Decentralized Adaptive Regulation

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***Abstract*—A new adaptive control scheme developed recently by Krstic *et al.* [11] is employed to design decentralized adaptive regulators. The interconnected system to be regulated consists of *N* coupled subsystems having arbitrarily relative degrees. Global stability is established for the closed-loop system and perfect regulation is ensured.**

I. Introduction

In the control of a large scale system, one usually faces poor knowledge on the plant parameters and interactions between subsystems. Thus the adaptive control technique in this case is an appropriate strategy to be employed. If some subsystems distribute distantly, it is difficult for a centralized controller to gather feedback signals from these subsystems. Also the design and implementation of the centralized controller are complicated. Therefore decentralized controllers, designed independently for local subsystems and using local available signals for feedback, are proposed to overcome such problems. The resulting decentralized controllers are also reliable in the sense that when some local controllers are out of order, the rest can still be in operation. Such decentralized controllers, however, should be robust against the ignored interactions.

In the context of decentralized adaptive control, only a limited number of results have been obtained by employing the traditional certainty equivalence principle to design the local adaptive controllers [1]–[9]. In [1]–[3], totally decentralized indirect adaptive controllers were designed with the use of parameter projections and global stability was established. The result in [4] was obtained by including a relative dead zone in each local estimator. To implement the dead zone, information exchange between subsystems is required. Therefore the local controllers are partially decentralized, and this result is only applicable to multivariable systems having easily obtainable feedback signals. In [5] and [6], robustness was achieved using σ-modification on the local estimators for subsystems with relative degrees $\rho_i^* \leq 2$ under the direct model reference control approach. The extension with this technique to an arbitrary relative degree case in [7] and [8], however, needs information exchange between subsystems and thus also sacrifices total decentrality. This is due to the requirement on the use of normalization and the augmented error when this traditional approach is applied to systems with relative degrees greater than two [10]. For the same reason, the result in [9] was established only for an interconnected system with all subsystems having relative degree one when output feedback is used.

Recently, a new approach using integrator backstepping was proposed to design adaptive controllers in [11]. Due to the use of nonlinear damping terms, normalization and the augmented error are not required for global stability and also better transient performance can be obtained. Based on this observation, here we apply the technique in [11] to design totally decentralized direct adaptive controllers for a class of interconnected systems with subsystems having arbitrarily relative degrees. It is shown that the global stability of the closed-loop system and perfect regulation are achieved in the presence of unmodeled interactions. A similar result on the relaxation of subsystem relative degrees was also obtained in [12] by using the concept of higher order tuning given in [13]. Again this result is due to the use of unnormalized estimation errors in the higher order estimators.

The remaining part of the paper is organized as follows. Section II gives the class of systems to be controlled, and Section III presents the decentralized controllers. The closed-loop system is analyzed and the main result is given in Section IV. Finally, the paper is concluded in Section V.

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II. Plant Models and Problem Formulation

A system consisting of N interconnected subsystems modeled as below is considered.

$$\dot{x}_{oi} = A_{o_i} x_{o_i} + b_{o_i} u_i + \sum_{j \neq i}^{N} \overline{f}_{ij}(t, y_j) \tag{1}$$

$$y_i = c_{o_i}^T x_{o_i}, \quad \text{for } i = 1, \cdots, N \tag{2}$$

where $x_{o_i} \in \Re^{n_i}$, $u_i \in \Re^1$, and $y_i \in \Re^1$ are the states, input, and output of the ith subsystem, respectively. $\overline{f}_{ij}(t, y_j) \in \Re^{n_i}$ denotes the nonlinear interactions from the jth subsystem to the ith subsystem. The matrices and vectors in (1) and (2) have appropriate dimensions, and their elements are constant but unknown. For each decoupled local system, we make the following assumptions.

