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Brief Paper

Decentralized Adaptive Control Using Integrator Backstepping*

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Key Words—Adaptive control; decentralized control; backstepping; stability; robustness.

Abstract—In this paper, the integrator backstepping approach together with an adaptive law using parameter projection is employed to design robust decentralized adaptive controllers. The techniques used allow us to relax the relative degree limitations on subsystems and the structural conditions on interactions between subsystems required in some earlier results. Further, no a priori knowledge of the unmodeled dynamics is required. Global stability is established for the closed-loop system and small in the mean tracking error is ensured. © 1997 Elsevier Science Ltd.

1. *Introduction*

Decentralized adaptive control has received a lot of attention in recent years. As the modeling error due to ignored interactions makes the stability analysis quite difficult, only a limited number of results have been obtained. With the traditional certainty equivalence principle, global stability of decentralized adaptive systems have been established but with various limitations, see for example, Ioannou and Kokotovic (1985), Ioannou (1986), Gavel and Siljak (1989), Wen and Hill (1992) and Wen (1995). For the direct model reference adaptive approach of Ioannou and Kokotovic (1985), Ioannou (1986) and Gavel and Siljak (1989), relative degrees of all the nominal subsystem models should be less than or equal to two. Recently, the concept of high-order tuners proposed by Morse (1991) was employed to relax the relative degree requirement by Ortega and Herrera (1993) and Ortega (1996). The integrator backstepping technique proposed by Krstic *et al.* (1994) was also applied to design decentralized adaptive regulators without restrictions on subsystem relative degrees in Wen (1994) and Jain and Khorrami (1995). However, due to the nature of the technique used, the interactions should satisfy certain structural conditions as commented in Ortega (1996). Also, adaptive tracking and the effect of external disturbances were not considered.

Recently, Zhang and Ioannou (1995) proposed to separate the backstepping control law design from an adaptive law and established the robustness of the resulting adaptive controller. A normalizing signal is introduced in their adaptive law so that the effect of the unmodeled dynamics normalized by the signal is uniformly bounded. When interactions are considered as some sources of modeling errors in the design of decentralized adaptive controllers, their effects may not be bounded by a local normalizing signal using local information in a subsystem. In this case, a normalizing signal requiring information exchange between subsystems may be expected as in Datta and Ioannou (1991, 1992) and therefore only partial decentralization is achieved. In Wen and Soh (1995), a different normalization signal is employed and a different system-theoretic analysis is presented. There, the boundedness of the unmodeled dynamics by the normalization signal is not required. In this paper, we apply the techniques in Wen and Soh (1995) to the design of decentralized adaptive controllers. In the design, no structural conditions on the interactions are imposed. Also, the robustness with respect to external disturbance is studied. With a lot of elaborations on the effects of interactions and subsystem modeling errors, global stability of the closed-loop system and small in the mean-tracking error property are established.

2. *System models*

In this paper, we consider the following class of interconnected systems.

$$y_i(t) = H_i(D)[1 + \bar{\varepsilon}_i \bar{H}_i(D)]u_i(t) + d_i(t) + \sum_{j=1}^{m} [\bar{\varepsilon}_{ij} \bar{H}_{ij}(D) u_j(t) + \bar{\bar{\varepsilon}}_{ij} \bar{\bar{H}}_{ij}(D) y_j(t)]. \tag{1}$$

for $i, j = 1, \ldots, m$, where y_i, u_i and d_i are respectively, the output, input and disturbance of the ith subsystem, and $H_i(D) = B_i(D)/A_i(D)$ is the reduced-order transfer function of subsystem i with

$$A_i(D) = D^{n_i} + a_i^{n_i-1} D^{n_i-1} + \cdots + a_i^0,$$
$$B_i(D) = b_i^{m_i} D^{m_i} + b_i^{m_i-1} D^{m_i-1} + \cdots + b_i^0,$$

where D denotes the differentiation operator, and $m_i < n_i$, $\bar{\varepsilon}_i, \bar{\varepsilon}_{ij}, \bar{\bar{\varepsilon}}_{ij}$ are constants, $\bar{H}_i(D)$ is the multiplicative uncertainty of the ith subsystem, $\bar{H}_{ij}(D)$ and $\bar{\bar{H}}_{ij}(D)$ denote the subsystem interactions if $i \neq j$ and unmodelled dynamics if $i = j$.

