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An Iterative Learning Controller with Initial State Learning

Y. Chen, C. Wen, Z. Gong, and M. Sun

***Abstract*—In iterative learning control (ILC), a common assumption is that the initial states in each repetitive operation should be inside a given ball centered at the desired initial states which may be unknown. This assumption is critical to the stability analysis, and the size of the ball will directly affect the final output trajectory tracking errors. In this paper, this assumption is removed by using an initial state learning scheme together with the traditional D-type ILC updating law. Both linear and nonlinear time-varying uncertain systems are investigated. Uniform bounds for the final tracking errors are obtained and these bounds are only dependent on the system uncertainties and disturbances, yet independent of the initial errors. Furthermore, the desired initial states can be identified through learning iterations.**

***Index Terms*—Learning control, reinitialization error, repetitive systems, tracking control, uncertainty nonlinear systems.**

I. INTRODUCTION

Learning can be regarded as a bridge between *knowledge* and *experience*. In control engineering, *knowledge* represents the modeling, environment, and related uncertainties information while *experience* is mainly from the system's repetitive operations, previous control efforts, and some resulting errors. When a system performs a given task repeatedly, we may find some new properties by employing control system theory. This was first started by Edwards and Owens [1] in which the process was called the *multipass process*, based on observations and analysis of a long-wall coal cutting process. Their main objectives were to propose the system analysis methods [1], [2]. Uchiyama [3] attempted to pursue a better control performance from the repetitive movement of the plant to be controlled. Also in 1984, Arimoto *et al.* [4], Casalino and Bartolini [5], and Craig [6] found that the performance of repetitive tasks can be improved by using the information gathered in the previous cycles. The phrase *learning* was first introduced in the control of the repetitive system or multipass process. In many practical control systems, the task is executed in a finite time interval while the same task will be repeatedly operated. In such cases, the idea of *iterative learning control* (*ILC*) is clearly applicable to improve the control performance from run to run and hence has received increasing attentions from the control community; some surveys can be found in [7]–[10]. It should be pointed out that the *repetitive control* (*RC*), [7] for example, and ILC are similar in nature [9]. However, the difference is that the ILC needs an initialization, i.e., the system should be started with the same initial condition at the beginning of each repetition, while the RC is supposed to track the periodical reference trajectory, i.e., the initial condition of current repetition is "automatically set" to the terminal condition of the previous repetition.

Robustness of ILC algorithms is an important issue in the presence of disturbances, uncertainties, and initialization errors. Arimoto *et al.* [11] presented a robustness analysis for time-varying mechanical systems with respect to initial state errors and differentiable state disturbances by using a small signal analysis method. Based on the nonlinear extension result of Hauser [12], Heinzinger *et al.* [13] analyzed robustness of ILC in terms of *stability of ILC*. The robustness of delayed nonlinear systems with a higher order ILC updating law was considered by Chen *et al.* [14]. Employing the passivity properties of nonlinear system dynamics, Arimoto [15] demonstrated the robustness of the P-type ILC algorithm, which was generalized in [16] and [17]. All ILC analyses mentioned above can guarantee the boundedness of the final tracking errors. However, these error bounds are not only directly related to the bounds of uncertainties and disturbances but also directly related to the initialization error bounds due to the unknown desired initial states. The concept of *closed-loop ILC* was proposed to employ the current iteration tracking error (CITE) error in the ILC updating law [17]–[19]. With this, we do have a measure to reduce the effect of initialization error on the final tracking error bounds by increasing the learning gain of CITE. This was achieved at the expense of using a high gain control. Also, one critical question raised is how the first (initial) point iteratively learns because the ILC is in fact a *point-wise* scheme as explained in [9]. Under the assumption that the input transmission term appears in the system's output equation, for example the system model used for ILC convergence analysis [20]–[22], [17], an impulsive initial input [23] could be used to compensate the output tracking error so that it could finally approach zero. But the use of an impulsive initial input is not practical.

Thus, how to totally eliminate the effect of the initialization errors on the final tracking error bounds is still an open problem. Although some efforts have been made [22], [24], [14], satisfactory results are still unavailable. In this paper, a new method employing an initial state learning scheme together with the traditional D-type ILC updating law is proposed. Both linear and nonlinear time-varying uncertain systems are considered. Through initial state learning, the desired initial states can be identified and thus the requirement that the initial state error should be inside a given ball is removed. In turn, it is shown that the bounds of the tracking errors are independent of the initial state errors. Simulation results are presented to illustrate the effectiveness of the proposed method.

II. LINEAR TIME-VARYING UNCERTAIN SYSTEMS

Consider a repetitive linear time-varying system with uncertainty and disturbance as follows:

$$\begin{cases} \dot{x}_i(t) = A(t)x_i(t) + B(t)u_i(t) + w_i(t) \\ y_i(t) = C(t)x_i(t) + v_i(t) \end{cases} \tag{1}$$

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where i denotes the ith repetitive operation of the system; $x_i(t) \in R^n$, $u_i(t) \in R^m$, and $y_i(t) \in R^r$ are the state, control input, and output of the system, respectively; $w_i(t)$, $v_i(t)$ are uncertainties or disturbances to the system; $t \in [t_0, T] \subseteq [0, T]$ is the time and t_0, T are given; and $A(t), B(t)$, and $C(t)$ are uncertain time-varying matrices with appropriate dimensions.