Assumption 2.1:

- A1: n_i is known;
- A2: The triple $(A_{o_i}, b_{o_i}, c_{o_i})$ are completely controllable and observable;
- A3: In the transfer function

$$G_i(s) = c_{o_i}^T (sI - A_{o_i})^{-1} b_{o_i} = \frac{N_i(s)}{D_i(s)} = \frac{b_i^{m_i} s^{m_i} + \cdots + b_i^1 s + b_i^0}{s^{n_i} + a_i^{n_i - 1} s^{n_i - 1} + \cdots + a_i^1 s + a_i^0} \tag{3}$$

  $N_i(s)$ is a Hurwitz polynomial. The sign of $b_i^{m_i}$ and the relative degree $\rho_i(= n_i - m_i)$ of $G_i(s)$ are known.

For the nonlinear interaction term $\overline{f}_{ij}(t, y_j)$, we have the following assumption.

Assumption 2.2:

- A4:

$$\|\overline{f}_{ij}(t, y_j)\| \leq \overline{\gamma}_{ij} |y_j| \tag{4}$$

  where $\|\cdot\|$ denotes the Euclidean norm and $\overline{\gamma}_{ij}$ are constants denoting the strength of the interaction.

The control objective is to design totally decentralized adaptive controllers for system (1) and (2) satisfying Assumptions A1–A4 such that the closed-loop system is stable and the states $x_{o_i}(t)$ are regulated to zero.

III. Decentralized Adaptive Controllers

In this section, the decentralized adaptive controllers are presented. The local controller of each subsystem is designed by ignoring the interactions with the other subsystems. Its design procedures are the same as that in [11] for a scalar system. Therefore, in this paper, the design details are omitted. Certain manipulation on the system models is necessary, however, to aid the stability analysis of later sections.

Clearly, there exists a nonsingular matrix T_i such that, under transformation $x_{o_i} = T_i x_i$, (1) and (2) can be transformed to the form of [11], i.e.,

$$\dot{x}_i = A_i x_i + a_i y_i + \begin{bmatrix} 0 \\ b_i \end{bmatrix} u_i + f_i \tag{5}$$

$$y_i = (e_i^1)^T x_i \quad \text{for } i = 1, \cdots, N \tag{6}$$

where

$$A_i = \begin{bmatrix} 0 & & \\ \vdots & I & \\ 0 & \cdots & 0 \end{bmatrix}, \quad a_i = \begin{bmatrix} -a_i^{n_i-1} \\ \vdots \\ -a_i^0 \end{bmatrix}, \quad b_i = \begin{bmatrix} b_i^{m_i} \\ \vdots \\ b_i^0 \end{bmatrix}$$

$$f_i = \sum_{j \neq i}^{N} T_i^{-1} \overline{f}_{ij}(t, y_j) \tag{7}$$

and e_i^k denotes the kth coordinate vector in $\Re^{n_i}$.

An 'estimate' of state x_i is given by

$$\hat{x}_i = \xi_i^{n_i} - \sum_{k=0}^{n_i-1} a_i^k \xi_i^k + \sum_{k=0}^{m_i} b_i^k v_i^k \tag{8}$$

where ξ_i^k and v_i^k are generated from the following two local filters

$$\dot{\eta}_i = A_i^* \eta_i + e_i^{n_i} y_i \tag{9}$$

$$\xi_i^k = (A_i^*)^k \eta_i, \qquad 0 \le k \le n_i - 1, \tag{10}$$

$$\xi_i^{n_i} = -(A_i^*)^{n_i} \eta_i \tag{11}$$

$$\dot{\lambda}_i = A_i^* \lambda_i + e_i^{n_i} u_i \tag{12}$$

$$v_i^k = (A_i^*)^k \lambda_i, \qquad 0 \le k \le m_i \tag{13}$$

where $A_i^* = A_i - l_i (e_i^1)^T$ with $l_i = [l_i^1, \cdots, l_i^{n_i}]^T$ and is chosen to be Hurwitz. It can be shown that the 'state estimation error' $\epsilon_i = x_i - \hat{x}_i$ satisfies

$$\dot{\epsilon}_i = A_i^* \epsilon_i + f_i. \tag{14}$$

Following [11], we now define

$$\theta_i^T = [a_i^T, b_i^T]$$
$$\omega_i^T = [\xi_i^{(2)} + (e_i^1)^T y_i, v_i^{(2)}]$$
$$\overline{\omega}_i^T = [\xi_i^{(2)} + (e_i^1)^T y_i, \overline{v}_i^{(2)}]$$