Suppose that y_i^* is a given reference set-point for output y_i. The control problem is to design a controller for plant (1) such that the closed loop system is stable in the sense that all signals in the system are bounded for arbitrary bounded y_i^* and initial conditions, and the tracking error is small in some sense. To solve the control problem, the following assumptions are made for the plant given in (1).

Assumption 2.1.

(A1) $B_i(D - \sigma_i^0)$ is Hurwitz where σ_i^0 is a known positive constant.

(A2) An upper bound for n_i, the nominal relative degree $n_i^* = n_i - m_i$ of subsystem i and the sign of the high-frequency gain $\mathrm{sgn}(b_i^{m_i})$ are known. Furthermore, the coefficients of $A_i(D)$ and $B_i(D)$ are inside a known compact convex region $\mathscr{C}_i$.

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(A3) $\bar{H}_i(D)$, $\bar{H}_{ij}(D)$ and $\bar{\bar{H}}_{ij}(D)$ are stable, and $H_i(D)$ $\bar{H}_i(D)$, $\bar{H}_{ij}(D)$ and $\bar{\bar{H}}_{ij}(D)$ are strictly proper.
(A4) $d_i(t)$ is bounded.
(A5) y_i^* and its first n_i^* derivatives are bounded piecewise continuous signals.

Remark 2.1. (i) While modeling errors satisfy (A3) and (A4), no a priori knowledge is required from them for the implementation of the adaptive controllers given in later sections.

(ii) From Assumption A2, there exists a constant $k_\theta^i > 0$ such that

$$\|\theta_i^1 - \theta_i^2\| \le k_\theta^i, \tag{2}$$

for any $\theta_i^1, \theta_i^2 \in \mathscr{C}_i$.

Assumption A2 also implies a known lower bound for $|b_i^{m_i}|$.

As in the single-loop case, for subsystem i, let us introduce a stable polynomial

$$F_i(D) = D^{n_i} + f_i^1 D^{n_i-1} + \cdots + f_i^{n_i-1} D + f_i^{n_i},$$

where the real parts of all its zeros are located on the left of $-\sigma_i^0$.

Define the following filtered variables

$$\begin{aligned} \dot{\xi}_i^{n_i} &= A_i^0 \xi_i^{n_i} + f_i y_i, \\ \dot{\xi}_i^k &= A_i^0 \xi_i^k + e_{n_i-k} y_i \quad 0 \le k \le n_i - 1, \\ \dot{v}_i^k &= A_i^0 v_i^k + e_{n_i-k} u_i \quad 0 \le k \le m_i, \end{aligned} \tag{3}$$

where

$$A_i^0 = \begin{pmatrix} -f_i & \begin{matrix} I_{(n_i-1)\times(n_i-1)} \\ 0\ldots0 \end{matrix} \end{pmatrix}, \quad f_i = (f_i^1, \ldots, f_i^{n_i})^{\mathrm{T}},$$

e_{n_i-k} is the $(n_i - k)$th coordinate column vector in $\mathbb{R}^{n_i}$ space.

To develop the decentralized adaptive controllers, the ith subsystem of the plant (1) can be reparameterized in the following form as shown in Zhang and Ioannou (1995):

$$y_i = \xi_i^{n_i,1} + \theta_i^{\mathrm{T}} \omega_i + \eta_i, \tag{4}$$

where

$$\theta_i^{\mathrm{T}} = [-a_i^{n_i-1}, \ldots, -a_i^0, b_i^{m_i}, \ldots, b_i^0],$$

$$\omega_i^{\mathrm{T}} = [\xi_i^{n_i-1,1}, \ldots, \xi_i^{0,1}, v_i^{m_i,1}, \ldots, v_i^{0,1}] \tag{5}$$

