REFERENCES

[1] S. M. Kang and L. O. Chua, "A global representation of multidimensional piecewise linear functions with linear partitions," *IEEE Trans. Circuits Syst.*, vol. 25, pp. 938–940, Nov. 1978.

[2] C. Kahlert and L. O. Chua, "A generalized canonical piecewise linear representation," *IEEE Trans. Circuits Syst.*, vol. 37, pp. 373–382, Mar. 1990.

[3] J.-N. Lin and R. Unbehauen, "A generalization of canonical piecewise-linear functions," *IEEE Trans. Circuits Syst.*, vol. 41, pp. 345–347, Apr. 1994.

[4] G. Guzelis and I. Goknar, "A canonical representation for piecewise affine maps and its applications to circuit analysis," *IEEE Trans. Circuits Syst.*, vol. 38, pp. 1342–1354, Nov. 1991.

[5] L. Breiman, "Hinging Hyperplanes for Regression, Classification and Function Approximation," *IEEE Trans. Inform. Theory*, vol. 39, pp. 999–1013, May 1993.

[6] J.-N. Lin and R. Unbehauen, "Canonical piecewise-linear approximations," *IEEE Trans. Circuits Syst.*, vol. 39, pp. 697–699, Aug. 1992.

The Stabilization and Synchronization of Chua's Oscillators via Impulsive Control

Z. G. Li, C. Y. Wen, Y. C. Soh, and W. X. Xie

***Abstract*—This paper considers the stabilization and synchronization of Chua's oscillators via an impulsive control with time-varying impulse intervals. Some less conservative conditions were derived in the sense that the Lyapunov function is only required to be nonincreasing along a subsequence of the switchings.**

***Index Terms*—Chua's oscillators, impulsive control, Lyapunov functions, stabilization.**

I. INTRODUCTION

A class of chaotic systems named Chua's oscillators has been widely used in chaotic secure communication systems, chaotic spread-spectrum communications [7], [16], [4], [18], [13] and some other fields [11], [5]. The dimensionless form of a Chua's oscillator is given by [3] and [2]

$$\begin{cases} \dot{x} = \alpha(y - x - f(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -\beta y - \gamma z \end{cases} \tag{1}$$

where $f(x)$ is the piecewise-linear characteristics of the Chua's diode and is given by

$$f(x) = bx + \tfrac{1}{2}(a-b)(|x+1| - |x-1|). \tag{2}$$

In (2), a and b are two constants and $a < b < 0$.

Recently, impulsive control has been widely used in the stabilization and synchronization of chaotic systems. Schweizer and Kennedy [15] and Hunt and Johnson [9] proposed two impulsive control schemes with varying impulse intervals. Due to the lack of efficient analysis tools for impulsive control systems, most impulse control schemes had been designed mainly by trial-and-error. To overcome this, Chua *et al.* [6] and Yang and Chua [17] applied the existing stability results of impulsive control systems in [1] and [14] to obtain some sufficient conditions for the stability of impulsive control systems and they also used these results to consider the stabilization and synchronization of Chua's oscillators via impulsive control. However, their results are conservative in the sense that the Lyapunov function is required to be nonincreasing along the whole sequence of the switchings. The stabilization and synchronization problems have also been considered by [12] and [8].

In this paper, we shall also consider the stabilization and synchronization of Chua's oscillators by using the results obtained in [10]. Our impulse intervals are time varying and a larger bound can be obtained. We derive some less conservative conditions in the sense that the Lyapunov function is not required to be nonincreasing along the whole sequence of the switchings as needed by [17].

The rest of this paper is organized as follows. In the following section, the stabilization of Chua's oscillators is considered. Section III studies the synchronization of Chua's oscillators. Two numerical examples are given in Section IV to illustrate the effectiveness of the main results. Finally, some concluding remarks are presented in Section V.