where

$$\xi_i^{(2)} = [\xi_i^{n_i-1,2}, \cdots, \xi_i^{0,2}],$$
$$v_i^{(2)} = [v_i^{m_i,2}, \cdots, v_i^{0,2}]$$

and

$$\overline{v}_i^{(2)} = [0, v_i^{m_i-1,2}, \cdots, v_i^{0,2}].$$

Suppose that the positive constants γ_i, c_i^k, d_i^k, $k = 1, \cdots, \rho_i$ and the positive definite matrix Γ_i of dimension $(n_i + m_i + 1) \times (n_i + m_i + 1)$ are the design parameters. Let $p_i = b_{m_i}^{-1}$ and $\hat{p}_i$, $\hat{\theta}_i$ be the estimates of p_i and θ_i respectively. Also define $z_i^1 = y_i$ and

$$\varphi_i = c_i^1 z_i^1 + d_i^1 z_i^1 + \xi_i^{n_i,2} + \overline{\omega}_i^T \hat{\theta}_i. \tag{15}$$

For illustration of the controller design procedures, we now give a brief description on the first step.

Step 1: It can be shown that

$$\begin{aligned} \dot{z}_i^1 &= x_i^2 - a_i^{n_i-1} y_i + f_i^1 \\ &= b_i^{m_i} v_i^{m_i,2} + \xi_i^{n_i,2} + \overline{\omega}_i^T \theta_i + \epsilon_i^2 + f_i^1 \end{aligned} \tag{16}$$

where x_i^2 is the second state in x_i, ϵ_i^2 is the second element of the 'state estimation error' ϵ_i, and f_i^1 is the first element of vector f_i defined in (7).

Let $z_i^2 = v_i^{m_i,2} - \alpha_i^1$, and substitute this to (16). If $v_i^{m_i,2}$ were the actual control and all the parameters in (16) were known, α_i^1 would be chosen so that $\dot{z}_i^1 = -c_i^1 z_i^1 + \epsilon_i^2 + f_i^1$. Since this is not the case, one may think to employ the integrator backstepping technique and replace the unknown parameters with their estimates as in [11]. To do this, a Lyapunov function given below is considered

$$V_i^1 = \frac{1}{2}(z_i^1)^2 + \frac{1}{2}(\theta_i - \hat{\theta}_i)^T \Gamma_i^{-1} (\theta_i - \hat{\theta}_i) + \frac{|b_i^{m_i}|}{2\gamma_i}(p_i - \hat{p}_i)^2 + \frac{1}{\bar{\bar{d}}_i^1} \epsilon_i^T P_i^0 \epsilon_i$$

where P_i^0 is positive definite and satisfies $P_i^0 A_i^* + A_i^{*T} P_i^0 = -I$, $\bar{\bar{d}}_i^1 > 0$ and satisfies $d_i^1 = \bar{\bar{d}}_i^1 + \bar{d}_i^1$ for a positive $\bar{d}_i^1$.

Remark 3.1: Splitting d_i^1 into $\bar{d}_i^1$ and $\bar{\bar{d}}_i^1$ is just to make the presentation easier in the stability analysis of the following section. $\bar{\bar{d}}_i^1$ and $\bar{d}_i^1$ deal with the terms having ϵ_i^2 and f_i^1 in the evaluation of $\dot{V}_i$, respectively.