$$= \left[\frac{D^{n_i-1}}{F_i(D)} y_i, \ldots, \frac{D}{F_i(D)} y_i, \frac{1}{F_i(D)} y_i, \frac{D^{m_i}}{F_i(D)} u_i, \ldots, \frac{D}{F_i(D)} u_i, \frac{1}{F_i(D)} u_i \right], \tag{6}$$

$$\eta_i(t) = \bar{\varepsilon}_i \frac{B_i}{F_i} \bar{H}_i u_i + \frac{A_i}{F_i} d_i + \frac{A_i}{F_i} \sum_{j=1}^{m} [\bar{\varepsilon}_{ij} \bar{H}_{ij} u_j + \bar{\bar{\varepsilon}}_{ij} \bar{\bar{H}}_{ij} y_j], \tag{7}$$

and $\xi_i^{j,k}, v_i^{j,k}$ are the kth element of the vectors ξ_i^j, v_i^j, respectively.

A first-order stable filter is introduced as follows:

$$\rho_i(t) = \frac{1}{\bar{F}_i} (|u_i(t)| + |y_i(t)|), \quad \rho_i(0) = 0, \tag{8}$$

where $\bar{F}_i = D + \sigma_i$ with $\sigma_i < \sigma_i^0$.

Now the bounds on ξ_i^k, v_i^k in (3), ω_i in (6), and η_i in (7) are given by the following lemma:

Lemma 2.1. For all members of the class of systems satisfying Assumption 2.1, there exist constants $c_\xi, c_v, c_\omega > 0$, $d_0 \ge 0$ and $\varepsilon_{ij} \ge 0$ such that for all t

$$\|\xi_i^k(t)\| \le c_\xi \rho_i(t), \quad \text{for } k = 0, 1, 2, \ldots, n_i,\ i = 1, 2, \ldots, m, \tag{9}$$

$$\|v_i^k(t)\| \le c_v \rho_i(t), \quad \text{for } k = 0, 1, 2, \ldots, m_i,\ i = 1, 2, \ldots, m, \tag{10}$$

$$\|\omega_i(t)\| \le c_\omega \rho_i(t) \quad \text{for } i = 1, 2, \ldots, m, \tag{11}$$

$$|\eta_i(t)| \le \sum_{j=1}^{m} \varepsilon_{ij} \sup_{0 \le \tau \le t} \rho_j(\tau) + d_0 \quad \text{for } i = 1, 2, \ldots, m. \tag{12}$$

If $\sup_{0\le\tau\le t} \rho_i(\tau) = \rho_i(t)$ and $\sup_{0\le\tau\le t} \rho_j(\tau) \le \sup_{0\le\tau\le t} \rho_i(\tau)$ for all $j \ne i$ and $t > t_i^0$, then (12) becomes

$$|\eta_i(t)| \le \varepsilon \rho_i(t) + d_0 \quad \text{for all } t \ge t_i^0, \tag{13}$$

where ε is a positive constant depending on ε_{ij}.

Proof. We can rewrite (7) as

$$\eta_i(t) = \bar{\varepsilon}_i \frac{B_i}{F_i} \bar{H}_i \frac{\bar{F}_i}{\bar{F}_i} u_i + \frac{A_i}{F_i} d_i + \frac{A_i}{F_i} \sum_{j=1}^{m} \left[\bar{\varepsilon}_{ij} \bar{H}_{ij} \frac{\bar{F}_j}{\bar{F}_j} u_j + \bar{\bar{\varepsilon}}_{ij} \bar{\bar{H}}_{ij} \frac{\bar{F}_j}{\bar{F}_j} y_j \right]. \tag{14}$$

