[7] ——, "Structural stable nonlinear regulation with step inputs," *Math. Syst. Theory*, vol. 17, pp. 319–333, 1984,.

[8] ——, "The semistable-center-unstable manifold near a critical element," *J. Math. Anal. Applications*, vol. 103, no. 2, pp. 321–331, Oct. 1984.

[9] J. Huang and W. J. Rugh, "On a nonlinear multivariable servomechanism problem," *Automatica*, vol. 26, no. 6, pp. 963–972, 1990.

[10] C. I. Byrnes and A. Isidori, "Nonlinear output regulation: Remarks on robustness," in *Proc. 27th Allerton Conf. Commun., Contr. Computing*, 1989.

[11] F. Delli Priscoli and A. Isidori, "Robust tracking for a class of nonlinear systems," in *Proc. 31st Conf. Decis. Contr.*, Tucson, AZ, Dec. 1992.

[12] F. Delli Priscoli, "A possible approach for achieving robust tracking of nonlinear systems," in *Proc. European Contr. Conf.*, Grenoble, France, July 2–5, 1991, pp. 1814–1819.

[13] C. A. Desoer and C. A. Lin, "Tracking and disturbance rejection of MIMO nonlinear systems with PI controller," *IEEE Trans. Automat. Contr.*, vol. 30, no. 9, pp. 861–867, 1985.

[14] C. A. Desoer and Y. T. Wang, "Linear time-invariant robust servomechanism problem: A self-contained exposition," *Contr. Dynamic Syst.*, vol. 16, pp. 81–129, 1980.

[15] B. A. Francis, "The linear multivariable regulator problem," *SIAM J. Contr. Optim.*, vol. 15, pp. 486–505, 1977.

[16] J. Huang and W. J. Rugh, "Stabilization on zero-error manifolds and the nonlinear servomechanism problem," *IEEE Trans. Automat. Contr.*, pp. 1009–1013, July 1992.

[17] A. Isidori and C. I. Byrnes, "Output regulation of nonlinear systems," *IEEE Trans. Automat. Contr.*, vol. 35, no. 2, pp. 131–140, 1990.

[18] J. Huang and C-F. Lin, "Robust tracking and disturbance rejection in nonlinear systems," in *Proc. 33rd Conf. Decis. Contr.*, Dec. 1994, pp. 2776–2781.

Indirect Robust Totally Decentralized Adaptive Control of Continuous-Time Interconnected Systems

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Abstract—**This paper presents a global stability result on continuous-time indirect totally decentralized adaptive control systems. The algorithms employed in the design of local adaptive controllers are earlier basic conventional adaptive control algorithms subject to parameter projection operation. For the first time, it is shown that without any restriction on signals such as persistence of excitation, global boundedness of the signals in the overall continuous-time feedback system is guaranteed. For implementation, no *a priori* knowledge on the interactions and unmodeled dynamics is required.**

I. Introduction

Decentralized adaptive control of unknown interconnected systems has attracted much research effort because this problem is important both theoretically and practically. The idea employed is that the input to each local controller is only from the available measurements in that local system. Perhaps the easiest deign method of local controllers is just simply to ignore the interactions between subsystems. Therefore each local adaptive controller should be robust not only to the subsystem modeling error but also the interactions. Thus one may employ the techniques of designing robust adaptive controllers for single-loop systems. It is nontrival, however, to analyze the overall closed-loop system due to the existence of interactions, and thus the results achieved so far are still limited.

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Most decentralized adaptive controllers published in literature are designed using direct model reference adaptive control schemes [1], [2], and the problems solved using these schemes are still limited. These results are only applicable to local systems of relative degree one and two. Recently, decentralized adaptive control schemes for local systems having arbitrary relative degrees appeared in [3] and [4]. The assumptions made for the local systems, however, are exactly the same as the earlier ideal assumptions for scalar systems.

In general, indirect adaptive controllers are more flexible with respect to the choice of controller design methodology and the choice of identification scheme, but the analysis of indirect adaptive systems is more difficult and complicated. This is particularly true in the analysis of totally decentralized indirect adaptive control system. So far, stability results on truly decentralized indirect adaptive control can only be found in [5]–[7]. One common feature of these works is that only discrete-time systems were considered, and the method of analysis is through induction which cannot be easily applied to continuous-time systems. Thus the stability problem for indirect continuous-time decentralized adaptive control has not been solved to date. In this paper, this problem is considered. It is shown that the local adaptive controllers designed using the proposed scheme can counteract the instability caused by ignored system modeling errors including fast parasitics, bounded disturbances, and weak interactions. Moreover, a local adaptive controller presented can preserve results established in earlier global convergence proofs [8] when the subsystem controlled by that local controller is decoupled from the rest and the modeling errors in that subsystem disappear. In the implementation of the local controllers, no prior knowledge required is needed from the unmodeled interactions and unmodeled dynamics.