II. STABILIZATION OF CHUA'S OSCILLATORS

In this section, we shall first present an existing result for the stability of the following impulsive control systems [14].

$$\begin{cases} \dot{X}(t) = AX(t) + \phi(X(t)), & t \neq \tau_k \\ \Delta X(t) = U(k, X(t)) = B_k X(t), & t = \tau_k,\ k = 1, 2, \ldots \\ X(t_0^+) = X_0, & t_0 < \tau_1 \end{cases} \tag{3}$$

where

$X \in R^n$	state vector;
$A, B_k (k = 1, 2, \ldots)$	$n \times n$ constant matrices;
$\phi: R^n \to R^n$	nonlinear function satisfying $\|\phi(X)\| \leq L\|X\|$;
L	positive number;

$$t_0 < \tau_1 < \tau_2 < \cdots < \tau_k < \tau_{k+1} < \cdot,\ \tau_k \to \infty \quad \text{as} \quad k \to \infty. \tag{4}$$

For matrices A and B ($A, B \in R^{n\times n}$), $A \geq B$ implies that $A - B$ is a nonnegative definite matrix.

Theorem 1 [10]: Suppose that an $n \times n$ matrix Γ is symmetric and positive definite, and $\lambda_{\min}$ and $\lambda_{\max}$, are, respectively, the smallest and the largest eigenvalues of Γ. Let

$$Q = \Gamma A + A^T \Gamma \tag{5}$$

and $Q \leq \gamma_1 \Gamma$ with γ_1 being a constant. Then, the origin of impulsive control system (3) is asymptotically stable if the following conditions hold:

1) The following inequality is true:

$$(I + B_k)^T \Gamma (I + B_k) \leq \gamma_2(k) \Gamma \tag{6}$$

where I is the identity matrix, $\gamma_2(k)$ $(k = 1, 2, \ldots)$ are positive constants.

2) There exists an $r > 1$ such that

$$\left(\gamma_1 + \frac{2L\lambda_{\max}}{\lambda_{\min}}\right)(\tau_{2k+3} - \tau_{2k+1}) \leq -\ln(r\gamma_2(2k+2)\gamma_2(2k+1)), \qquad k = 0, 1, \ldots. \tag{7}$$

Manuscript received February 26, 2000; revised November 1, 2000. This paper was recommended by Associate Editor M. Gilli.

Z. G. Li is with the Center for Signal Processing Nanyang Technological University, Nanyang Avenue, Singapore 639798.

C. Y. Wen, Y. C. Soh, and W. X. Xie are with the School of Electrical and Electronic Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798 (e-mail: ECYWEN@ntu.edu.sg).

Publisher Item Identifier S 1057-7122(01)09649-0.

3) The following inequalities hold:

$$\gamma_1 + \frac{2L\lambda_{\max}}{\lambda_{\min}} \geq 0 \tag{8}$$

$$\sup_i \left(\gamma_2(i) \exp\left(\left(\gamma_1 + \frac{2L\lambda_{\max}}{\lambda_{\min}} \right) (\tau_{i+1} - \tau_i) \right) \right) = \epsilon_1 < \infty. \tag{9}$$

We shall now consider the stabilization of Chua's oscillator (1). Let $X^T = (x, y, z)$, then we can rewrite (1) into the form

$$\dot{X} = AX + \psi(X) \tag{10}$$

where

$$A = \begin{bmatrix} -\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & -\gamma \end{bmatrix} \tag{11}$$

$$\psi(X) = \begin{bmatrix} -\alpha f(X) \\ 0 \\ 0 \end{bmatrix}. \tag{12}$$

Introducing the following impulsive control:

$$U(k, X(t)) = BX, \qquad t = \tau_k, \quad k = 1, 2, \ldots \tag{13}$$

where B is a symmetric matrix satisfying $\rho(I+B) < 1$ and $\rho(I+B)$ denotes the spectral radius of matrix $(I+B)$, $\tau_k (k = 1, 2, \ldots)$ are time varying and

$$\tau_{2i+1} - \tau_{2i} \leq \epsilon_1 (\tau_{2i} - \tau_{2i-1}). \tag{14}$$

In (14), ϵ_1 is a given positive constant. Denote

$$\Delta_1 = \sup\{\tau_{2i} - \tau_{2i-1}\} < \infty \tag{15}$$

$$\Delta_2 = \sup\{\tau_{2i+1} - \tau_{2i}\} < \infty. \tag{16}$$

Then, we have

Theorem 2: The origin of Chua's oscillator (1) under impulsive control (13) and (14) is asymptotically stable if