Then the stability of $F(s)$ and Assumptions A3 and A4 yield that

$$|\eta_i(t)| \le \sum_{j=1}^{m} \bar{\varepsilon}_{ij}' \sup_{0\le\tau\le t} \bar{\rho}_j(\tau) + \sum_{j=1}^{m} \bar{\bar{\varepsilon}}_{ij}' \sup_{0\le\tau\le t} \bar{\bar{\rho}}_j(\tau) + d_0, \tag{15}$$

where $\bar{\rho}_i(t) = |u_i/\bar{F}_i|$ and $\bar{\bar{\rho}}_i(t) = |y_i/\bar{F}_i|$ with $\bar{\rho}_i(0) = \bar{\bar{\rho}}_i(0) = 0$. As $\bar{\rho}_i(t) \le \rho_i(t)$ and $\bar{\bar{\rho}}_i(t) \le \rho_i(t)$, we have (12), and (13) is easily verified. From the choices of σ_i and σ_i^0, (9)–(11) can be established as in Ioannou and Tsakalis (1986). □

Remark 2.2. (i) In the proof of Lemma 2.1, the effects of some exponentially decaying terms due to nonzero initial conditions have been absorbed by d_0.

(ii) The constant ε_{ij} indicates the strength of the interactions between subsystems i and j when $i \ne j$, and the unmodelled dynamics of the ith subsystem to the nominal model when $i = j$.

(iii) In terms of the bounding signals, the bound for the modeling error in (12) allows the effects of the unmodeled dynamics and interactions to have infinite memory and thus is looser than those given in some existing literature such as Ioannou and Kokotovic (1985) and Ioannou (1986). The class of modeling errors considered can be enlarged to include any nonlinear unmodeled dynamics satisfying (12) and (8).

3. *Robust decentralized adaptive control scheme*

In this section, a robust adaptive control scheme is proposed to design decentralized adaptive controllers based on (4). Each local adaptive controller consists of two modules: a parameter estimator and a control law developed from the integrator backstepping.

3.1. *Parameter estimator.* The following estimation algorithm with projection is used to estimate the unknown parameters of the nominal plant model:

$$\dot{\hat{\theta}}_i(t) = \mathscr{P} \left\{ \frac{\omega_i(t) e_i(t)}{m_{s_i}^2(t)} \right\}, \tag{16}$$

where $\hat{\theta}_i(t)$ is the estimate of θ_i, the normalization signal m_{s_i} is defined as

$$m_{s_i} = (1 + \rho_i^2)^{1/2}, \tag{17}$$

the prediction error $e_i(t)$ is given as

$$e_i(t) = y_i(t) - \xi_i^{n_i,1} - \omega_i^{\mathrm{T}}(t)\hat{\theta}_i(t) \tag{18}$$

and $\mathscr{P}\{.\}$ denotes a projection operation proposed in Pomet and Praly (1992). Such an operation can ensure that the estimated parameter vector $\hat{\theta}_i(t) \in \mathscr{C}$ for all t if $\hat{\theta}_i(0) \in \mathscr{C}$.

Now, define

$$\tilde{e}_i(t) = \frac{e_i(t)}{m_{s_i}}.$$

Suppose M_0 is a positive constant s.t. $d_0/M_0 \le \delta$. Then some useful properties of the estimator in (16)–(18) can be stated as in the following lemma.

Lemma 3.1. The estimator (16)–(18), applied to the plant given in (1), has the following properties:

(i) If $\rho_i(t) > M_0$, $\sup_{0\le\tau\le t}\rho_i(\tau) = \rho_i(t)$ and $\sup_{0\le\tau\le t}\rho_j(\tau) \le \sup_{0\le\tau\le t}\rho_i(\tau)\ \forall j \ne i$ for all $t > t_i^0$, then

$$|\tilde{e}_i(t)| \le (k_1 c_\omega + \varepsilon + \delta) \quad \text{for } t \ge t_i^0, \tag{19}$$

$$\left|\frac{\omega_i^{\mathrm{T}}(t)\tilde{\theta}_i(t)}{m_{s_i}(t)}\right| \le k_1 c_\omega \quad \text{for } t \ge 0, \tag{20}$$

where k_1 is a constant depending on k_θ defined as $k_\theta = \max\{k_\theta^1, k_\theta^2, \dots, k_\theta^m\}$ and

$$\int_{t_i^0}^{t} \tilde{e}_i^2(\tau)\,\mathrm{d}\tau \le k_2 + \gamma_1(t - t_i^0) + \gamma_2(t - t_i^0) \quad \text{for } t \ge t_i^0, \tag{21}$$