After calculating $\dot{V}_i^1$ as in [11], we choose

$$\alpha_i^1 = -\hat{p}_i \varphi_i$$
$$\dot{\hat{p}}_i = \gamma_i \operatorname{sgn}(b_i^{m_i}) \varphi_i z_i^1$$
$$\dot{\hat{\theta}}_i = \Gamma_i \overline{\omega}_i z_i^1.$$

With the above choices, (16) becomes

$$z_i^1 = -c_i^1 z_i^1 - d_i^1 z_i^1 + b_i^{m_i} z_i^2 + \overline{\omega}_i^T (\theta_i - \hat{\theta}_i) + b_i^{m_i} \varphi_i (p_i - \hat{p}_i) + \epsilon_i^2 + f_i^1 \tag{17}$$

and we can obtain

$$\dot{V}_i^1 \le -c_i^1 (z_i^1)^2 + b_i^{m_i} z_i^1 z_i^2 + (\theta_i - \hat{\theta}_i)^T \Gamma_i^{-1} (\Gamma_i \overline{\omega}_i z_i^1 - \dot{\hat{\theta}}_i) - \frac{1}{\bar{\bar{d}}_i^1} \Omega_i(\epsilon_i) + z_i^1 f_i^1$$

where

$$\Omega_i(\epsilon_i) = (\epsilon_i^1)^2 + \tfrac{3}{4}(\epsilon_i^2)^2 + (\epsilon_i^3)^2 + \cdots + (\epsilon_i^{n_i})^2.$$

Following Steps 2 to ρ_i in [11], the ith local adaptive controller $(i = 1, \cdots, N)$ can be designed and is given as

$$\dot{\hat{p}}_i = \gamma_i \operatorname{sgn}(b_i^{m_i}) \varphi_i z_i^1 \tag{18}$$

$$\dot{\hat{\theta}}_i = \Gamma_i \left\{ \overline{\omega}_i z_i^1 - \sum_{j=2}^{\rho_i} \frac{\partial \alpha_i^{j-1}}{\partial y_i} \omega_i z_i^j + [0, \cdots, 0, z_i^1 z_i^2, 0, \cdots, 0]^T \right\} \tag{19}$$

$$\begin{aligned} u_i = {} & -c_i^{\rho_i} z_i^{\rho_i} - d_i^{\rho_i} \left(\frac{\partial \alpha_i^{\rho_i-1}}{\partial y_i} \right)^2 z_i^{\rho_i} - z_i^{\rho_i-1} - \beta_i^{\rho_i} \\ & + \frac{\partial \alpha_i^{\rho_i-1}}{\partial y_i} \omega_i^T \hat{\theta}_i - \sum_{j=2}^{\rho_i-1} z_i^j \frac{\partial \alpha_i^{j-1}}{\partial \theta_i} \Gamma_i \frac{\partial \alpha_i^{\rho_i-1}}{\partial y_i} \omega_i \\ & + \frac{\partial \alpha_i^{\rho_i-1}}{\partial \theta_i} \Gamma_i \left\{ \overline{\omega}_i z_i^1 - \sum_{j=2}^{\rho_i} \frac{\partial \alpha_i^{j-1}}{\partial y_i} \omega_i z_i^j \right. \\ & \left. + [0, \cdots, 0, z_i^1 z_i^2, 0, \cdots, 0]^T \right\} \end{aligned} \tag{20}$$

where $z_i^1 z_i^2$ appears in the (n_i+1)th entry of the row vector and

$$\alpha_i^1 = -\hat{p}_i \varphi_i$$
$$z_i^2 = v_i^{m_i,2} - \alpha_i^1$$
$$\alpha_i^2 = -c_i^2 z_i^2 - d_i^2\left(\frac{\partial \alpha_i^1}{\partial y_i}\right)^2 z_i^2 - \beta_i^2 + \frac{\partial \alpha_i^1}{\partial y_i}\omega_i^T\hat{\theta}_i - \hat{b}_i^{m_i} z_i^1 + \frac{\partial \alpha_i^1}{\partial \theta_i}\Gamma_i\left\{\varpi_i z_i^1 - \frac{\partial \alpha_i^1}{\partial y_i}\omega_i z_i^2 + [0,\cdots,0,\, z_i^1 z_i^2,\, 0,\cdots,0]^T\right\}$$