II. System Models

We consider the following m-input m-output interconnected continuous-time systems

$$y_i(t) = H_i(D)[1 + \bar{\epsilon}_i \bar{H}_i(D)]u_i + d_i(t) + \sum_{j=1}^{m}[\bar{\epsilon}_{ij}\bar{H}_{ij}u_j + \bar{\bar{\epsilon}}_{ij}\bar{\bar{H}}_{ij}y_j] \tag{1}$$

for $i, j = 1, \cdots, m$, where y_i, u_i, and d_i are the output, input, and disturbance of the ith subsystem, $H_i(D) = \frac{B_i(D)}{A_i(D)}$ and is the reduced order transfer function of subsystem i with

$$A_i(D) = D^{n_i} + a^i_{n_i-1}D^{n_i-1} + \cdots + a^i_0$$
$$B_i(D) = b^i_{m_i}D^{m_i} + b^i_{m_i-1}D^{m_i-1} + \cdots + b^i_0$$

D denotes the differentiation operator and $m_i < n_i$, $\bar{\epsilon}_i$, $\bar{\epsilon}_{ij}$, $\bar{\bar{\epsilon}}_{ij}$ are constants, $\bar{H}_i(D) = \frac{\bar{B}_i(D)}{\bar{A}_i(D)}$ and is the multiplicative uncertainty of the ith subsystem, $\bar{H}_{ij}$ and $\bar{\bar{H}}_{ij}$ denote the subsystem interactions if $i \neq j$ and unmodeled dynamics if $i = j$.

Assumption 2.1:

A1) n_i is known and the coefficients of $A_i(D)$ and $B_i(D)$ are inside a known compact convex region in which the estimated models $\frac{\hat{B}_i(D,t)}{\hat{A}_i(D,t)}$ of $\frac{B_i}{A_i}$ are uniformly controllable and observable.

A2) $\bar{H}_i(D)$ is stable and satisfies $\partial_{\bar{A}_i} - \partial_{\bar{B}_i} \geq 1 - (n_i - m_i^*)$, where m_i^* is the true degree of $B_i(D)$. $\bar{H}_{ij}, \bar{\bar{H}}_{ij}$ are (strictly) proper and stable, and $d_i(t)$ is bounded.

Remark 2.1: Note that there is no constraint on the pole locations of unmodeled dynamics and interactions except the stability requirement. While modeling errors satisfy A2), no *a priori* knowledge is required from them for the implementation of the adaptive controllers given in later sections.

Following a standard procedure as in [9], we introduce a stable filter $1/F_i$ to system (1) and obtain, after some manipulation

$$y_i(t) = (F_i - A_i)y_{if}(t) + B_i u_{if}(t) + \eta_{if} = \phi_i^T(t)\theta_*^i + \eta_{if}(t) \tag{2}$$

where

$$y_{if}(t) = \frac{1}{F_i(D)}y_i(t),$$
$$u_{if}(t) = \frac{1}{F_i(D)}u_i(t),$$
$$F_i(D) = D^{n_i} + f_{n_i-1}^i D^{n_i-1} + \cdots + f_0^i,$$
$$\phi_i(t)^T = [D^{n_i-1}y_{if}(t), \cdots, y_{if}(t), D^{n_i-1}u_{if}(t), \cdots, u_{if}(t)],$$
$$\theta_*^{i^T}(t) = [f_{n_i-1}^i - a_{n_i-1}^i, \cdots, f_0^i - a_0^i, 0, \cdots, 0, b_{m_i}^i, \cdots, b_0^i]$$

and

$$\eta_{if} = \bar{\epsilon}_i \frac{B_i \bar{B}_i}{V_i \bar{A}_i} V_i u_{if} + \frac{A_i}{F_i} d_i(t) + \sum_{j=1}^{m} \left[\bar{\epsilon}_{ij} \bar{H}_{ij} \frac{A_i}{F_i} \frac{F_j}{V_j} V_j u_{jf} + \bar{\bar{\epsilon}}_{ij} \bar{\bar{H}} \bar{\bar{H}}_{ij} \frac{A_i}{F_i} \frac{F_j}{V_j} V_j y_{jf} \right]. \tag{3}$$

In (3), $V_j(D)$ is an arbitrary Hurwitz polynomial of degree $n_j - 1$. Clearly, η_{if} satisfies

$$|\eta_{if}(t)| \leq \sum_{j=1}^{m} \epsilon_{ij} \sup_{0 \leq \tau \leq t} \|\phi_j(\tau)\| + d_0 \tag{4}$$

where d_0 is a constant uniformly bounding $\frac{A_i}{F_i} d_i(t)$ and exponentially decaying initial conditions for all $i = 1, \cdots, m$, $\epsilon_{ij} \geq 0$. Also from Assumption A2), there exists a known convex compact region $\mathcal{C}_i$ such that $\theta_*^i \in \mathcal{C}_i$.