Given the realizable desired output trajectory $y_d(t)$, the tracking error $e_i(t)$ at ith repetition is that $e_i(t) \triangleq y_d(t) - y_i(t)$. Then the problem is formulated as follows. Starting from an arbitrary continuous initial control input $u_0(t)$ and an arbitrary initial state $x_0(t_0)$, which may be different from the unknown desired $x_d(t_0)$, obtain the next control input $u_1(t)$ and initial state $x_1(t_0)$, and the subsequent series $\{u_i(t), x_i(t_0) \mid i = 2, 3, \cdots\}$ for system (1), in such a way that when $i \to \infty$, $y_i(t) \to y_d(t)$ and $x_i(t_0) \to x_d(t_0)$. Furthermore, $y_i(t) - y_d(t)$ and $x_i(t) - x_d(t)$ are independent of the initialization error $x_i(t_0) - x_d(t_0)$.

To solve the above problem, we shall use the D-type ILC updating law [4], i.e.,

$$u_{i+1}(t) = u_i(t) + L(t)\dot{e}_i(t) \tag{2}$$

where $L(t) \in R^{m \times r}$ is a continuous learning gain matrix, together with an initial state learning algorithm given by

$$x_{i+1}(t_0) = x_i(t_0) + B(t_0)L(t_0)e_i(t_0). \tag{3}$$

For the analysis of the ILC process, the following norms are introduced in this paper:

$$\|f\| = \max_{1 \le i \le n} |f_i|, \qquad \|G\| = \max_{1 \le i \le m} \left(\sum_{j=1}^{n} |g_{ij}| \right)$$
$$\|h(t)\|_\lambda = \sup_{t \in [t_0, T]} e^{-\lambda t} \|h(t)\|, \qquad \lambda > 0$$

where $f = [f_1, \cdots, f_n]^T$ is a vector, $G = [g_{ij}] \in R^{m \times n}$ is a matrix, and $h(t)\,(t \in [t_0, T])$ is a real function. It should be noted that the λ-norm is equivalent to the infinity-norm because $\|h(\cdot)\|_\lambda \le \|h(\cdot)\|_\infty \le \|h(\cdot)\|_\lambda e^{\lambda T}$, where the infinity-norm $\|h(t)\|_\infty \triangleq \sup_{t \in [0, T]} \|h(t)\|$.

To restrict our discussion, the following assumptions are made.

A1) The uncertainty and disturbance terms $w_i(t)$ and $v_i(t)$ are bounded as follows, $\forall t \in [t_0, T]$ and $\forall i$:

$$\|w_{i+1}(t) - w_i(t)\|_\lambda \le b_w, \quad \|v_{i+1}(t) - v_i(t)\|_\lambda \le b_v.$$

A2) For $t \in [t_0, T]$, matrices $B(t)$ and $C(t)B(t)$ have full column ranks.

A3) $B(t)$ and $L(t)$ are differentiable over $[t_0, T]$. Furthermore, it is required that $L(t_0) \neq 0$, $B(t_0) \neq 0$.

Assumption A1) puts the boundedness restrictions on the differences of the uncertainties and disturbances between two successive system repetitions. A3) is a reasonable assumption which makes the initial state correction possible. In this paper, a common fundamental knowledge is that for a given bounded desired output $y_d(t)$, there exists a unique bounded input $u_d(t)$, $t \in [t_0, T]$ such that when $u(t) = u_d(t)$, the system has a unique bounded state $x_d(t)$ and $y_d(t) = C(t)x_d(t), t \in [t_0, T]$.

The general solution of state (1) can be written in the following form:

$$x_i(t) = \Phi(t, t_0)\left\{ x_i(t_0) + \int_{t_0}^{t} \Phi(t_0, \tau)[B(\tau)u_i(\tau) + w_i(\tau)]\, d\tau \right\} \tag{4}$$

where $\Phi(t, t_0)$ stands for the state transition matrix of system (1). For brevity of our discussion, in the sequel, the following notations are used:

$$\phi(t_0, t) \triangleq \frac{d}{dt}(\Phi(t_0, t)B(t)L(t)), \qquad b_\phi \triangleq \sup_{t \in [t_0, T]} \|\phi(t_0, t)\|$$
$$b_C \triangleq \sup_{t \in [t_0, T]} \|C(t)\|, \qquad b_\Phi \triangleq \sup_{t \in [t_0, T]} \|\Phi(t, t_0)\|$$
$$\varphi(t) \triangleq d(B(t)L(t))/dt, \qquad b_\varphi \triangleq \sup_{t \in [t_0, T]} \|\varphi(t)\|$$
$$b_{BL} \triangleq \sup_{t \in [t_0, T]} \|B(t)L(t)\|.$$