$$0 \leq \upsilon + 2|\alpha a| \leq -\frac{1}{(1+\epsilon_1)\Delta_1} \ln(\xi\, d_1^2) \tag{17}$$

where $\xi > 1$, υ is the largest eigenvalue of $(A + A^T)$ and

$$d_1 = \rho^2(I+B).$$

Proof: Choose $\Gamma = I$. Note that $\gamma_2(i) = d_1$ and

$$\|\psi(X)\| \leq |a\alpha|\, \|X\|$$
$$\sup\{\gamma_2(i) \exp((\upsilon + 2|a\alpha|)(\tau_{i+1} - \tau_i))\}$$
$$\leq \exp((\upsilon + 2|a\alpha|) \max\{\Delta_1, \Delta_2\}).$$

Moreover

$$\begin{aligned}
&(\upsilon + 2|a\alpha|)(\tau_{2k+3} - \tau_{2k+1}) \\
&\quad = (\upsilon + 2|a\alpha|)[(\tau_{2k+3} - \tau_{2k+2}) + (\tau_{2k+2} - \tau_{2k+1})] \\
&\quad \leq (\upsilon + 2|a\alpha|)(\Delta_1 + \Delta_2) \\
&\quad \leq (\upsilon + 2|a\alpha|)(1+\epsilon_1)\Delta_1.
\end{aligned}$$

From (17), we have

$$(\upsilon + 2|a\alpha|)(\tau_{2k+3} - \tau_{2k+1}) \leq -\ln(\xi\, d_1^2).$$

Using Theorem 1, we know that the origin of Chua's oscillator under impulse control (13) and (14) is asymptotically stable. □

Remark 1: Consider the case that $\epsilon_1 < 1$. For any $\xi > 1$ satisfying $\xi\, d_1 \leq 1$, which is required by [17], we choose that

$$\Delta_1 = -\frac{2\ln(\xi\, d_1)}{(1+\epsilon_1)(\upsilon + 2|a\alpha|)}. \tag{18}$$

It can be shown that (17) holds. Thus, the origin of Chua's oscillator under impulsive control (13) and (14) is asymptotically stable. Note that Δ_1 is greater than the upper bound $\Delta_{\max}$ defined in [17, (39)]. Thus, a larger bound can be obtained by using the proposed approach here. □

Remark 2: Let the Lyapunov function be $V(t, X) = X^T X$. It can be easily shown that the Lyapunov function is only required to be nonincreasing along an odd or an even subsequence of the switchings. This relaxes the requirement in [17] that the Lyapunov function is required to be nonincreasing along the whole switching sequence. □

III. Synchronization of Chua's Oscillators

In this section, we study the impulsive synchronization of two Chua's oscillators, which are called the driven system and the driving system, respectively [17]. In an impulsive synchronization configuration, the driving system is given by (1), whereas the driven system is given by

$$\dot{\tilde{X}} = A\tilde{X} + \psi(\tilde{X}) \tag{19}$$

where $\tilde{X} = (\tilde{x}, \tilde{y}, \tilde{z})^T$ is the state variables of the driven system and A and ψ are defined in (11) and (12).

At discrete instants $\tau_i (i = 1, 2, \ldots)$ defined in (14), the state variables of the driving system are transmitted to the driven system and then the state variables of the driven system are subject to jumps at these instants. In this sense, the driven system is modeled by the following impulsive equations:

$$\begin{cases} \dot{\tilde{X}} = A\tilde{X} + \psi(\tilde{X}), & t \neq \tau_i \\ \Delta\tilde{X}|_{t=\tau_i} = -Be, & i = 1, 2, \ldots \end{cases} \tag{20}$$

where B is a 3×3 symmetric matrix satisfying $\rho(I+B) < 1$, and $e^T = (e_x, e_y, e_z) = (x - \tilde{x}, y - \tilde{y}, z - \tilde{z})$ is the synchronization error. Let

$$\Psi(X, \tilde{X}) = \psi(X) - \psi(\tilde{X}) = \begin{bmatrix} -\alpha f(x) + \alpha f(\tilde{x}) \\ 0 \\ 0 \end{bmatrix} \tag{21}$$

then the error system of the impulsive synchronization is given by

$$\begin{cases} \dot{e} = Ae + \Psi(X, \tilde{X}), & t \neq \tau_i \\ \Delta e|_{t=\tau_i} = Be, & i = 1, 2, \ldots. \end{cases} \tag{22}$$

Similar to the stabilization of Chua's oscillators, we can obtain the following result.