Suppose y_i^* is a given reference set-point for output y_i. The control problem is to design a controller for the ith subsystem described by (1) under Assumption 2.1 such that the overall closed-loop system is stable in the sense that all signals in the system are bounded for arbitrary bounded y_i^* and initial conditions, and the tracking errors are small in some sense. In addition, when the ith subsystem is decoupled from the rest and its unmodeled dynamics and disturbances disappear, the ith local adaptive controller should retain the properties of earlier unmodified conventional adaptive controllers without any additional requirement. The feedback signals for the ith local controller are only local measurements in the ith subsystem.

III. Adaptive Control Scheme

An indirect adaptive control scheme is proposed to design a local controller for each subsystem in this section. The local adaptive controller consists of two modules: an parameter estimator and a linear controller designed based on certainty equivalence principle.

A. Parameter Estimator

The following estimation algorithm is introduced to the ith local estimator

$$\dot{\hat{\theta}}^i(t) = \mathcal{P}\left\{ \frac{\phi_i(t)e_i(t)}{1 + \phi_i^T(t)\phi_i(t)} \right\} \tag{5}$$

where

$$\hat{\theta}^i = [f_{n_i-1}^i - \hat{a}_{n_i-1}^i, \cdots, f_0^i - \hat{a}_0^i, 0, \cdots, 0, \hat{b}_{m_i}^i, \cdots, \hat{b}_0^i]^T$$

$\hat{a}_{n_i-1}^i, \cdots, \hat{a}_0^i, \hat{b}_{m_i}^i, \cdots, \hat{b}_0^i$ are the estimates of unknown parameters $a_{n_i-1}^i, \cdots, a_0^i, b_{m_i}^i, \cdots, b_0^i$, and $e_i(t)$ is a prediction error defined as

$$e_i(t) = y_i(t) - \phi_i^T(t)\hat{\theta}^i(t). \tag{6}$$

$\mathcal{P}\{\cdot\}$ denotes a projection operation as defined in [10]. Such an operation can ensure the estimated parameter vector $\hat{\theta}^i(t) \in \mathcal{C}_i$ for all t if $\hat{\theta}^i(0) \in \mathcal{C}_i$. Some useful properties of the estimator in (5) and (6) are summarized as follows.

Lemma 3.1: Suppose M_0 is a positive constant s.t. $\dot{d}_0/M_0 \leq \delta$ where d_0 is given in (4). The estimator (5) and (6), applied to the ith subsystem given in (1), has the following properties:

1) If at time $t_0^i, \|\phi_i(t_0^i)\| = M_0$ and for all $t \geq t_0^i$, $\sup_{0 \leq \tau \leq t} \|\phi_i(\tau)\| = \|\phi_i(t)\|$ and $\sup_{0 \leq \tau \leq t} \|\phi_j(\tau)\| \leq \|\phi_i(t)\| \forall j \neq i$, then the normalized prediction error defined as

$$\tilde{e}_i(t) = \frac{e_i(t)}{(1 + \phi_i^T(t)\phi_i(t))^{1/2}}$$

satisfies

a)

$$|\tilde{e}_i(t)| \leq k_\theta + \epsilon + \delta \text{ for } t \geq t_0^i \tag{7}$$

where $k_\theta = \max_{i \in \{1, \cdots, m\}} \{k_{\theta_i}\}$ and k_{θ_i} denotes the size of region $\mathcal{C}_i$

$$\epsilon = \max_{i \in \{1, \cdots, m\}} \sum_{j=1}^{m} \epsilon_{ij} \tag{8}$$

b)

$$\int_{t_0^i}^{t} \tilde{e}_i^2(\tau)\, d\tau \leq k + \alpha_1(t - t_0^i) + \alpha_2(t - t_0^i) \quad \text{for } t \geq t_0^i \tag{9}$$

where

$$k = \frac{1}{2}k_\theta^2, \quad \alpha_1 = (k_\theta + 2\epsilon)\epsilon, \quad \alpha_2 = (k_\theta + 2\delta)\delta \tag{10}$$