$$\int_{t_i^0}^{t} \left|\frac{\omega_i^{\mathrm{T}}(\tau)\tilde{\theta}_i(\tau)}{m_{s_i}(\tau)}\right|^2 \mathrm{d}\tau \le 2k_2 + k_3\gamma_1(t - t_i^0) + k_4\gamma_2(t - t_i^0)$$

$$\text{for } t \ge t_i^0, \tag{22}$$

where

$$k_2 = \tfrac{1}{2}k_\theta^2, \quad \gamma_1 = (k_1 + 2\varepsilon)\varepsilon, \quad \gamma_2 = (k_1 + 2\delta)\delta, \tag{23}$$

and k_3, k_4 are generic positive constants.

(ii)

$$\|\dot{\hat{\theta}}_i(t)\| \le c_\omega \beta |\tilde{e}_i(t)|. \tag{24}$$

Proof. From (4) and (18), we get

$$e_i(t) = -\tilde{\theta}_i^{\mathrm{T}}(t)\omega_i(t) + \eta_i(t), \tag{25}$$

where

$$\tilde{\theta}_i = \hat{\theta}_i - \theta_i.$$

Then applying (11) and (13) gives

$$|e_i(t)| \le k_\theta c_\omega \|\omega_i(t)\| + \varepsilon\rho_i(\tau) + d_0. \tag{26}$$

Once (26) is obtained, the results of the lemma can be readily established by using (11), (13) and following some standard steps as in Wen and Hill (1992). □

Remark 3.1. γ_1 can be made arbitrary small by reducing ε, and γ_2 by making M_0 a sufficiently large number. M_0 is used here for the purpose of analysis only. It is not a design parameter.

3.2. *Control law synthesis.* The local control law developed is the same as that in Zhang and Ioannou (1995) where the backstepping technique was employed. For the completeness of this paper, we now present the control law.

Let

$$\bar{\omega}_i^{\mathrm{T}} = [\xi_i^{n_i-1,1}, \dots, \xi_i^{0,1}, 0, v_i^{m_i-1,1}, \dots, v_i^{0,1}] \tag{27}$$

and define

$$\alpha_i^0 = \frac{1}{\hat{b}_i^{m_i}}(-\xi_i^{n_i,1} - \bar{\omega}_i^{\mathrm{T}}\hat{\theta}_i + y_i^*), \tag{28}$$

$$z_i^1 = v_i^{m_i,1} - \alpha_i^0$$
$$= \frac{1}{\hat{b}_i^{m_i}}(e_{i,y_i} - \omega_i^{\mathrm{T}}\hat{\theta}_i - \eta_i), \tag{29}$$

where e_{i,y_i} is the tracking error

$$e_{i,y_i} = y_i - y_i^*.$$

Then α_i^k, z_i^k are iteratively generated through the following equations:

$$z_i^k = v_i^{m_i,k} - \alpha_i^{k-1}, \tag{30}$$

$$\alpha_i^k = k_i^1 v_i^{m_i,1} + \sum_{j=0}^{n_i}\frac{\partial\alpha_i^{k-1}}{\partial\xi_i^j}A_i^0\xi_i^j + \sum_{j=0}^{m_i}\frac{\partial\alpha_i^{k-1}}{\partial v_i^j}A_i^0 v_i^j$$
$$+ w_i^k\omega_i^{\mathrm{T}}\hat{\theta}_i - c_i^k z_i^k - d_i^k(s_i^k)^2 z_i^k - z_i^{k-1}, \tag{31}$$

$$w_i^k = -\left(\sum_{j=0}^{n_i-1}\frac{\partial\alpha_i^{k-1}}{\partial\xi_i^j}e_{n_i-j} + \frac{\partial\alpha_i^{k-1}}{\partial\xi_i^{n_i}}f_i\right), \tag{32}$$

$$(s_i^k)^2 = (w_i^k)^2 + \sum_{j=0}^{n_i}\left\|\frac{\partial^2\alpha_i^{k-1}}{\partial\xi_i^j\partial\hat{\theta}_i}\right\|_e^2 + \sum_{j=0}^{m_i}\left\|\frac{\partial^2\alpha_i^{k-1}}{\partial v_i^j\partial\hat{\theta}_i}\right\|_e^2, \tag{33}$$

where $k = 1, 2, \dots, n_i^*$, $z_i^0 = 0$ and

$$\|A\|_e = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}|a_{ij}|,$$

where a_{ij} is the (i,j) element of $A \in \mathbb{R}^{n_1\times n_2}$.