Theorem II.1: For the repetitive linear time-varying uncertain system (1) with Assumptions A1)–A3), given the desired trajectory $y_d(t)$ over the fixed time interval $[t_0, T]$, by using the ILC updating law (2) and the initial state learning formula (3), then, the λ-norm of the output tracking error is bounded, if

$$\|I_r - C(t)B(t)L(t)\| < 1, \qquad \forall t \in [t_0, T]. \tag{5}$$

For a sufficiently large λ

$$\lim_{i \to \infty} \|e_i(t)\|_\lambda \le \frac{b_v + O_1(\lambda^{-1})}{1 - \rho - O_2(\lambda^{-1})}, \qquad \forall t \in [t_0, T] \tag{6}$$

where

$$\rho \triangleq \sup_{t \in [t_0, T]} \|I_r - C(t)B(t)L(t)\| \tag{7}$$
$$O_1(\lambda^{-1}) \triangleq b_w b_C b_\Phi \|\Phi(t_0, t)\|_\lambda / \lambda \tag{8}$$
$$O_2(\lambda^{-1}) \triangleq b_C b_\Phi b_\phi / \lambda. \tag{9}$$

In the case that the uncertainties and disturbances in the ILC iterations tend to be the same, i.e., $b_w \to 0$ and $b_v \to 0$, we have $e_i(t) \to 0$, i.e., $y_i(t) \to y_d(t)$, and also $x_i(t) \to x_d(t), u_i(t) \to u_d(t)$ as $i \to \infty$ for all $t \in [t_0, T]$.

Proof: Using (4) together with (1)–(3), the tracking error $e_{i+1}(t)$ can be expressed as

$$\begin{aligned} e_{i+1}(t) &= y_d(t) - y_{i+1}(t) \\ &= y_d(t) - v_{i+1}(t) - C(t)\Phi(t, t_0)x_i(t_0) \\ &\quad - C(t)\Phi(t, t_0)B(t_0)L(t_0)e_i(t_0) \\ &\quad - C(t)\Phi(t, t_0)\int_{t_0}^{t} \Phi(t_0, \tau)(B(\tau)u_i(\tau) + w_i(\tau))\, d\tau \\ &\quad - C(t)\Phi(t, t_0)\int_{t_0}^{t} \Phi(t_0, \tau)B(\tau)L(\tau)\dot{e}_i(\tau)\, d\tau \\ &\quad - C(t)\Phi(t, t_0)\int_{t_0}^{t} \Phi(t_0, \tau)(w_{i+1}(\tau) - w_i(\tau))\, d\tau. \end{aligned} \tag{10}$$

Integrating the term $\dot{e}_i(\cdot)$ in (10) by parts yields

$$\begin{aligned} &-C(t)\Phi(t, t_0)\int_{t_0}^{t} \Phi(t_0, \tau)B(\tau)L(\tau)\dot{e}_i(\tau)\, d\tau \\ &\quad = -C(t)\Phi(t, t_0)\Phi(t_0, t)B(t)L(t)e_i(t) \\ &\qquad + C(t_0)\Phi(t, t_0)\Phi(t_0, t_0)B(t_0)L(t_0)e_i(t_0) \\ &\qquad + C(t)\Phi(t, t_0)\int_{t_0}^{t} \left[\frac{d}{d\tau}(\Phi(t_0, \tau)B(\tau)L(\tau))\right] e_i(\tau)\, d\tau \\ &\quad = -C(t)B(t)L(t)e_i(t) + C(t_0)\Phi(t, t_0)B(t_0)L(t_0)e_i(t_0) \\ &\qquad + C(t)\Phi(t, t_0)\int_{t_0}^{t} \phi(t_0, \tau)e_i(\tau)\, d\tau. \end{aligned} \tag{11}$$

By substituting (11) in (10), we get

$$\begin{aligned} e_{i+1}(t) &= [I_r - C(t)B(t)L(t)]e_i(t) - (v_{i+1}(t) - v_i(t)) \\ &\quad - C(t)\Phi(t, t_0)\int_{t_0}^{t} \Phi(t_0, \tau)(w_{i+1}(\tau) - w_i(\tau))\, d\tau \\ &\quad + C(t)\Phi(t, t_0)\int_{t_0}^{t} \phi(t_0, \tau)e_i(\tau)\, d\tau. \end{aligned} \tag{12}$$

Taking the norm of (12), we have

$$\|e_{i+1}(t)\| \le \rho\|e_i(t)\| + b_v + b_w b_C b_\Phi \int_{t_0}^{t} \|\Phi(t_0,\tau)\|\,d\tau + b_C b_\Phi b_\phi \int_{t_0}^{t} \|e_i(t)\|\,d\tau. \tag{13}$$