2)

$$\|\dot{\hat{\theta}}(t)\| \leq |\tilde{e}(t)|. \tag{11}$$

Proof: We only give the proof of 1-b) here. Let $\tilde{\theta}^i = \hat{\theta}^i - \theta_*^i$. Then (2) and (6) yield

$$e_i(t) = -\phi^T(t)\tilde{\theta}^i(t) + \eta_{if}(t). \tag{12}$$

Now consider the function $v_i(t) = \frac{1}{2}\tilde{\theta}^{i^T}(t)\tilde{\theta}^i(t)$. The projection operation, (5), (12), (4) and the assumptions of the lemma give that

$$\dot{v}_i(t) \leq -\tilde{e}_i^2(t) + \frac{|\eta_{if}(t)||e_i(t)|}{1 + \phi_i^T(t)\phi_i(t)} \leq -\tilde{e}_i^2 + \frac{\epsilon\|\phi_i(\tau)\| + d_0}{(1 + \phi_i^T(t)\phi_i(t))^{1/2}}\tilde{e}_i(t)$$

$$\int_{t_0^i}^{t} \tilde{e}_i^2(\tau)\, d\tau \leq -\int_{t_0^i}^{t} \dot{v}_i d\tau + (\alpha_1 + \alpha_2)\int_{t_0^i}^{t} d\tau \leq k + \alpha_1(t - t_0^i) + \alpha_2(t - t_0^i).$$

Remark 3.2:

1) In (10), α_1 can be made small by reducing ϵ, i.e., restricting the strength of interactions and unmodeled dynamics, α_2 is made small by a sufficiently large number M_0. M_0 is not a design parameter. As in [6], it is used for the purpose of analysis only and its role will be made clear later.
2) When the ith subsystem is decoupled from the rest and its unmodeled dynamics and disturbance disappear, $\eta_{if} = 0$. In this case, the properties given in Lemma 3.1 for the ith subsystem are exactly the same as those of earlier unmodified conventional estimators [8].

B. Controller Synthesis

For the module of controller synthesis we use a pole assignment strategy. The control $u_i(t)$ is given by

$$\hat{L}_i(D)u_{if}(t) = \hat{P}_i(D)(y^*_{if}(t) - y_{if}(t)) \tag{13}$$

where $y^*_i(t)$ is the setpoint and $y^*_{if} = \frac{1}{F_i}y^*_i$, $\hat{L}_i$, and $\hat{P}_i$ are polynomials in D of the form

$$\hat{L}_i(D) = D^{n_i} + l^i_{n_i-1}D^{n_i-1} + \cdots + l^i_0,$$
$$\hat{P}(D) = p^i_{n_i-1}D^{n_i-1} + \cdots + p^i_0$$

and determined from the following diophantine equation

$$\hat{A}_i(t)\hat{L}_i(t) + \hat{B}_i(t)\hat{P}_i(t) = A^*_i. \tag{14}$$

In (14), A^*_i is a monic polynomial in D of degree $2n_i$ and its zeros are chosen to be the required closed-loop poles. A guideline for choosing A^*_i can be found in [9]. $\hat{A}_i, \hat{B}_i$ are the estimates of A_i, B_i. From Assumption A1), (14) gives a bounded solution for $\hat{L}_i, \hat{P}_i, \forall t$.

Clearly, (13) gives a strictly proper control law and can be implemented as

$$u_i(t) = (F_i - \hat{L}_i)u_{if} - \hat{P}_i(y_{if} - y^*_{if}).$$

IV. Stability Analysis

In this section, we will study the adaptive system consists of the plant in Section II and the adaptive controller in Section III. An equation describing the ith loop of the closed-loop system can be obtained by combining (13) with (6)

$$D\phi_i(t) = \hat{A}^i_c\phi_i(t) + b^i_1 e_i(t) + b^i_2 r_i(t) \tag{15}$$

where

$$b^i_1 = [1, 0, \cdots, 0]^T, \quad b^i_2 = [0, \cdots, 0, 1, \cdots, 0]^T$$
$$r_i(t) = \frac{\hat{P}_i}{F_i}y^*_i(t) \tag{16}$$

and $\hat{A}^i_c$ is a matrix having the similar structure as $\bar{A}^i$ in [6].

From Lemma 3.1, we can show that $\exists c > 0, \sigma > 0$ such that the transition matrix of the homogeneous part of (15), denoted as $\Phi_i(t, \tau)$, satisfies

$$\|\Phi_i(t, \tau)\| \le ce^{-\sigma(t-\tau)} \quad \text{for } t \ge \tau \ge t^i_0 \tag{17}$$

for all $\epsilon \le \bar{\epsilon}^*, \delta \le \bar{\delta}^*$ under the assumptions of Lemma 3.1, where bounds $\bar{\epsilon}^*, \bar{\delta}^*$ are sufficiently small numbers to ensure $\alpha_1 + \alpha_2 \le \bar{\alpha}^*$. α_1, α_2 are given in (10), $\bar{\alpha}^*$ is a sufficiently number.