for $3 \le k < \rho_i$

$$\alpha_i^k = -c_i^k z_i^k - d_i^k\left(\frac{\partial \alpha_i^{k-1}}{\partial y_i}\right)^2 z_i^k - z_i^{k-1} - \beta_i^k + \frac{\partial \alpha_i^{k-1}}{\partial y_i}\omega_i^T\hat{\theta}_i - \sum_{j=2}^{k-1} z_i^j \frac{\partial \alpha_i^{j-1}}{\partial \theta_i}\Gamma_i \frac{\partial \alpha_i^{k-1}}{\partial y_i}\omega_i + \frac{\partial \alpha_i^{k-1}}{\partial \theta_i}\Gamma_i\left\{\varpi_i z_i^1 - \sum_{j=2}^{k}\frac{\partial \alpha_i^{j-1}}{\partial y_i}\omega_i z_i^j + [0,\cdots,0,\, z_i^1 z_i^2,\, 0,\cdots,0]^T\right\}$$
$$z_i^{k+1} = v_i^{m_i,k+1} - \alpha_i^k$$

$\beta_i^l,\, l = 2,\cdots,\rho_i$ denotes some known terms and its detailed structure can be found in [11].

Remark 3.2: When going through the details of the design procedures, we note that in the equations concerning $\dot{z}_i^j,\, j = 1, 2,\cdots,\rho_i$, just function f_i^1 from the interactions appears and is always together with ϵ_i^2. This is because only $\dot{y}_i$ from the plant model (5) and (6) was used in the calculation of $\dot{\alpha}_i^k$ for steps $k+1,\ k = 1, 2,\cdots,\rho_i - 1$.

IV. Stability Analysis

In this section, the stability of the overall closed-loop system consisting of the interconnected plant and decentralized controllers will be established. First, a mathematical model for each local closed-loop control system is derived from (17) and the rest of the design steps.

$$\begin{bmatrix}\dot{z}_i^1\\ \dot{z}_i^2\\ \dot{z}_i^3\\ \vdots\\ \dot{z}_i^{\rho_i}\end{bmatrix} = A_i^c\begin{bmatrix}z_i^1\\ z_i^2\\ z_i^3\\ \vdots\\ z_i^{\rho_i}\end{bmatrix} + [\omega_i^T(\theta_i - \hat{\theta}_i) + \epsilon_i^2 + f_i^1]\begin{bmatrix}1\\ -\frac{\partial \alpha_i^1}{\partial y_i}\\ -\frac{\partial \alpha_i^2}{\partial y_i}\\ \vdots\\ -\frac{\partial \alpha_i^{\rho_i-1}}{\partial y_i}\end{bmatrix} + (p_i - \hat{p}_i)\begin{bmatrix}b_i^{m_i}\varphi_i\\ 0\\ 0\\ \vdots\\ 0\end{bmatrix} + (b_i^{m_i} - \hat{b}_i^{m_i})\begin{bmatrix}-v_i^{m_i,2}\\ z_i^1\\ 0\\ \vdots\\ 0\end{bmatrix} \quad (21)$$

where A_i^c is a matrix having the same structure as in the scalar system given in [11].

To show the system stability, the variables of the η-filters in (9) and the zero dynamics of subsystems should be included in the Lyapunov function. Under a similar transformation as in [11], the variables ζ_i associated with the zero dynamics of the ith subsystem can be shown to satisfy

$$\dot{\zeta}_i = A_i^{b_i}\zeta_i + \bar{b}_i z_i^1 + \overline{f}_i \quad (22)$$

where the eigenvalues of the $m_i \times m_i$ matrix $A_i^{b_i}$ are the zeros of the Hurwitz polynomial $N_i(s)$, $\bar{b}_i \in \Re^{m_i}$ and $\overline{f}_i \in \Re^{m_i}$ denoting the effects of the transformed interactions.