Multiplying $e^{-\lambda t}$ on both sides of (13) and then taking the λ-norm gives

$$\|e_{i+1}(t)\|_\lambda \le \rho\|e_i(t)\|_\lambda + b_v + b_w b_C b_\Phi \|\Phi(t_0,t)\|_\lambda O(\lambda^{-1}) + b_C b_\Phi b_\phi \|e_i(\tau)\|_\lambda O(\lambda^{-1}) \tag{14}$$

where

$$O(\lambda^{-1}) = \frac{1-e^{-\lambda(t-t_0)}}{\lambda} \le \frac{1}{\lambda}, \qquad \forall t \in [t_0,T]. \tag{15}$$

Referring to (8) and (9), we can simply write (14) as

$$\|e_{i+1}(t)\|_\lambda \le \bar{\rho}\|e_i(t)\|_\lambda + \varepsilon \tag{16}$$

$$\|e_i(t)\|_\lambda \le \bar{\rho}^i\|e_0(t)\|_\lambda + \frac{1-\bar{\rho}^i}{1-\bar{\rho}}\varepsilon \tag{17}$$

where

$$\bar{\rho} = \rho + O_2(\lambda^{-1}) \tag{18}$$

$$\varepsilon = b_v + O_1(\lambda^{-1}). \tag{19}$$

Clearly, $\exists\lambda^* > 0$ such that $\bar{\rho} < 1$, $\forall\lambda \ge \lambda^*$. Thus

$$\lim_{i\to\infty}\|e_i(t)\|_\lambda \le \frac{\varepsilon}{1-\bar{\rho}} = \frac{b_v + O_1(\lambda^{-1})}{1-\rho-O_2(\lambda^{-1})}. \tag{20}$$

When b_w and b_v tend to zero, as $i \to \infty$, $e_i(t) \to 0$, i.e., $y_i(t) \to y_d(t)$. Obviously, we also have $x_i(t) \to x_d(t)$, and $u_i(t) \to u_d(t)$ $\forall t \in [t_0,T]$ as $i \to \infty$. □

Remark II.1: Assumption A1) is less restrictive than the conventionally proposed one such as in [16]. In the case that at every ILC iteration the uncertainty and disturbance are all the same, i.e., they are repeatable, the final tracking error bound will be zero.

From (6), it can be seen that the initialization error has no effect on the final tracking error bound through the initial state learning scheme given in (3) together with the D-type ILC updating law (2). This property still holds for nonlinear systems by using the same ILC updating law (2) and initial state learning scheme (3).

III. Nonlinear Uncertain Systems

The repetitive nonlinear time-varying uncertain system is described by

$$\begin{cases} \dot{x}_i(t) = f(x_i(t),t) + B(t)u_i(t) + w_i(t) \\ y_i(t) = C(t)x_i(t) + v_i(t). \end{cases} \tag{21}$$

Now, with the same assumptions, notations, and definitions as in Section II if not otherwise indicated, we intend to show that with the same ILC updating law (2) and initial state learning scheme (3), a similar conclusion can be made for the above nonlinear time-varying uncertain system (21). Before presenting Theorem II.1, we need one more assumption.

A4) $f(\cdot,\cdot) : R^n \times [t_0,T] \mapsto R^n$ is a piecewise continuous function and satisfies a Lipschitz continuity condition, i.e., $\forall t \in [t_0,T]$

$$\|f(x_{i+1}(t),t) - f(x_i(t),t)\| \le k_f\|x_{i+1}(t) - x_i(t)\|$$

where $k_f > 0$ is the Lipschitz constant.

Theorem III.1: For the repetitive nonlinear time-varying uncertain system (21) with Assumptions A1)–A4), given the desired trajectory $y_d(t)$ over the fixed time interval $[t_0,T]$, by using the ILC updating law (2) and the initial state learning scheme (3), if (5) is satisfied, then the λ-norm of the output tracking error is bounded. For a sufficiently large λ

$$\lim_{i\to\infty}\|e_i(t)\|_\lambda \le \frac{2b_v + 2b_C b_w T + O_3(\lambda^{-1})}{1-\rho-O_4(\lambda^{-1})}, \qquad \forall t \in [t_0,T] \tag{22}$$

where

$$O_3(\lambda^{-1}) = \frac{b_C b_w k_f T O(\lambda^{-1})}{1-k_f O(\lambda^{-1})} \tag{23}$$

$$O_4(\lambda^{-1}) = b_C b_\varphi O(\lambda^{-1}) + \frac{b_C k_f O(\lambda^{-1})(b_\varphi O(\lambda^{-1}) + b_{BL})}{1-k_f O(\lambda^{-1})}. \tag{24}$$

If the uncertainties and disturbances of successive ILC iterations tend to be the same, i.e., $b_w \to 0$ and $b_v \to 0$, we have $e_i(t) \to 0$, i.e., $y_i(t) \to y_d(t)$, and also $x_i(t) \to x_d(t)$, $u_i(t) \to u_d(t)$ as $i \to \infty$ for all $t \in [t_0,T]$.