Now notice that for $i = 1, 2, \cdots, m$ and for any bounded initial conditions $\phi_i(0)$, set points y^*_i and disturbances $d_i(t)$, there always exists a number M_0 such that $\|\phi_i(0)\| \le M_0, \|r_i(t)\|_\infty \le M_0$ and $\frac{d_0}{M_0} \le \delta$ for a sufficiently small δ given in Lemma 3.1, where $r_i(t)$ is given by (16). In this section, such an intermediate number is used to aid our analysis. Clearly, the closed-loop system is stable if $\|\phi_i(t)\| \le M_0$ for all $i = 1, 2, \cdots, m$ and $t > 0$. Thus the only situation which can cause instability is that $\|\phi_i(t)\| \ge M_0$ for some i and $t \in \Re^+$. In this case, there must exist a time instant t^i_0 such that $\|\phi_i(t^i_0)\| = M_0$. To start the stability analysis in this situation, we now examine a special case that the trajectory $\|\phi_i(t)\|$ of the ith subsystem satisfies certain conditions.

Lemma 4.1: Suppose that for all $t \ge t^i_0$ $\sup_{0\le\tau\le t}\|\phi_i(\tau)\| = \|\phi_i(t)\|$ and $\sup_{0\le\tau\le t}\|\phi_j(\tau)\| \le \|\phi_i(t)\| \forall j \ne i$. Then consider the adaptive system consisting of interconnected continuous-time systems modeled by (1) and decentralized adaptive controllers designed in (5), (6), (13), and (14). Under Assumption 2.1, there exists a constant ϵ^*_1 such that for all $\epsilon \le \epsilon^*_1$ the closed-loop system ensures that

$$\|\phi_k(t)\| \le M \quad \forall k = 1, \cdots, m \text{ and } t \ge 0 \tag{18}$$

where $M = \sqrt{c_1 M^2_0 + c_2}$ and c_1, c_2 are constants.

Proof: By examining the general solution of (15) and following the procedures as in [6] and [9], the results can be proved after the use of Schwartz inequality, Grownwall Lemma, and Lemma 3.1. □

It is the estimator properties in Lemma 3.1 that gives the result of Lemma 4.1. These properties, however, are not sufficient to establish the global boundedness of signals in a general case. Thus we need to further explore the local parameter estimators, and this gives Lemma 4.2 as follows.

Lemma 4.2: Suppose M_0 is a positive constant s.t. $d_0/M_0 \le \delta$. The estimator (5) and (6), applied to plants given in (1), has the following additional properties.

If $\|\phi_i(t)\| \ge M_0$ for all $t \ge t^i_0$, $\|\phi_k(\tau_1)\| \le \sqrt{c_1 M^2_0 + c_2}\forall k = 1, \cdots, m$ and for $\tau_1 \in [0, t^i_1]$, also for all $t > t^i_1$ if $\sup_{0\le\tau\le t}\|\phi_i(\tau)\| = \|\phi_i(t)\|$ and $\sup_{0\le\tau\le t}\|\phi_j(\tau)\| \le \|\phi_i(t)\| \ \forall j \ne i$, then

1)
$$|\bar{e}_i(t)| \le k_\theta + \epsilon(\sqrt{c_1} + \sqrt{c_2}) + \delta \quad \text{for } t \ge t^i_0$$

and

2)
$$\int_{t^i_0}^t \bar{e}^2_i(\tau)d\tau \le k + \bar{\alpha}_1(t - t^i_0) + \alpha_2(t - t^i_0) \quad \text{for } t \ge t^i_0$$

where

$$\bar{\alpha}_1 = (k_\theta + 2(\sqrt{c_1} + \sqrt{c_2})\epsilon)(\sqrt{c_1} + \sqrt{c_2})\epsilon.$$

Proof: The proof of property 2) is presented. Note that for all $k = 1, \cdots, m$

$$\sup_{0\le\tau\le t^i_1}\|\phi_k(\tau)\| \le \sqrt{c_1 M^2_0 + c_2}.$$

From the proof of Lemma 3.1 and assumptions of Lemma 4.2, we can have

$$\begin{aligned}\int_{t^i_0}^t \bar{e}^2_i(\tau) &\le -\int_{t^i_0}^t \dot{v}_i\, d\tau + \bar{\alpha}_1\int_{t^i_0}^{t^i_1} d\tau \\ &\quad + \alpha_1\int_{t^i_1}^t d\tau + \alpha_2\int_{t^i_0}^t d\tau \\ &\le k + \bar{\alpha}_1(t - t^i_0) + \alpha_2(t - t^i_0).\end{aligned}$$

□

Remark 4.1: Note that the properties in the above lemma are quite similar to that in Lemma 3.1 except that the constants c_1 and c_2 appear here.

We can now state our main result as follows.

Theorem 4.1: Consider the adaptive system consisting of interconnected continuous-time systems modeled by (1) and decentralized adaptive controllers designed in (5), (6), (13), and (14). Under Assumption 2.1, there exists a constant ϵ^* such that for all $\epsilon \leq \epsilon^*$ the closed-loop system is globally stable in the sense that $y_i(t)$ and $u_i(t) i = 1, \cdots, m$ as well as all the states in the system are bounded for all $t \geq 0$ and for all finite initial states, any bounded y_i^* and arbitrarily bounded external disturbances.