The local control signal u_i is set to be $\alpha_i^{n_i^*}$, i.e.,

$$u_i = \alpha_i^{n_i^*}. \tag{34}$$

Remark 3.2. From (16), (18) and (34), we note that each local adaptive controller only employs the local measurements in the subsystem it controls. This is in contrast with some decentralized adaptive control schemes which require the information exchange among subsystems, see, for example, Hill *et al.* (1988) and Datta and Ioannou (1991, 1992).

4. *Stability analysis*

The decentralized adaptive control system designed is now analyzed in this section. From (29), we have

$$y_i = \hat{b}_i^{m_i}z_i^1 + y_i^* + \eta_i + \omega_i^{\mathrm{T}}\tilde{\theta}_i. \tag{35}$$

Also, following the derivation in Zhang and Ioannou (1995) gives

$$u_i = \sum_{k=1}^{n_i^*}\bar{L}_i^k z_i^k + G_i^0 y_i + G_i^0\sum_{j=1}^{m}[\bar{\varepsilon}_{ij}\bar{H}_{ij}u_j$$
$$+ \bar{\bar{\varepsilon}}_{ij}\bar{\bar{H}}_{ij}y_j] + \bar{\varepsilon}_i G_i^{n_i^*}\bar{H}_i u_i + G_i^0 d_i + \tilde{H}_i^{n_i^*}, \tag{36}$$

where $\tilde{L}_i^j, j = 1,2,\ldots,n_i^*, \tilde{H}_i^{n_i^*}$ are some bounded functions and G_i^l is defined as

$$G_i^l = \sum_{k=0}^{n_i+m_i-l} p_i^k \frac{D^k}{F_i(D)B_i(D)} \tag{37}$$

for some bounded functions p_i^k.

In the following analyses, all $c_i, i = 1,2,\ldots$ denote generic positive constants without further clarification.

Define

$$u_i^1 = u_i - (G_i^0 d_i + G_i^0 y_i^* + \tilde{H}_i^{n_i^*}), \quad \tilde{u}_i^1 = \frac{u_i^1}{m_{s_i}},$$

$$\tilde{e}_{i,y_i} = \frac{e_{i,y_i}}{m_{s_i}}.$$

Then the following result can be obtained.

Lemma 4.1. Under the conditions of Lemma 3.1.1, we have

$$|\tilde{u}_i^1| + |\tilde{e}_{i,y_i}| \le k_{uy,1}(k_1 + \varepsilon + \delta) + \bar{k}_{uy,1}\frac{||y_i^*||_\infty}{m_{s_i}(t_i^0)} \tag{38}$$

for constants $k_{uy,1} > 0$, $\bar{k}_{uy,1} > 0$, and

$$\int_{t_i^0}^{t}[\tilde{u}_i^{1^2}(\tau) + \tilde{e}_{i,y_i}^2(\tau)]\,d\tau \le k_{uy,2}(k_2 + \gamma_1(t - t_i^0)$$
$$+\gamma_2(t - t_i^0)) + \bar{k}_{uy,2}\left(\frac{||y_i^*||_\infty}{m_{s_i}(t_i^0)}\right)^2, \tag{39}$$

where $k_{uy,2}$ and $\bar{k}_{uy,2}$ are positive constants.

Proof: The results can be established by following similar analysis to Wen and Soh (1995). □

From Lemma 4.1, the stability of the system can be established under a special case. This is presented in the following lemma.