Proof: The idea is similar to that of the proof of Theorem II.1. The formula for the tracking error at $(i+1)$th repetition is

$$\begin{aligned} e_{i+1}(t) &= e_i(t) - (y_{i+1} - y_i(t)) \\ &= e_i(t) - C(t)(x_{i+1} - x_i(t)) - (v_{i+1} - v_i(t)). \end{aligned} \tag{25}$$

Integrating (21) gives

$$\begin{aligned} x_{i+1}(t) - x_i(t) &= B(t_0)L(t_0)e_i(t_0) \\ &\quad + \int_{t_0}^{t}(f(x_{i+1}(\tau),\tau) - f(x_i(\tau),\tau))\,d\tau \\ &\quad + \int_{t_0}^{t}(w_{i+1}(\tau) - w_i(\tau))\,d\tau + \int_{t_0}^{t} B(\tau)L(\tau)\dot{e}_i(\tau)\,d\tau \\ &= B(t)L(t)e_i(t) \\ &\quad + \int_{t_0}^{t}(f(x_{i+1}(\tau),\tau) - f(x_i(\tau),\tau))\,d\tau \\ &\quad + \int_{t_0}^{t}(w_{i+1}(\tau) - w_i(\tau))\,d\tau - \int_{t_0}^{t}\varphi(\tau)e_i(\tau)\,d\tau. \end{aligned} \tag{26}$$

Substituting (26) into (25) and taking norm yields

$$\begin{aligned} \|e_{i+1}(t)\| &\le \rho\|e_i(t)\| + b_v + b_C b_w T \\ &\quad + b_C k_f \int_{t_0}^{t}\|x_{i+1}(\tau) - x_i(\tau)\|\,d\tau \\ &\quad + b_C b_\varphi \int_{t_0}^{t}\|e_i(\tau)\|\,d\tau. \end{aligned} \tag{27}$$

Taking the λ-norm for (27), we have

$$\begin{aligned} \|e_{i+1}(t)\|_\lambda &\le \rho\|e_i(t)\|_\lambda + b_v + b_C b_w T \\ &\quad + b_C k_f\|x_{i+1}(t) - x_i(t)\|_\lambda O(\lambda^{-1}) \\ &\quad + b_C b_\varphi\|e_i(t)\|_\lambda O(\lambda^{-1}). \end{aligned} \tag{28}$$

Taking the λ-norm for (26) and assuming that λ is large enough to ensure

$$\lambda > k_f(1-e^{-\lambda T}) \tag{29}$$

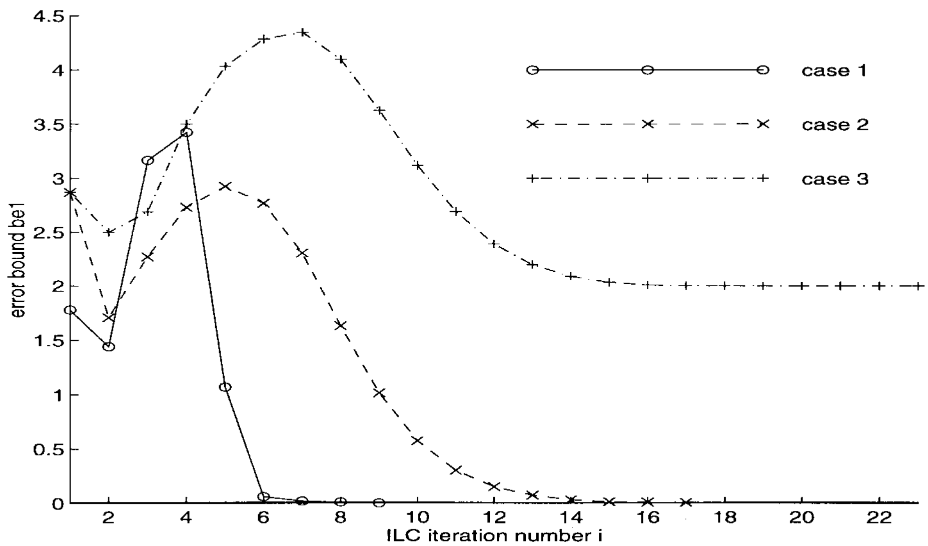

Fig. 1. The tracking error bound $b_{e_{1_i}}$.