Theorem 3: The impulsive synchronization of two Chua's oscillators, given in (22), is asymptotically stable if (17) holds.

Proof: Similar to the proof of Theorem 2, we can prove this result by using the following inequality:

$$\|\psi(X) - \psi(\tilde{X})\| \leq |a\alpha|\, \|e\|.$$

□

Remark 3: Impulsive control (13) and (14) can be generalized as follows:

There exists a finite positive integer m_0 such that

$$\begin{aligned}
&\tau_{l+1+km_0} - \tau_{l+km_0} \\
&\quad \leq \epsilon_l \left(\tau_{1+(k+1)m_0} - \tau_{(k+1)m_0}\right), \\
&\qquad 1 \leq l \leq m_0 - 1 \quad k = 0, 1, 2, \ldots.
\end{aligned} \tag{23}$$

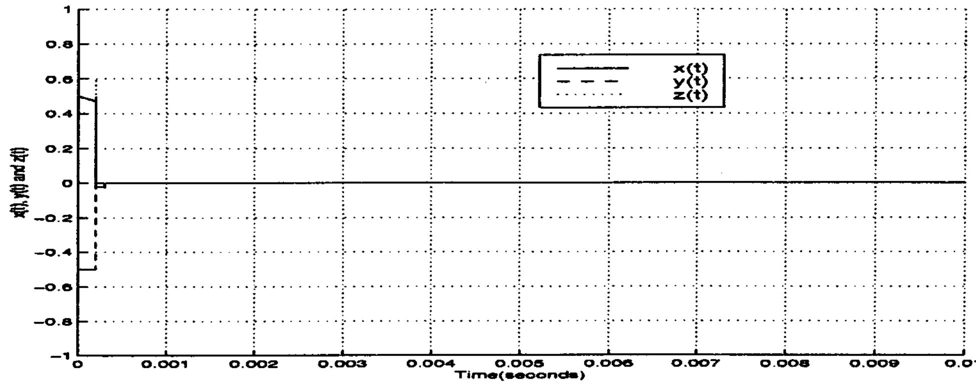

Fig. 1. The time responses $X(t) = [x(t)\ y(t)\ z(t)]^T$ of the system.

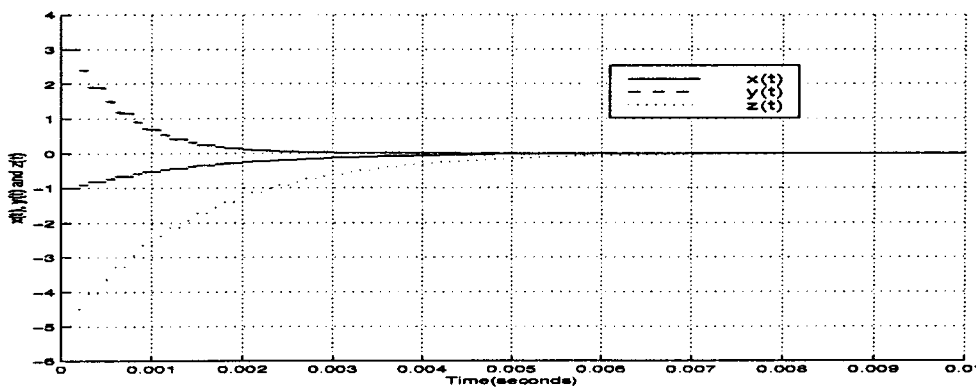

Fig. 2. The time responses $X(t) = [x(t)\ y(t)\ z(t)]^T$ of the system.