Now define a Lyapunov function of the overall decentralized adaptive control system as

$$V = \sum_{i=1}^{N} V_i$$

where

$$V_i = \sum_{j=1}^{\rho_i}\left(\frac{1}{2}(z_i^j)^2 + \frac{1}{\bar{\bar{d}}_i^j}\epsilon_i^T P_i^0 \epsilon_i\right) + \frac{|b_i^{m_i}|}{2\gamma_i}(p_i - \hat{p}_i)^2 + \frac{1}{2}(\theta_i - \hat{\theta}_i)^T\Gamma_i^{-1}(\theta_i - \hat{\theta}_i) + \frac{1}{k_i^{\eta_i}}\eta_i^T P_i^0 \eta_i + \frac{1}{k_i^{\zeta_i}}\zeta_i^T P_i^b \zeta_i \quad (23)$$

and $P_i^{b_i}$ satisfies $P_i^{b_i}(A_i^{b_i}) + (A_i^{b_i})^T P_i^{b_i} = -I$, $k_i^{\eta_i}, k_i^{\zeta_i}$ are constants satisfying

$$k_i^{\eta_i} \ge \frac{8\|P_i^0 e_i^{n_i}\|^2}{c_i^1}$$

$$k_i^{\zeta_i} \ge \frac{8\|P_i^{b_i}\bar{b}_i\|^2}{c_i^1}.$$

Under the same manipulation as in [11] and using (5), (14), (9) and (21)–(22), we can obtain

$$\begin{aligned}\dot{V}_i \le &-\left(\frac{c_i^1}{2}(z_i^1)^2 - \sum_{j=1}^{\rho_i}\frac{1}{4\bar{d}_i^j}(f_i^1)^2\right)\\ &- \sum_{j=1}^{\rho_i}\frac{1}{\bar{\bar{d}}_i^j}\left(\frac{3}{4}\|\epsilon_i\|^2 - \|\epsilon_i\|2\|P_i^0 f_i\|\right) - \sum_{j=2}^{\rho_i} c_i^j (z_i^j)^2\\ &- \sum_{j=1}^{\rho_i}\bar{d}_i^j\left(z_i^j\frac{\partial \alpha_i^{j-1}}{\partial y_i} + \frac{1}{2\bar{\bar{d}}_i^j}f_i^1\right)^2 - \frac{1}{2k_i^{\eta_i}}\|\eta_i\|^2\\ &- \frac{1}{2k_i^{\zeta_i}}\|\zeta_i\|^2 + \frac{2}{k_i^{\zeta_i}}\|\zeta_i\|\,\|P_i^{b_i}\overline{f}_i\|\\ \le &-\left[\frac{c_i^1}{2}(z_i^1)^2 - \|f_i\|^2\sum_{j=1}^{\rho_i}\left(\frac{1}{4\bar{d}_i^j} + \frac{2}{\bar{\bar{d}}_i^j}\|P_i^0\|^2\right)\right.\\ &\left.- \frac{4}{k_i^{\zeta_i}}\|P_i^{b_i}\|\,\|\overline{f}_i\|\right] - \sum_{j=1}^{\rho_i}\frac{1}{4\bar{\bar{d}}_i^j}\|\epsilon_i\|^2\\ &- \sum_{j=2}^{\rho_i} c_i^j (z_i^j)^2 - \sum_{j=1}^{\rho_i}\bar{d}_i^j\left(z_i^j\frac{\partial \alpha_i^{j-1}}{\partial y_i} + \frac{1}{2\bar{d}_i^j}f_i^1\right)^2\\ &- \frac{1}{2k_i^{\eta_i}}\|\eta_i\|^2 - \frac{1}{4k_i^{\zeta_i}}\|\zeta_i\|^2\end{aligned} \quad (24)$$

where the inequality $ab \le (a^2 + b^2)/2$ was used. From (4) in Assumption A4, we can show that

$$\|f_i\|^2\sum_{j=1}^{\rho_i}\left(\frac{1}{4\bar{d}_i^j} + \frac{2}{\bar{\bar{d}}_i^j}\|P_i^0\|^2\right) + \frac{4}{k_i^{\zeta_i}}\|P_i^{b_i}\|\,\|\overline{f}_i\| \le \sum_{j\ne i}^{N} d_i\gamma_{ij}|z_j^1|^2 \quad (25)$$

where γ_{ij} are constants depending on $\overline{\gamma}_{ij}$ and indicating the coupling strength from the jth subsystem to the ith subsystem, d_i, $i = 1, 2,\cdots,N$ are constants depending on $\bar{d}_i^j$, $\bar{\bar{d}}_i^j$, $\|P_i^0\|$, $k_i^{\zeta_i}$, and $\|P_i^{b_i}\|$. Now taking the summation of the first term in (24) into account and using (25), we get