Proof: We first outline some motivation steps to clarify the development of the proof.

1) Overall, we try to establish a uniform bound M for all the trajectories $\phi_k(t)$, $k = 1, 2, \cdots, m$ progressively in time starting from $t = 0$. This is in contrast with some other approaches where a subsystem trajectory is studied over the whole time period $[0, \infty)$. The flow of the proof is to use Lemma 4.1 to derive the bound M for all the subsystem trajectories in the beginning period and then to apply Lemma 4.2 to propagate the same bound M also for all the trajectories for the future time in an interval by interval basis. Fig. 1 is used to aid our analysis.
2) From the compactness of the integer set $\{1, 2, \cdots, m\}$ and the continuity of $\|\phi_k(t)\|, k = 1, 2, \cdots, m$, there always exists an integer $l \in \{1, 2, \cdots, m\}$ such that
$$\|\phi_l(t)\| = \max_{\{1,2,\cdots,m\}} \|\phi_i(t)\| \tag{19}$$
over a time interval. To establish the bound M for all the trajectories, we study a function $\Phi_M(t)$ defined as
$$\Phi_M(t) = \max_{\{1,2,\cdots,m\}} \|\phi_k(t)\|. \tag{20}$$
Clearly, $\Phi_M(t)$ is continuous in time and also depends on integer l at different time intervals. By dividing the time horizon $\Re_+$ into two subsequences
$$\Re_1 := \{t \in \Re_+ \mid \Phi_M(t) > M_0, \}$$
$$\Re_2 := \{t \in \Re_+ \mid \Phi_M(t) \leq M_0\}$$
and constraining the initial time $t = 0$ in $\Re_2$, i.e., $\|\phi_k(0)\| \leq M_0$ for all $k = 1, 2, \cdots, m$, we can conclude the result by showing that $\Phi_M(t)$ is bounded for $t \in \Re_1$. Also this division allows that all the trajectories to be studied at different time intervals inside $\Re_1$ have the same "initial" value, i.e., $\|\phi_l(t_0^l)\| = M_0$.

Now the formal proof starts from the initial time zero and suppose that the first time for $\Phi_M(t)$ to cross the constant line M_0 occurs to the trajectory $\|\phi_i(t)\|$ at time t_0^i where i is an arbitrary member of the set $\{1, 2, \cdots, m\}$. From its continuity, $\|\phi_i(t)\|$ will satisfy the assumptions of Lemma 4.1 for some $t > t_0^i$. Therefore from Lemma 4.1, we have for all $\epsilon \leq \epsilon_1^*$
$$\sup_{0 \leq \tau \leq t} \|\phi_k(\tau)\| \leq M \quad \forall k = 1, 2, \cdots, m \quad \text{and} \quad t > t_0^i$$
and thus
$$\sup_{0 \leq \tau \leq t} \Phi_M(\tau) \leq M \quad \text{for } t > t_0^i.$$
If the conditions of Lemma 4.1 are satisfied by $\|\phi_i(t)\|$ for all the remaining time in $\Re_1$, then the results are proved. Clearly this is not always possible and suppose that the assumptions of Lemma 4.1 are violated by $\|\phi_i(t)\|$ when $t > t_1^i$ where t_1^i is arbitrary but satisfies $t_1^i > t_0^i$. For the violation, there are only two possible cases in a time interval at the right side of t_1^i when function $\Phi_M(t)$ is considered.

Case 1)
$$\sup_{0 \leq \tau \leq t} \Phi_M(\tau) \begin{cases} \leq M & \text{for } t \leq t_1^i \\ \neq \Phi_M(t) & \text{for } t_1^i < t. \end{cases}$$
This situation implies that for $t > t_1^i$, $\|\phi_i(t)\|$ does not satisfy the condition that $\sup_{0 \leq \tau \leq t} \|\phi_i(\tau)\| = \|\phi_i(t)\|$ of Lemma 4.1. As for the other condition that $\sup_{0 \leq \tau \leq t} \|\phi_k(\tau)\| \leq \|\phi_i(t)\|, \forall k \neq i$, whether it is satisfied will not affect our following analysis.

For $t > t_1^i$ in this case, we can automatically have
$$\sup_{0 \leq \tau \leq t} \Phi_M(\tau) = \Phi_M(t_M) \leq M$$
where $t_M \leq t_1^i$. Therefore
$$\sup_{0 \leq \tau \leq t} \|\phi_k(\tau)\| \leq \sup_{0 \leq \tau \leq t} \Phi_M(\tau) \leq M$$
for $k = 1, 2, \cdots, m$.