Equation (17) can be extended as follows:

$$0 \le 1 + 2|a\alpha| \le -\frac{\ln(\xi\, d_1^{m_0})}{\left(1 + \sum_{l=1}^{m_0-1} \epsilon_l\right)} \tag{24}$$

where $\xi > 1$ and $\xi\, d_1^{m_0} \le 1$. □

Remark 4: Denote

$$\Delta_l = \sup_k \{\tau_{l+1+km_0} - \tau_{l+km_0}\}, \qquad 1 \le l \le m_0. \tag{25}$$

For any ϵ_l $(1 \le l \le m_0 - 1)$ satisfying $0 < \epsilon_l < 1$ and ξ satisfying $\xi > 1$ and $\xi\, d_1 \le 1$, an upper bound of Δ_{m_0} can be obtained as follows:

$$\Delta_{m_0} \le \frac{m_0 \ln(\xi\, d_1)}{\left(1 + \sum_{i=1}^{m_0-1} \epsilon_l\right)(1 + 2|a\alpha|)}. \tag{26}$$

Obviously, Δ_{m_0} is greater than $\Delta_{\max}$ obtained in [17]. □

IV. Illustrative Examples

Two examples will be used to illustrate the effectiveness of the obtained results.

Example 1: We shall consider the stabilization of Chua's oscillators. Choose the parameters of Chua's oscillators as $\alpha = 15$, $\beta = 20$, $\gamma = 0.5$, $a = -120/7$ and $b = -75/7$. These parameters are the same as those in [17].

Similar to [17], we consider the following two cases.

Strong Control: We choose $\epsilon_1 = 0.5$, $m_0 = 2$ and matrix B as

$$B = \begin{bmatrix} \theta & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{27}$$

where θ is a constant satisfying $-2 < \theta < 0$. It is easy to show that

$$d_1 = (\theta + 1)^2, \qquad \upsilon = 20.162\,18.$$

For any ξ satisfying $\xi > 1$ and $0 < \xi(\theta + 1)^2 \le 1$, we choose that

$$\Delta_1 = -\frac{2(\ln(\xi) + 2\ln|\theta + 1|)}{1.5(\upsilon + 2|a\alpha|)} \tag{28}$$

$$\Delta_2 = -\frac{\ln(\xi) + 2\ln|\theta + 1|}{1.5(\upsilon + 2|a\alpha|)}. \tag{29}$$

Note that

$$\begin{aligned} &\int_{\tau_2}^{\tau_3} (\upsilon + 2|a\alpha|)\, dt + \ln\left(\xi(\theta + 1)^2\right) \\ &\quad = (\upsilon + 2|a\alpha|)\Delta_1 + \ln\left(\xi(\theta + 1)^2\right) \\ &\quad = -1/3 \ln\left(\xi(\theta + 1)^2\right) > 0. \end{aligned}$$

Thus, [17, Theorem 3] cannot be used to study this example. However, the conditions of Theorem 2 hold. Using Theorem 2, we know that the origin of Chua's oscillators is asymptotically stable.

Consider a special case with $\theta = -1.05$ and $\xi_0 = 300$. It can be shown that

$$\Delta_1 = 7.1771 \times 10^{-4} \quad \text{and} \quad \Delta_2 = 3.5885 \times 10^{-4}.$$

The simulation results with $\Delta_1 = 6 \times 10^{-4}$ and $\Delta_2 = 3 \times 10^{-4}$ are given in Fig. 1. It is shown that the state $X(t)$ approaches the origin very fast under the strong control.

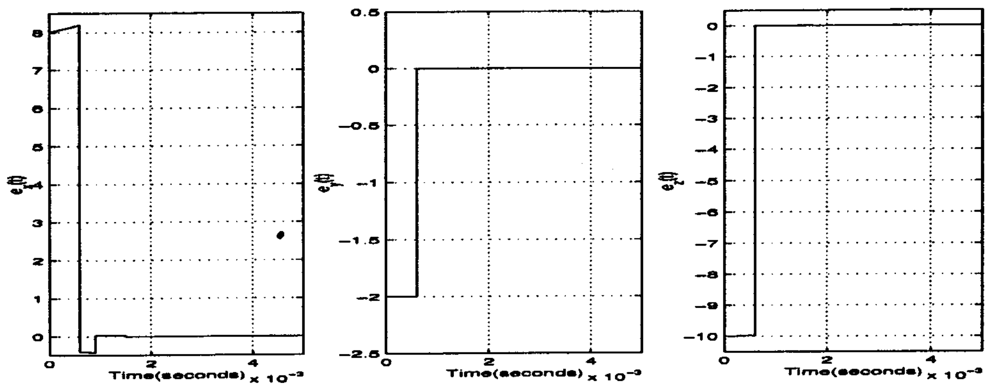

Fig. 3. The time responses $X(t) - \tilde{X}(t)$ of the system under strong control.