$$\sum_{i=1}^{N} -\left[\frac{c_i^1}{2}(z_i^1)^2 - \|f_i\|^2\sum_{j=1}^{\rho_i}\left(\frac{1}{4\bar{d}_i^j} + \frac{2}{\bar{\bar{d}}_i^j}\|P_i^0\|^2\right) - \frac{4}{k_i^{\zeta_i}}\|P_i^{b_i}\|\,\|\overline{f}_i\|\right] \le -z_1^T S z_1 \quad (26)$$

where

$$z_1^T = [z_1^1, z_2^1, \cdots, z_N^1]$$

and

$$S = \{s_{ij}\}_{N \times N} = \begin{cases} \frac{c_i^1}{2} & i = j \\ -(d_i\gamma_{ij} + d_j\gamma_{ji}) & i \neq j. \end{cases} \quad (27)$$

Thus if matrix S is positive definite, then

$$\dot{V} = \sum_{i=1}^{N} \dot{V}_i \leq 0 \quad (28)$$

and the right side of (28) is zero only at the origin.

This proves the uniform stability and the uniform boundedness of z_i^1 (i.e., y_i), $z_i^2, \cdots, \hat{p}_i, \hat{\theta}_i, \epsilon_i, \eta_i, \zeta_i, x_i, \lambda_i$, and u_i. Following the similar argument as in [11], it can be shown that both V and $\dot{V}$ are bounded as well as $\dot{V}$ is integrable over $[0, \infty]$. Therefore, $\dot{V}$ tends to zero and thus the system states x_i converge to zero from (24) and (28).

This establishes the main result of the paper which is stated as in the following theorem.

Theorem 4.1: Consider the closed-loop decentralized adaptive control system consisting of plant (1), (2) and decentralized controllers (18)–(20). If the interactions are such that the matrix S in (27) is positive definite, then the origin is a globally uniformly stable equilibrium and all the states of the system asymptotically to zero.

Remark 4.1: From Theorem 4.1, the allowable size of the interactions defined in class (1) and (2) can be arbitrarily large by suitable selection of the controller gains c_i^1. This is consistent with the result established in [14] where a nonadaptive decentralized scheme was proposed to stabilize a similar class of interconnected systems with arbitrarily bounded interactions.

V. Conclusion

This paper provides a solution to the longstanding problem on the relaxation of subsystem relative degrees in direct decentralized adaptive control (see [5], [6], [9]) by applying the new adaptive control scheme developed for single-loop systems in [11]. It is shown that global stability is ensured, and all states in the closed-loop system are guaranteed to converge to zero. This implies that the adaptive controller in [11] has a certain degree of robustness with respect to ignored interactions between subsystems. Thus this paper partially solves the robustness problem of the controller in [11].

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'First Come, First Served' Can be Unstable!

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***Abstract*—We consider flexible manufacturing systems using the 'first come, first served' (FCFS or FIFO) scheduling policy at each machine. We describe and discuss in some detail simple deterministic examples which have adequate capacity but which, under FCFS, can exhibit instability; unboundedly growing WIP taking the form of a repeated pattern of behavior with the repetitions on an increasing scale.**

I. Introduction

We consider network models involving multiple flows with buffering/queuing at each node (processor). Specifying a queue discipline (i.e., a scheduling policy for the processing at nodes) then defines the dynamics for the system. A queue discipline is called stable if the queue lengths (WIP) remain uniformly bounded in time for any realization—configuration, initial state—with input rates subject to the obvious capacity limitations. We quote from [3] the observation that, "We have been unable to resolve whether FCFS is stable—a significant open question." It is the point of this note to resolve that question,[1] to show by examples that the popular 'first come, first

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[1] Subsequent to the original submission of this paper, we learned of related work by Bramson [1] considering a rather different configuration and demonstrating there almost sure instability in a stochastic context, i.e., with the probability 1 the total WIP has infinite liminf. A subsequent paper [2] shows that when even subject to a stronger capacity condition (replacing