We note that it is not necessary to study the subsystem trajectory that gives $\Phi_M(t)$ in this case.

Case 2)[1]
$$\sup_{0 \leq \tau \leq t} \Phi_M(\tau) \begin{cases} \leq M & \text{for } t \leq t_1^i \\ = \Phi_M(t) & \text{for } t_1^i < t \end{cases}$$
and
$$\Phi_M(t) = \|\phi_j(t)\| \quad \text{for } t_1^i < t. \tag{21}$$
It can be noted that when $t > t_1^i$, the following assumption of Lemma 4.1 is not satisfied by $\|\phi_i(t)\|$ in this case
$$\sup_{0 \leq \tau \leq t} \|\phi_j(\tau)\| \leq \sup_{0 \leq \tau \leq t} \|\phi_i(\tau)\|.$$
Again, the condition that $\sup_{0 \leq \tau \leq t} \|\phi_i(\tau)\| = \|\phi_i(t)\|$ will not affect the result in this situation.

Obviously, this case has the following implications
$$\|\phi_j(t)\| = \sup_{0 \leq \tau \leq t} \|\phi_j(\tau)\|$$
and
$$\|\phi_j(t)\| \geq \sup_{0 \leq \tau \leq t} \|\phi_k(\tau)\| \quad \forall k \neq j$$
$t > t_1^i$.

As $t_1^i \in \Re_1$, then there exists a time instant $t_{0_0}^j$ satisfying $\|\phi_j(t_{0_0}^j)\| = M_0$ and the interval $[t_{0_0}^j, t_1^i] \subset \Re_1$ from the continuity of $\|\phi_j(t)\|$. Thus, the conditions in Lemma 4.2 are satisfied by $\|\phi_j(t)\|$. By studying $\|\phi_j(t)\|$ from the time instant $t_{0_0}^j$, applying Lemma 4.2 and following the same steps as in the proof of Lemma 4.1, we can show that
$$\sup_{0 \leq \tau \leq t} \|\phi_j(\tau)\| \leq M \quad \forall t > t_1^i$$
for $\epsilon \leq \epsilon^*$, where $\epsilon^* = \frac{\epsilon_1^*}{\sqrt{c_1} + \sqrt{c_2}}$ and therefore
$$\sup_{0 \leq \tau \leq t} \Phi_M(\tau) \leq M,$$
$$\sup_{0 \leq \tau \leq t} \|\phi_k(\tau)\| \leq M \text{ for } t > t_1^i \text{ and all } k = 1, 2, \cdots, m.$$

[1] This case is not shown in the right side neighborhood of t_1^i in Fig. 1, but it is given when $t \geq t_{1_1}^i$.

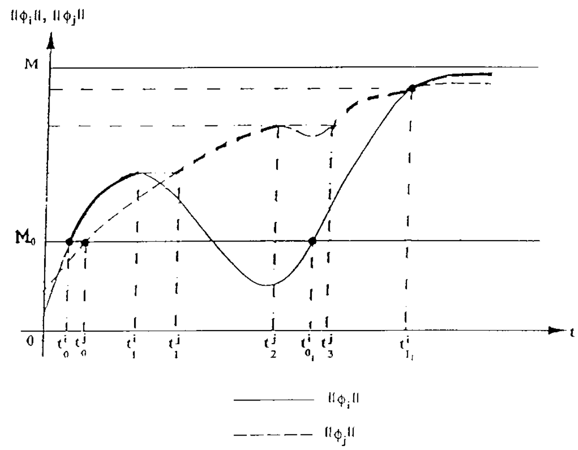

Fig. 1. Trajectories of $\|\phi_i\|$ and $\|\phi_j\|$.

After the examination of the two cases, it has been shown that when the assumptions of Lemma 4.1 are violated after certain time instant t_1^i, the bound M for all the subsystem trajectories are propagated to a new time instant T where T is such that for $t > T$, one of the following changes happens to $\Phi_M(t)$.

1) $\Phi_M(t) = \sup_{0\leq\tau\leq t}\Phi_M(t)$ right after Case 1).
2) $\Phi_M(t) \neq \sup_{0\leq\tau\leq t}\Phi_M(t)$ right after Case 2).
3) $\Phi_M(t)$ still equals $\sup_{0\leq\tau\leq t}\Phi_M(t)$, but is generated by a different subsystem trajectory $\|\phi_l(t)\|$ right after Case 2).

As we can note for $t > T$, changes 1) and 3) will get $\Phi_M(t)$ to Case 2 whereas change 2) will make $\Phi_M(t)$ into Case 1). Thus no matter how $\Phi_M(t)$ changes for $t > T$, it will still fall into one of the two cases except that t_1^i is shifted to the new time point T. Then we can repeatedly apply the argument in the two cases to establish the boundedness of all trajectories by M for $\epsilon \leq \epsilon^*$ in a method of interval by interval deduction. For further illustration, four more time intervals are presented in Fig. 1. For the four intervals, the following can be easily noted.