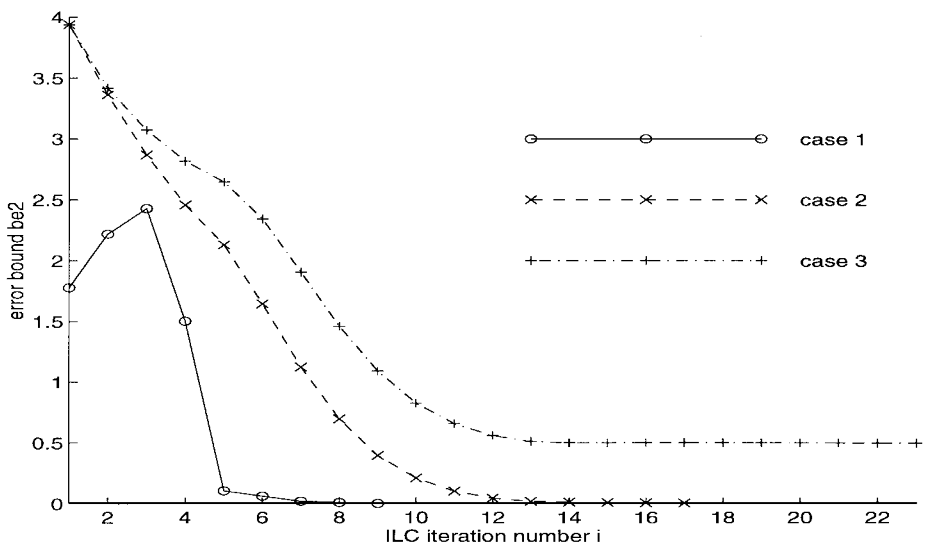

Fig. 2. The tracking error bound $b_{e_{2_i}}$.

the relationship between $\|x_{i+1}(t)-x_i(t)\|_\lambda$ and $\|e_i(t)\|_\lambda$ is given by

$$\|x_{i+1}(t)-x_i(t)\|_\lambda \leq \frac{b_w T + (b_\varphi O(\lambda^{-1}) + b_{BL})\|e_i(t)\|_\lambda}{1 - k_f O(\lambda^{-1})}. \tag{30}$$

By substituting (30) into (28), then $\|e_{i+1}(t)\|_\lambda$ can be expressed simply as

$$\|e_{i+1}(t)\|_\lambda \leq \tilde{\rho}\|e_i(t)\|_\lambda + \tilde{\varepsilon} \tag{31}$$

where

$$\tilde{\rho} = \rho + O_4(\lambda^{-1}) \tag{32}$$
$$\tilde{\varepsilon} = b_v + b_C b_w T + O_3(\lambda^{-1}). \tag{33}$$

Obviously, a sufficiently large λ that satisfies (29) and the condition $\tilde{\rho} < 1$ simultaneously exist. This completes the current proof by referring to the proof of Theorem II.1. □

IV. Simulation Illustrations

The following uncertain time-varying nonlinear system is used for the simulation studies:

$$\begin{cases} \begin{bmatrix} \dot{x}_{1_i} \\ \dot{x}_{2_i} \end{bmatrix} = \begin{bmatrix} \alpha_1 \sin(x_{2_i}) & 1 + \alpha_1 \sin(x_{1_i}) \\ -2-5t & -3-2t \end{bmatrix} \begin{bmatrix} x_{1_i} \\ x_{2_i} \end{bmatrix} \\ \qquad + \begin{bmatrix} u_{1_i} \\ 2u_{2_i} \end{bmatrix} + \begin{bmatrix} w_{1_i} \\ w_{2_i} \end{bmatrix} \\ \begin{bmatrix} y_{1_i}(t) \\ y_{2_i}(t) \end{bmatrix} = \begin{bmatrix} 4x_{1_i}(t) \\ x_{2_i}(t) \end{bmatrix} + \begin{bmatrix} v_{1_i}(t) \\ v_{2_i}(t) \end{bmatrix} \end{cases}$$

where i is the system repetition number and the time $t \in [0, 1]$. The uncertainties and output disturbances are

$$\begin{bmatrix} w_{1_i}(t) \\ w_{2_i}(t) \end{bmatrix} \triangleq \alpha_2 \begin{bmatrix} \cos(2\pi f_0 t) \\ 2\cos(4\pi f_0 t) \end{bmatrix}$$
$$\begin{bmatrix} v_{1_i}(t) \\ v_{2_i}(t) \end{bmatrix} \triangleq \alpha_2 \begin{bmatrix} \sin(2\pi f_0 t) \\ 2\sin(4\pi f_0 t) \end{bmatrix}$$

where $f_0 = 1/(20h)$ Hz. The RK-4 method is used to numerically integrate the state equation with a fixed time step $h = 0.01$ s. The