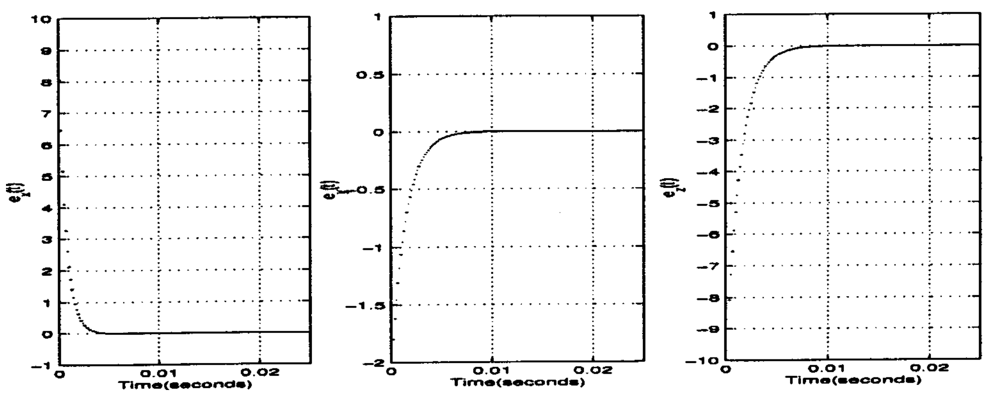

Fig. 4. The time responses $X(t) - \tilde{X}(t)$ of the system under weak control.

Weak Control: We choose $\epsilon_1 = 0.5$ and matrix B as

$$B = \begin{bmatrix} \theta & 0 & 0 \\ 0 & -0.1 & 0 \\ 0 & 0 & -0.1 \end{bmatrix} \tag{30}$$

where θ is a constant satisfying $-2 < \theta < 0$. It is easy to show that

$$d_1 = \begin{cases} (\theta+1)^2, & \text{if } (\theta+1)^2 > 0.81 \\ 0.81, & \text{otherwise.} \end{cases}$$

For any ξ satisfying $\xi > 1$ and $0 < \xi \max\{(\theta+1)^2, 0.81\} \leq 1$, we choose that

$$\Delta_1 = -\frac{2(\ln(\xi) + \ln(d_1))}{1.5(\upsilon + 2|a\alpha|)} \tag{31}$$

$$\Delta_2 = -\frac{\ln(\xi) + \ln(d_1)}{1.5(\upsilon + 2|a\alpha|)}. \tag{32}$$

Similarly, it can be shown that $\int_{\tau_2}^{\tau_3} (\upsilon + 2|a\alpha|)\, dt + \ln(\xi d_1) > 0$. Thus, [17, Theorem 3] cannot be used here. But using Theorem 2, we know that the origin of Chua's oscillators is asymptotically stable.

Also consider a special case with $\theta = -0.2$ and $\xi = 1.1$. It can be shown that

$$\Delta_1 = 2.8793 \times 10^{-4} \quad \text{and} \quad \Delta_2 = 1.4396 \times 10^{-4}.$$

The simulation results with $\Delta_1 = 2 \times 10^{-4}$ and $\Delta_2 = 1 \times 10^{-4}$ in this case are illustrated as in Fig. 2. Obviously, the convergence speed under the strong control is faster than that under the weak control.

Example 2: We consider the synchronization of Chua's oscillators with the parameters being the same as those in our first example. The initial state of two Chua's oscillators are given as follows:

$$X = [10,\ 20,\ 10]^T \quad \tilde{X} = [15,\ 10,\ 5]^T.$$

Similar to the stabilization problem, we shall also consider both the strong control and the weak control. The simulation results are given in Figs. 3 and 4. It is also shown that the convergence speed under the strong control is faster than that under the weak control.

V. Conclusion

In this brief, we have presented an impulsive control with time varying impulse intervals for the stabilization and synchronization of Chua's oscillators. Some less conservative conditions were derived in the sense that the Lyapunov function is only required to be nonincreasing along a subsequence of the switchings.

