\begin{aligned} w(t,t_0,w_0) &= w_0 \prod_{i=1}^{2k} d_i \exp(\lambda(\tau_1) - \lambda(t_0)) \\ &\quad \times \exp(\lambda(t) - \lambda(\tau_1)) \\ &\le w_0 \prod_{i=1}^{2k} d_i \exp(\lambda(\tau_1) - \lambda(t_0)) \\ &\quad \times \exp(\lambda(\tau_{2k+1}) - \lambda(\tau_1)) \\ &\le \frac{w_0}{r^k} \exp(\lambda(\tau_1) - \lambda(t_0)) \end{aligned}$$

Therefore, (12) holds.

We shall now consider the case that there exists a $d_k = 0$. In this case, we have

$$w(t,t_0,w_0) = 0, \quad \forall t \ge \tau_k^+ \tag{13}$$

This implies that (12) holds for all $t \ge \tau_k^+$. The proof of (12) in the case of $t \le \tau_k$ is similar to the above process.

Hence, choosing $\delta = \xi/2 \times \max\{1,\epsilon_0\} \exp(\lambda(t_0) - \lambda(\tau_1))$, the stability of the trivial solution $w = 0$ of (6) follows.

Note that $k \to \infty$ as $t \to \infty$. From Case 2 and Case 3, we know that

$$\lim_{t\to\infty} w(t,t_0,w_0) = 0.$$

Thus, the trivial solution $w = 0$ of (6) is asymptotically stable.

To use Theorem 1, we shall prove that 2) of Theorem 1 holds. Consider the following two cases:

Case 1) $\epsilon_0 = 0$. Obviously, 2) holds for any $\rho_0 > 0$;

Case 2) $\epsilon_0 \ne 0$. Since α is a strictly increasing function with $\alpha(0) = 0$, then for any given ρ, there exists a $\rho_0 > 0$, such that

$$\alpha(\rho_0) \le \frac{\beta(\rho)}{\epsilon_0} \tag{14}$$

From (9), we know that when $X \in S(\rho_0)$, we have

$$V(t,X) < \alpha(\rho_0)$$

It follows that

$$\begin{aligned} V(t, X + I_k(X)) &\le d_k V(t,X) \\ &\le d_k \exp(\lambda(\tau_{k+1}) - \lambda(\tau_k)) V(t,X) \\ &\le \sup_k\{d_k \exp(\lambda(\tau_{k+1}) - \lambda(\tau_k))\} V(t,X) \\ &< \epsilon_0 \alpha(\rho_0) \\ &< \beta(\rho) \end{aligned}$$

Using (9), we obtain

$$\beta(\|X + I_k(X)\|) \le V(t, X + I_k(X)) \le \beta(\rho)$$

$$\begin{aligned} w(t,t_0,w_0) &= w_0 \prod_{i=1}^{2k-1} d_i \exp(\lambda(\tau_1) - \lambda(t_0)) \exp(\lambda(t) - \lambda(\tau_1)) \\ &\le \begin{cases} w_0 \prod_{i=1}^{2k} d_i \exp(\lambda(\tau_1) - \lambda(t_0)) \exp(\lambda(\tau_{2k+1}) - \lambda(\tau_1)), & d_{2k} \ge 1 \\ w_0 \epsilon_0 \prod_{i=1}^{2k-2} d_i \exp(\lambda(\tau_1) - \lambda(t_0)) \exp(\lambda(\tau_{2k-1}) - \lambda(\tau_1)), & d_{2k} < 1 \end{cases} \\ &\le \max\{1,\epsilon_0\} \frac{w_0}{r^{k-1}} \exp(\lambda(\tau_1) - \lambda(t_0)) \end{aligned}$$

Since β is a strictly increasing continuous function, it follows that

$$\|X + I_k(X)\| < \rho$$

This implies that $X + I_k(X) \in S(\rho)$.Thus, the origin of (1), from Theorem 1, is asymptotically stable. □

Remark 1: If the conditions of Corollary 3.2.1 [1] hold, then $\epsilon_0 \le 1$. This implies that our bound given in (12) is the same as that given in Corollary 3.2.1 [1]. □

Remark 1: Equation (7) can be generalized to the following condition.

There exist a finite integer $m_0 > 0$ and an$r > 1$ such that

$$\lambda(\tau_{m_0(k+1)+1}) + \ln(rd_{m_0(k+1)} \ldots d_{m_0k+1}) \le \lambda(\tau_{m_0k+1}), \quad k = 0, 1, \ldots. \tag{15}$$

Similar to the choice of a Lyapunov function, the choice of m_0 in (15) depends on the actual system considered. □

III. Design of Impulsive Control

In this section, we will use the stability results obtained in Section II to design impulsive control for a class of nonlinear systems. A formal definition of impulsive control, which is slightly modified from [6], is given first as follows.