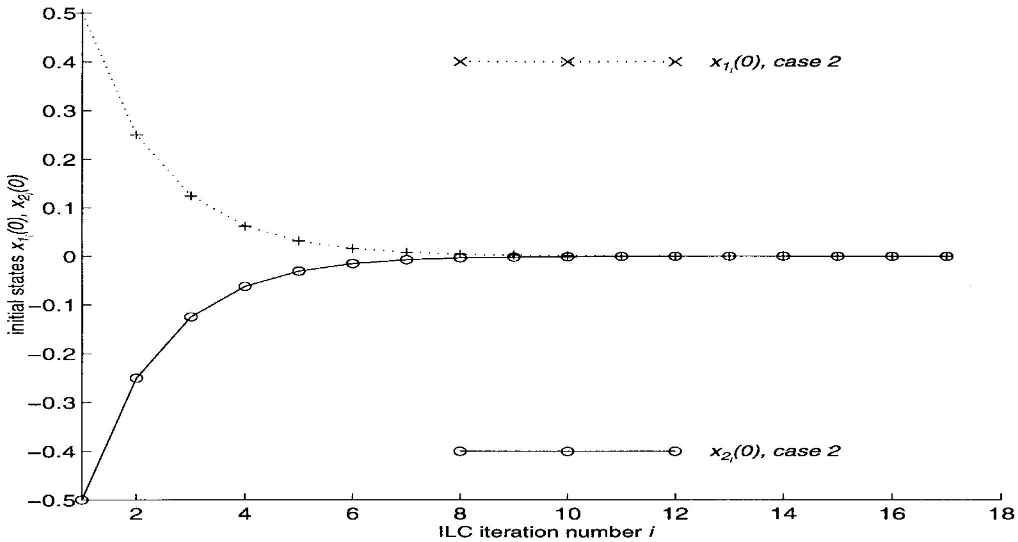

Fig. 3. The learning processes of initial states $x_{1_i}(0),\ x_{2_i}(0)$.

desired tracking trajectories are $y_{1_d}(t) = y_{2_d}(t) \triangleq 12t^2(1-t)$. Referring to (1)–(3), we know $B = \mathrm{diag}[1, 2]$ and $C = \mathrm{diag}[4, 1]$. So, the best learning matrix is $L^* = (CB)^{-1} = \mathrm{diag}[0.25, 0.5]$. However, the system dynamics is assumed unknown. It is reasonable to use the ILC updating law (2) with a modified learning gain matrix $L = \alpha_3 L^*$. And also in the initial state learning scheme (3), the best learning coefficient matrix $BL^* = \mathrm{diag}[0.25, 1]$ is replaced with a modified one $BL = \alpha_4 BL^*$. The coefficients α_3 and α_4 are freely chosen to accommodate the inaccurate knowledge of B and C. Without loss of generality, here we assume that at the first ILC iteration, the initial states are $x_{1_1}(0) = \alpha_5,\ x_{2_1}(0) = -\alpha_5$. Let $\alpha = [\alpha_1, \alpha_2, \cdots, \alpha_5]$. Clearly, when $\alpha = [0, 0, 1, 0, 0]$, the system reduces to the one considered in [4], [20]. In our simulation studies, the following three cases with different α's were examined.

- *Case 1:* $\alpha = [1, 0, 1, 0, 0]$. This is an ideal case without any initialization error, uncertainty, and disturbance.
- *Case 2:* $\alpha = [1, 1, 0.5, 0.5, 0.5]$. This implies an initialization error exists and an initial state learning scheme is applied. Some uncertainties also exist in B and C.
- *Case 3:* $\alpha = [1, 1, 0.5, 0, 0.5]$. The initial state learning scheme is switched off. The amplitudes of disturbance and uncertainty are twice of the initialization error, which are the same as in Case 2.

Let the final tracking error bound be $b_{e_{j_i}} \triangleq \sup_{t\in[0,T]} \mid e_{j_i}(t) \mid$, $j = 1, 2$. Then the simulation results are shown in Figs. 1–3. For all the three cases, the ILC termination conditions are all the same, i.e., the simulation will stop if $b_{e_{j_i}} < 0.01,\ \forall j = 1, 2$.

From Figs. 1–3, we can observe that the initial state learning scheme is effective. The initial states finally track the desired ones. In Case 3, we note that the final tracking error bounds are directly contributed by initialization errors. This can be explained from Remark II.1 because $b_v = b_w = 0$.

V. Conclusion

In this paper, an initial state learning scheme is proposed to completely eliminate the effect of the initialization errors on the final tracking error bounds through ILC with a traditional D-type updating law. Both linear and nonlinear time-varying uncertain systems have been studied. It is shown that the final tracking errors are uniformly bounded and these bounds are only dependent on the system uncertainties and disturbances, but independent of the initialization errors. Furthermore, the desired initial states can be identified through learning iterations.

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Robustness of Supervisors for Discrete-Event Systems

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***Abstract*—Supervisory control in the context of ω-languages is considered. The nominal supervisor design problem is to find a nonblocking supervisor for a nominal plant such that the closed-loop infinite behavior equals a specified closed-loop behavior. The robustness of solutions to the nominal problem is defined with respect to variations in the plant. It is shown there exists a supervisor solving the nominal problem which maximizes the set of plants for which the closed-loop languages for all other plants in the set satisfy lower and upper bounds in the sense of language containment. Computational issues are discussed, and the theoretical results are illustrated with an example.**

***Index Terms*—Discrete-event systems, robustness, supervisory control.**

I. Introduction

This paper concerns robustness of supervisors for discrete-event systems (DES's) modeled by pairs of languages corresponding to their finite and infinite (ω-language) behaviors, an extension of the finite-string language framework [4] that allows for the representation of nonterminating processes and liveliness specifications [2]–[5]. We consider the robustness of nonblocking supervisors designed for nominal plants to satisfy specifications for the nominal closed-loop infinite behavior. The objective is to design the supervisor so as to maximize (if possible) the set of plants for which: 1) the supervisor remains nonblocking and 2) the closed-loop behavior (applying the same supervisor) remains within specified lower and upper bounds.