Lemma 4.2. Suppose that $\rho_i(t_i^0) = M_0$ and for all $t > t_i^0$ $\rho_i(t) > M_0$, $\sup_{0\le\tau\le t}\rho_i(\tau) = \rho_i(t)$ and $\sup_{0\le\tau\le t}\rho_j(\tau) \le \rho_i(t)$ for all $j \ne i$. Then consider the decentralized adaptive system consisting of local estimators (16)–(18) and controllers (34). Under Assumptions 2.1, there exists a constant ε_1^* such that for all $\varepsilon \le \varepsilon_1^*$ the closed-loop system ensures that

$$\sup_{0\le\tau\le t} \rho_i(\tau) \le M \quad \forall i = 1,2,\ldots,m, \tag{40}$$

where $M = \sqrt{c_1 M_0^2 + c_2}$.

Proof. From the definition of $\rho_i(t)$, we have

$$\rho_i(t) = e^{-\sigma_i(t-t_i^0)}\rho_i(t_i^0) + \int_{t_i^0}^{t} e^{-\sigma_i(t-\tau)}(|u_i(\tau)| + |y_i(\tau)|)\,d\tau$$
$$\le M_0 + \int_{t_i^0}^{t} e^{-\sigma_i(t-\tau)}\{[|\tilde{u}_i^1| + |\tilde{e}_{i,y_i}|]\rho_i(\tau) + |\tilde{u}_i^1| + |\tilde{e}_{i,y_i}|$$
$$+|y_i^*| + |G_i^0 d_i + G_i^0 y_i^* + \tilde{H}_i^{n_i^*}|\}d\tau. \tag{41}$$

Suppose that the intermediate number M_0 is also such that

$$||y_i^*||_\infty + ||G_i^0 d_i + G_i^0 y_i^* + \tilde{H}_i^{n_i^*}||_\infty \le M_0$$
$$\text{for all } i = 1,2,\ldots,m.$$

Clearly, such an M_0 always exists for any bounded y_i^*, d_i and $\tilde{H}_i^{n_i^*}$. Now applying the Schwarz inequality and squaring both sides of (41), we get,

$$\rho_i^2(t) \le c_3 M_0^2 + c_3\int_{t_i^0}^{t} e^{-\sigma(t-\tau)}\{[(\tilde{u}_i^1)^2 + (\tilde{e}_{i,y_i})^2]\rho_i^2(\tau)$$
$$+[|\tilde{u}_i^1| + |\tilde{e}_{i,y_i}|]^2 + M_0^2\}d\tau \tag{42}$$

where $\sigma = \min\{\sigma_1,\sigma_2,\ldots,\sigma_m\}$.Multiplying both sides of (42) by $e^{\sigma t}$ gives

$$e^{\sigma t}\rho_i^2(t) \le s_i^2(t) + c_3\int_{t_i^0}^{t} e^{\sigma\tau}\rho_i^2(\tau)[(\tilde{u}_i^1)^2 + (\tilde{e}_{i,y_i})^2]d\tau, \tag{43}$$

where

$$s_i^2(t) = e^{\sigma t}c_3 M_0^2 + c_3\int_{t_i^0}^{t} e^{\sigma\tau}[(|\tilde{u}_i^1| + |\tilde{e}_{i,y_i}|)^2 + M_0^2]d\tau. \tag{44}$$

Then applying the Bellman–Grownwall lemma to (43) and using Lemma 4.2 with the fact that $||y_i^*||_\infty/m_{s_i}(t_i^0) \le 1$ yields

$$\rho_i(t)^2 \le e^{-\sigma t}s_i^2(t) + c_3\int_{t_i^0}^{t} e^{-\sigma t}[(\tilde{u}_i^1)^2$$
$$+(\tilde{e}_{i,y_i})^2]s_i^2(\tau)e^{\int_\tau^t c_3[(\tilde{u}_i^1)^2+(\tilde{e}_{i,y_i})^2]d\tau_1}d\tau \le c_1 M_0^2 + c_2, \tag{45}$$

for $\varepsilon \le \varepsilon_1^*$ and $\delta \le \delta^*$ where ε_1^* and δ^* are sufficiently small constants satisfying

$$c_3 k_{uy,2}(\gamma_1^* + \gamma_2^*) < \sigma, \tag{46}$$

with γ_1^*, γ_2^* depending on ε_1^* and δ^*. Clearly, c_1 and c_2 are independent of ε if ε is replaced by its bound ε_1^*, a generic constant satisfying (46). □

To establish the stability result for the general case, we explore the parameter estimator further and this gives Lemma 4.3 as follows.