References

[1] D. D. Bainov and P. S. Simeonov, *Systems With Impulse Effect: Stability, Theory and Applications*. New York: Halsted, 1989.
[2] L. O. Chua, "The genesis of Chua's circuit," *Arch. Elektron. Ubertragung.*, vol. 46, no. 3, pp. 250–257, 1992.
[3] ——, "Global unfolding of Chua's circuit," *IEICE Trans. Fund.*, vol. 76, no. 9, pp. 704–734, 1993.
[4] ——, "Chua's circuit—An overview ten years later," *J. Circuit Syst. Comput.*, vol. 4, no. 1, pp. 117–159, 1994.
[5] L. O. Chua, C. A. Desor, and E. S. Kuh, *Linear and Nonlinear Circuits*. New York: McGraw-Hill, 1987.
[6] L. O. Chua, M. Itoh, L. Kocarev, and K. Eckert, "Chaos synchronization in Chua's circuit," *J. Circuit Syst. Comput.*, vol. 3, no. 1, pp. 93–108, 1993.
[7] K. Halle, C. W. Wu, M. Itoh, and L. O. Chua, "Spread spectrum communication through modulation of chaos," *IEICE Trans. Commun.*, vol. 3, no. 2, pp. 469–477, 1993.
[8] B. Hunt, E. Ott, and A. Yorke, "Differentiable generalized synchronism of chaos," *Phys. View Lett.*, vol. 55, no. 21, pp. 4029–4033, 1997.
[9] E. R. Hunt and G. Johnson, "Keeping chaos at bay," *IEEE Spectrum*, vol. 30, pp. 32–36, Nov. 1993.

[10] Z. G. Li, C. Y. Wen, and Y. C. Soh, "Analysis and design of impulsive control systems," *IEEE Trans. Automat. Contr.*, vol. 46, pp. 894–899, June 2001.
[11] R. Madan, *Special Issue on Chua's Circuit: A Paradigm for Chaos, Part I, Vol. 3 J. Circuit, Systems, and Computers*. Singapore: World Scientific, 1993.
[12] E. Ott, C. Grebogi, and J. A. Yorke, "Controlling chaos," *Phys. View Lett.*, vol. 64, no. 11, pp. 1196–1199, 1990.
[13] U. Parlitz, L. O. Chua, L. Kocarev, K. S. Halle, and A. Shang, "Transmission of digital signals by chaotic synchronization," *Int. J. Bifurcation Chaos*, vol. 2, no. 4, pp. 973–977, 1992.
[14] A. M. Samoilcnko and N. A. Perestyuk, *Impulsive Differential Equations*. Singapore: World Scientific, 1995.
[15] J. Schweizer and M. P. Kennedy, "Predictive poincare control: A control theory for chaotic systems," *Phys. Rev. E.*, vol. 52, no. 5, pp. 4865–4876, 1995.
[16] C. W. Wu and L. O. Chua, "A simple way to synchronize chaotic systems with applications to secure communication systems," *Int. J. Bifurcation Chaos*, vol. 3, no. 6, pp. 1619–1627, 1993.
[17] T. Yang and L. O. Chua, "Impulsive stabilization for control and synchronization of chaotic systems: Theory and application to secure communication," *IEEE Trans. Circuits Syst. I*, vol. 44, pp. 976–988, Oct. 1997.
[18] T. Yang, C. W. Wu, and L. O. Chua, "Cryptography based on chaotic systems," *IEEE Trans. Circuits Syst. I*, vol. 44, pp. 469–472, May 1997.

Novel Robust Stability Criteria for Interval-Delayed Hopfield Neural Networks

Xiaofeng Liao, Kwok-Wo Wong, Zhongfu Wu, and Guanrong Chen

***Abstract*—In this paper, some novel criteria for the global robust stability of a class of interval Hopfield neural networks with constant delays are given. Based on several new Lyapunov functionals, delay-independent criteria are provided to guarantee the global robust stability of such systems. For conventional Hopfield neural networks with constant delays, some new criteria for their global asymptotic stability are also easily obtained. All the results obtained are generalizations of some recent results reported in the literature for neural networks with constant delays. Numerical examples are also given to show the correctness of our analysis.**

***Index Terms*—Interval Hopfield neural networks, Lyapunov functionals, robust stability, time delays.**

I. Introduction

It is well-known that time delays are frequently encountered in practical information processing systems. They are very often the source of oscillation and instability in neural networks [7]. In view of this, the stability issue of time-delay neural networks is a topic of great practical importance, which has gained increasing interest in the potential applications in signal processing, image processing and other fields [13]–[16], [18], [1919].