Lin considered the problem of designing a single supervisor for a given finite collection of plants such that the closed-loop behaviors of all of the plants will satisfy the given specifications using the same supervisor [6]. In our formulation the nominal closed-loop behavior for a single nominal plant is a constraint on the admissible supervisors. We show that one can design a supervisor that maximizes the (infinite) set of plants for which the nominal specifications will be satisfied.

Due to lack of space, the proofs are not included in this paper but may be found in [8].

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II. Preliminaries

For a given finite set of events, Σ, let L, S, M denote, respectively, languages in Σ^*, Σ^ω, $\Sigma^* \cup \Sigma^\omega$; $\mathrm{pre}(M)$ denotes the set of all finite length prefixes of strings in M. L is said to be prefix closed if $\mathrm{pre}(L) = L$; $\lim\colon 2^{\Sigma^*} \to 2^{\Sigma^\omega}$ is defined for L as $\lim(L) = \mathrm{pre}^{-1}(L) \cap \Sigma^\omega$, where $\mathrm{pre}^{-1}(L) = \{\nu \in (\Sigma^* \cup \Sigma^\omega) | \mathrm{pre}(\nu) \subset L\}$; clo: $2^{\Sigma\omega} \to 2^{\Sigma\omega}$ is defined for S as $\mathrm{clo}(S) = \lim(\mathrm{pre}(S))$. S is said to be ω-closed if $\mathrm{clo}(S) = S$, and a language $T \subset \Sigma^\omega$ is said to be ω-closed w.r.t. S if $T = \mathrm{clo}(T) \cap S$. For a string $t \in \Sigma^*$ we denote the set $\{\nu \in \Sigma^* \cup \Sigma^\omega | t\nu \in M\}$ by $M/t \cdot \Sigma_L(t)$, the active set of L after $t \in L$, is defined as $\Sigma_L(t) = \Sigma \cap (\mathrm{pre}(L)/t)$. Following Thistle [5], we define a DES G as pair of languages $(L_G, S_G) \subset \Sigma^* \times \Sigma^\omega$, such that L_G is prefix-closed and $\mathrm{pre}(S_G) \subset L_G \cdot L_G$ and S_G describe, respectively, the finite and infinite logical behavior of the system. Δ^Σ denotes the set of all DES's defined over Σ.

A *control structure* is defined as a set $\Gamma \subset 2^\Sigma$ such that: 1) Γ is closed under the operation of intersection of sets and 2) if $\gamma \in \Gamma$ and $\gamma \subset \gamma' \in 2^\Sigma$, then $\gamma' \in \Gamma$. This definition corresponds to the control structure assumed in [4] where Σ is partitioned into a set Σ_c of controllable events and a set Σ_u of uncontrollable events and $\gamma \in 2^\Sigma$ is an element of Γ if and only if $\gamma \supset \Sigma_u$. A control input $\gamma \in \Gamma$ represents the set of next events allowed to occur in G. Throughout the paper we assume a control structure Γ is given.

A supervisor is defined as a map $f\colon \Sigma^* \to \Gamma$ that applies a control input γ to a DES as a function of the observed sequence of past events. f/G denotes the DES resulting from G under control of f, where $L_{f/G}$ is the set of finite strings of G that subsist under control law f, and $S_{f/G} = \lim(L_{f/G}) \cap S_G$. We say that f is a *nonblocking supervisor* for G if $\mathrm{pre}(S_{f/G}) = L_{f/G}$. We assume without loss of generality that f is a total function, which implies $\Sigma^* = \mathrm{dom}(f) \supset L_G$, i.e., f is *complete* for any $G \in \Delta^\Sigma$ [5].

Given a DES $G \in \Delta^\Sigma$, a language $R \subset \Sigma^\omega$ is *-controllable with respect to L_G if $\forall s \in R$, $\exists \gamma \in \Gamma$ such that $\gamma \cap \Sigma_{L_G}(S) = \Sigma_R(S)$; a language $T \subset \Sigma^*$ is *-controllable with respect to L_G if $\mathrm{pre}(T)$ is *-controllable with respect to L_G; T is ω-controllable with respect to G if $\mathrm{pre}(T) = \mathrm{pre}_G(T)$ where $\mathrm{pre}_G(T) = \{t \in \mathrm{pre}(T) | \exists T' \in \Sigma^\omega, T' \neq \emptyset, T'$ is *-controllable w.r.t. L_G/t and ω-closed w.r.t. S_G/t.

The following proposition summarizes some basic important results from [5] regarding supervisors.

Proposition 1 [5]: Given a DES G and a language $K \subset S_G$, the following statements are equivalent: 1) there exists a nonblocking supervisor f for G such that $S_{f/G} = K$; 2) K is *-controllable with respect L_G to and ω-closed with respect to S_G; and 3) K is ω-controllable with respect to G and ω-closed with respect to S_G.