Definition 4: Consider a plant P whose state variable is denoted by $X \in R^n$, a set of control instants $T = \{\tau_k\}, \tau_k \in R_+, \tau_k < \tau_{k+1}, k = 1, 2, \ldots$, and control laws $U(k, X) \in R^n, k = 1, 2, \ldots$. At each τ_k, X is changed impulsively, i.e., $X(\tau_k^+) = X(\tau_k^-) + U(k, X)$, such that the system is stable and certain specifications are achieved.

In this note, we consider the impulsive control design for the following nonlinear systems:

$$\begin{cases} \dot{X}(t) = AX(t) + \phi(X(t)) \\ Y(t) = CX(t) \end{cases} \tag{16}$$

where

$X \in R^n$	state vector;
A	$n \times n$ constant matrix;
$y \in R^m$	output vector;
C	$m \times n$ constant matrix;
$\phi : R^n \to R^n$	nonlinear function satisfying $\|\phi(X)\| \le L\|X\|$ with L being a positive number.

The control instant is defined by

$$0 < \tau_1 < \tau_2 < \cdots < \tau_k < \tau_{k+1} < \cdots; \quad \tau_k \to \infty \text{ as } k \to \infty$$

and the time varying control $U(k, X)$ is given by

$$U(k, X(\tau_k)) = B_k Y(\tau_k), \qquad k = 1, 2, \ldots \tag{17}$$

then, we can obtain a nonlinear impulsive control system as follows:

$$\begin{cases} \dot{X}(t) = AX(t) + \phi(X(t)) \\ Y(t) = CX(t), & t \ne \tau_k \\ X(\tau_k^+) = X(\tau_k) + U(K, X) \\ U(k, X(\tau_k)) = B_k Y(\tau_k), & k = 1, 2, \ldots \end{cases}. \tag{18}$$

To use the results obtained in Section II, the above system is rewritten as

$$\begin{cases} \dot{X}(t) = AX(t) + \phi(X(t)), & t \ne \tau_k \\ \Delta X(t) = U(k, X(t)) = B_k CX(t), & t = \tau_k, \quad k = 1, 2, \ldots \\ X(t_0^+) = X_0 \end{cases} \tag{19}$$

where $U(k, X(t)$ corresponds to $I_k(x)$ defined in (1).

Then, we can obtain the result on the design of impulsive controls as follows.

Theorem 3: Suppose that an $n \times n$ matrix Γ is symmetric and positive definite, and $\lambda_{\min}$ and $\lambda_{\max}$ are respectively the smallest and the largest eigenvalues of Γ. Let

$$Q = \Gamma A + A^{\mathrm{T}}\Gamma \tag{20}$$

and $Q \le \gamma_1 \Gamma$ with γ_1 being a constant. Then the origin of impulsive control system (19) is asymptotically stable if the following conditions hold:

1)
$$(I + B_kC)^{\mathrm{T}}\Gamma(I + B_kC) \le \gamma_2(k)\Gamma \tag{21}$$
where I is the identity matrix, $\gamma_2(k)(k = 1, 2, \ldots)$ are positive constants;

2) there exists an $r > 1$ such that
$$\left(\gamma_1 + \frac{2L\lambda_{\max}}{\lambda_{min}}\right)(\tau_{2k+3} - \tau_{2k+1}) \le -\ln(r\gamma_2(2k+2)\gamma_2(2k+1)), \quad k = 0, 1, \ldots; \tag{22}$$

3)
$$\gamma_1 + \frac{2L\lambda_{\max}}{\lambda_{\min}} \ge 0 \tag{23}$$
$$\sup_i \left\{\gamma_2(i) \exp\left[\left(\gamma_1 + \frac{2L\lambda_{\max}}{\lambda_{\min}}\right)(\tau_{i+1} - \tau_i)\right]\right\} = \epsilon_0 < \infty. \tag{24}$$

Proof: Let

$$V(X) = X^{\mathrm{T}}\Gamma X$$

Similar to the proof of [6, Th. 3], we have

$$D^+V(x) \le \left(\gamma_1 + \frac{2L\lambda_{\max}}{\lambda_{\min}}\right)V(x), \qquad t \ne \tau_k$$

and

$$V(X + U(k, X)) \le \gamma_2(k)V(X), \qquad t = \tau_k$$

Then, we can obtain the following comparison system:

$$\begin{cases} \dot{w}(t) = \left(\gamma_1 + \frac{2L\lambda_{\max}}{\lambda_{\min}}\right)w(t); & t \ne \tau_k \\ w(\tau_k^+) = \gamma_2(k)w(\tau_k) \\ w(t_0^+) = w_0 \ge 0 \end{cases}. \tag{25}$$

Note that

$$\lambda(\tau_{2k+3}) - \lambda(\tau_{2k+1}) = \left(\gamma_1 + \frac{2L\lambda_{\max}}{\lambda_{\min}}\right)(\tau_{2k+3} - \tau_{2k+1}).$$

From Theorem 2, we know that the result holds. □

Remark 3: We do not require that BC is symmetric. Moreover, we do not require that $\|I + B_k C\| \le 1$. Thus, our result can be used for a wider class of nonlinear systems as compared to [6]. □