Recently, a number of global stability conditions for neural networks with constant delays have been proposed (see [2]–[10], [13]–[16]). However, the stability of a neural network may often be destroyed by its unavoidable uncertainty due to the existence of modeling errors, external disturbance and parameter fluctuation during the implementation on very-large-scale-integration (VLSI) chips. Thus, it is important to investigate the stability and robustness of the network against such errors and fluctuation. In order to overcome thisdifficulty, Liao and Yu [8] have extended the model of delayed Hopfield neural networks to interval dynamical systems with time delays. The resultant networks are interval delayed Hopfield neural networks (IDHNN). Some robust stability criteria for these systems have been derived. The main objective of this paper is to investigate some novel conditions for the robust stability of IDHNN systems by means of constructing several Lyapunov functionals. In fact, the delay-independent robust stability conditions derived in this paper generalize the delay-independent results obtained from analyzing IDHNN with constant delays as reported in [8].

The organization of the remaining part of this paper is as follows. In the next section, we introduce the interval delayed Hopfield neural networks (IDHNN) to take account for the time delay in axonal signal transmission. In Section III, the interval dynamic approach [11] is employed to analyze the existence and uniqueness of the equilibrium point. By constructing suitable Lyapunov functionals differed from those reported in literature [3]–[10], [14], [15], the robust stability of HNN with constant delays is analyzed in detail for IDHNN. As the monotonicity or smoothness conditions of the activation functions are released, the stability requirements are not as strict as those reported in [8]. In particular, the criteria found are independent of the magnitude of the time delays. Moreover, our analysis does not require the symmetry of interconnection weight matrix. In addition, numerical examples are illustrated to support the results of the analysis. Details can be found in Section IV. Finally, conclusions are drawn in Section V.

II. Preliminaries

In this paper, we consider the following Hopfield neural networks with time delay:

$$\frac{du_i(t)}{dt} = -a_i u_i(t) + \sum_{j=1}^{n} w_{ij} f_j(u_j(t)) + \sum_{j=1}^{n} w_{ij}^{\tau} f_j \cdot (u_j(t-\tau_{ij}(t))) + I_i \qquad i = 1, 2, \ldots, n \tag{1}$$

where

nonnegative number a_i	neuron charging time constants;
nonnegative number τ_{ij}	axonal signal-transmission delays;
w_{ij}, w_{ij}^{τ}	weights of the neuron interconnections;
f_j	activation function of the neurons;
I_i	external constant inputs.

The usual assumption is that the activation functions are continuous, differentiable, monotonically increasing, and bounded, such as the sigmoid-type function. However, in this paper, we do not assume monotonicity or smoothness of the activation functions f_j, $j = 1, 2, \ldots, n$. Instead, we assume the following conditions are satisfied.

H_1) There exist constants L_j, $0 < L_j < +\infty$, $j = 1, 2, \ldots, n$ such that the incremental ratio for

$$f_j\colon R \to R$$

satisfies

$$0 \le \frac{f_j(x_j) - f_j(y_j)}{x_j - y_j} \le L_j, \qquad x_j \ne y_j.$$

Manuscript received April 5, 2000; revised January 9 2001, and July 5 2001. This work was supported in part by the post-doctoral Foundation of China and in part by the City University under Grant 7000773. This paper was recommended by Associate Editor M. J. Ogorjalek.

X. Liao is with the Department of Computer Science and Engineering, Chongqing University, Chongqing 400044, China, and also with the Department of Electronic Engineering, City University of Hong Kong, Hong Kong (e-mail: xfliao@cqu.edu.cn).

K.-W. Wong and G. Chen are with the Department of Electronic Engineering, City University of Hong Kong, Hong Kong.

Z. Wu is with the Department of Computer Science and Engineering, Chongqing University, Chongqing 400044, China.

Publisher Item Identifier S 1057-7122(01)09646-5.