1) Change 1) occurs to $\Phi_M(t)$ at $t = t_1^j$. Then for $t > t_1^j$, $\Phi_M(t)$ falls into Case 2).
It can be seen that $\Phi_M(t) = \|\phi_j(t)\|$ for $t > t_1^j$. Thus the argument in Case 2) can be applied with now $t_{0_0}^j$ replaced by t_0^j.
2) Change 2) happens to $\Phi_M(t)$ at $t = t_2^j$. Thus when $t > t_2^j$, $\Phi_M(t)$ belongs to Case 1). and the argument in Case 1) can be employed.
3) When $T = t_3^j$. $\Phi_M(t)$ falls into Case 2) again and $\Phi_M(t) = \|\phi_j(t)\|$ for $t > T$. The argument in Case 2) can be applied and $\|\phi_j(t)\|$ is still examined from t_0^j since $[t_0^j, t_3^j] \subset \Re_1$.
4) For $t > t_{1_1}^i$, this is still Case 2) but $\Phi_M(t) = \|\phi_i(t)\|$. Thus the analysis in Case 2) is applied to $\|\phi_i(t)\|$ with "initial" time $t_{0_1}^i$ because $\|\phi_i(t_{0_1}^i)\| = M_0$ and $[t_{0_1}^i, t_{1_1}^i] \subset \Re_1$.

From the proof above, we can conclude that $\|\phi_k(t)\| \leq M\ \forall k = 1, 2, \cdots, m$ and $\epsilon \leq \epsilon^*$. Now note that M only depends on M_0 which is a uniform bound for initial values of $\phi_i(0)$, bounds of y_i^* and $d_i(t), i = 1, \cdots, m$. As $\phi_i(0), y_i^*$ and $d_i(t), i = 1, \cdots, m$ are bounded, thus M and therefore $\phi_i(t), i = 1, \cdots, m$ are bounded. Once establishing the boundedness of $\phi_i(t)$, we can have $u_i(t)$ and $y_i(t)$ bounded. □

Remark 4.2: If the ith subsystem is decoupled from the rest subsystems and it has no modeling errors, we can still obtain the results that basic adaptive control algorithms can achieve for ideal plants [8]. One of them is that perfect tracking can be achieved since the prediction error tends to zero.

V. Conclusion

In this paper, we studied a totally decentralized indirect continuous time adaptive control system. Each local controller is designed by ignoring the interactions from other subsystems and consists of a gradient estimator, subject to parameter projection as the only modification plus a pole assignment controller. It has been shown that the above decentralized adaptive controllers can stabilize an interconnected system with weak interactions and modeling errors including bounded disturbances and small amount unmodeled dynamics. It is also clear that those results established in earlier global convergence analysis of ideal situations are preserved for those subsystems which are decoupled from the interconnected system and also satisfy the "ideal assumptions" [8].

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References

[1] P. A. Ioannou, "Decentralized adaptive control of interconnected systems," *IEEE Trans. Automat. Contr.*, vol. AC-31, pp. 291–298, 1986.
[2] D. T. Gavel and D. D. Siljak, "Decentralized adaptive control: Structural conditions for stability," *IEEE Trans. Automat. Contr.*, vol. 34, pp. 413–426, 1989.
[3] R. Ortega and H. Herrera, "A solution to the decentralized adaptive stabilization problem," in *Proc. 31st IEEE CDC*, 1992.
[4] C. Wen, "Decentralized adaptive regulation," *IEEE Trans. Automat. Contr.*, vol. 39, pp. 2163–2166, 1994.
[5] L. Praly and E. Trulsson, "Decentralized indirect adaptive control," *RAIRO APPI*, vol. 20, pp. 643–668, 1986.
[6] C. Wen and D. J. Hill, "Global boundedness of discrete-time adaptive control just using estimator projection," to appear *Automatica*, 1992.
[7] C. Wen and D. J. Hill, "Decentralized adaptive control of linear time-varying systems," in *Proc. 11th World Congr. Automat. Contr.*, Tallinn, USSR, 1990, vol. 4, pp. 131–136.
[8] G. C. Goodwin and K. S. Sin, *Adaptive Filtering Prediction and Control.* Englewood Cliffs, NJ: Prentice-Hall, 1984.
[9] R. H. Middleton and G. C. Goodwin, *Digital Control and Estimation—A Unified Approach.* Englewood Cliffs, NJ: Prentice-Hall, 1990.
[10] J.-B. Pomet and L. Praly, "Adaptive nonlinear regulation: Estimation from the Lyapunov equation," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 729–740, 1992.