Remark 4: A necessary condition to achieve the asymptotic stability of the origin of system (19) is that C is nonsingular. □

Remark 5: Equation (23) implies that the original system (16) is unstable. □

Remark 6: Similar to the impulsive control proposed in [6], a time invariant control can also be used. That is, $B_1 = B_2 = \cdots = B_k = B_{k+1} = \cdots$. □

Remark 7: Note that

$$\sup_i \left(\gamma_2(i) \exp\left(\gamma_1 + \frac{2L\lambda_{\max}}{\lambda_{\min}} (\tau_{i+1} - \tau_i) \right) \right) \le \frac{1}{r} < 1$$

holds in [6]. This implies that (24) is also required in [6]. □

Remark 8: Note that when

$$\left(\gamma_1 + \frac{2L\lambda_{\max}}{\lambda_{\min}} \right) (\tau_{k+1} - \tau_k) \le -\ln(r_0 \gamma_2), \qquad r_0 > 1 \tag{26}$$

holds for all k and some $r_0 > 1$ [6], we can obtain (22) as follows:

$$\begin{aligned} &\left(\gamma_1 + \frac{2L\lambda_{\max}}{\lambda_{\min}} \right) (\tau_{2k+3} - \tau_{2k+1}) \\ &\quad = \left(\gamma_1 + \frac{2L\lambda_{\max}}{\lambda_{\min}} \right) (\tau_{2k+3} - \tau_{2k+2}) \\ &\qquad + \left(\gamma_1 + \frac{2L\lambda_{\max}}{\lambda_{\min}} \right) (\tau_{2k+2} - \tau_{2k+1}) \\ &\quad \le -2\ln(r_0 \gamma_2) \\ &\quad = -\ln\left(r_0^2 \gamma_2^2 \right) \end{aligned}$$

However, (26) cannot be derived from (22) as (22) is only required along a subsequence of τ_k. Thus, Theorem 3 is less conservative than [6, Th. 3]. □

Remark 9: Condition (22) can be generalized to (27) given below. There exist a finite integer $m_0 > 0$ and an $r > 1$ such that

$$\begin{aligned} &\left(\lambda_3 + \frac{2L\lambda_{\max}}{\lambda_{\min}} \right) (\tau_{m_0(k+1)+1} - \tau_{m_0 k+1}) \\ &\quad \le -\ln(r\lambda_4^{m_0}), \quad k = 0, 1, \ldots, \quad r > 1 \end{aligned} \tag{27}$$

Similar to the choice of a Lyapunov function for a particular system, the choice of m_0 is related to the actual system considered. □

IV. Conclusion

We have considered the problem of impulsive control based on the theory of impulsive differential equations. Some sufficient conditions were derived to ensure the asymptotic stability of an impulsive differential system. The results are also applied to design an impulsive control for a class of nonlinear systems. Our method is helpful to improve the existing technologies used in chaotic secure communication systems and chaotic spread spectrum communications.

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Robust Stability of a Class of Hybrid Nonlinear Systems

Z. G. Li, Y. C. Soh, and C. Y. Wen

***Abstract*—In this note, we analyze the discrete behavior to identify all kinds of cycles of hybrid nonlinear systems and then study the continuous behavior along each kind of cycle. Based on these analysis, we construct some continuous functions to bound Lyapunov functions along all subsystems and identify a subsequence of time points where the Lyapunov functions are nonincreasing. We use these results to derive some new sufficient conditions for the robust stability of a class of hybrid nonlinear systems with polytopic uncertainties. These conditions do not require the Lyapunov functions to be nonincreasing along each subsystem nor the whole sequence of the switchings. Furthermore, they do not require the knowledge of continuous trajectory.**

***Index Terms*—Hybrid nonlinear systems, robust stability.**

I. Introduction

Recently, the stability of hybrid dynamic systems (HDS) has been studied by many researchers. Piece-wise Lyapunov functions have been applied to consider the stability of some nonimpulsive HDS in [2], [12], [8]. The Lyapunov function is required to be nonincreasing and these results can only be used to study the stability of a limited class of HDS without impulsive effects. However, HDS do have impulsive effects because of the switchings [1]. So, it is of practical and theoretical interest to consider the stability of a wider class of HDS which can compose of unstable systems with impulsive effects. Hou *et al.* [7] and Li *et al.* [9] have derived some sufficient conditions for the stability of such impulsive HDS without perturbations and the Lyapunov function is only required to be nonincreasing along a subsequence of the switchings. However, no method has been presented to identify the nonincreasing subsequence in [9]. Moreover, these results cannot be used to study the robustness of HDS because the knowledge of the continuous trajectory is needed.

In this note, we consider the robust stability of a class of infinite switching hybrid nonlinear systems (HNS) with polytopic uncertain-

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Z. G. Li is with the Center for Signal Processing, School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798.
Y. C. Soh and C. Y. Wen are with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798 (e-mail: EYCSOH@ntu.edu.sg).
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