Lemma 4.3. If $\rho_i(t_i^0) = M_0$, $\rho_i(t) > M_0$ for all $t \ge t_i^0$, and $\sup_{0\le\tau\le t}\rho_j(\tau) \le \sqrt{c_1 M_0^2 + c_2}$ for all $t \in [0, t_1]$ and $j = 1,2,\ldots,m$, and $\sup_{0\le\tau\le t}\rho_i(\tau) = \rho_i(t)$, $\sup_{0\le\tau\le t}\rho_j(\tau) \le \sup_{0\le\tau\le t}\rho_i(\tau), \forall j \ne i$ for all $t \ge t_1$, then

$$|\tilde{u}_i^1| + |\tilde{e}_{i,y_i}| \le k_{uy,1}[k_1 + \varepsilon(\sqrt{c_1} + \sqrt{c_2}) + \delta]$$
$$+\bar{k}_{uy,1}\frac{||y_i^*||_\infty}{m_{s_i}(t_i^0)} \quad \text{for } t \ge t_i^0 \tag{47}$$

and

$$\int_{t_i^0}^{t}[(\tilde{u}_i^1(\tau))^2 + (\tilde{e}_{i,y_i}(\tau))^2]d\tau \le k_{uy,2}(k_2 + \bar{\gamma}_1(t - t_i^0)$$
$$+\gamma_2(t - t_i^0)) + \bar{k}_{uy,2}\left(\frac{||y_i^*||_\infty}{m_{s_i}(t_i^0)}\right)^2 \quad \text{for } t \ge t_i^0, \tag{48}$$

where

$$\bar{\gamma}_1 = [k_1 + 2\varepsilon(\sqrt{c_1} + \sqrt{c_2})]\varepsilon(\sqrt{c_1} + \sqrt{c_2}). \tag{49}$$

Proof. By noting the condition of the lemma and using (13), we have

$$\frac{|\eta_i(t)|}{m_{s_i}(t)} \le \varepsilon(\sqrt{c_1} + \sqrt{c_2}) + \delta \quad \forall t \in [t_i^0, t_1]. \tag{50}$$

Then the results can be established by following similar analyses as in Lemmas 3.1 and 4.1. □

Remark 4.1. (i) Note that the properties in the above lemma is quite similar to Lemma 4.1 except that the constants c_1 and c_2 appear here.

(ii) All the generic constants in Lemmas 3.1, 4.1 and 4.3 are uniform for $i = 1, 2, \ldots, m$.

From Lemma 4.3, we get our main stability result as stated in the following theorem.

Theorem 4.1. Consider the decentralized adaptive system consisting of plant (1), local adaptive controllers (16), (18) and (34). Under Assumption 2.1, there exists a constant ε^* such that for all $\varepsilon \le \varepsilon^*$,

(i) The closed loop system is globally stable in the sense that all signals remain bounded $\forall t$ for all finite initial states, any bounded y_i^* and arbitrarily bounded external disturbances;

(ii) The tracking error $e_{i,y_i}(t)$ satisfies

$$\int_{t_i^0}^{t} e_{i,y_i}^2(\tau)\,\mathrm{d}\tau \le \beta_1 + \beta_2(\varepsilon + d_0)(t - t_i^0) \quad \text{for all } t_i^0 \ge 0, \tag{51}$$

where β_1, β_2 are constants.

Proof. (i) To show the boundedness of all the trajectories $\rho_i, i = 1, 2, \ldots, m$, we consider a function $\rho(t)$ defined as

$$\rho(t) = \max\{\rho_1(t), \rho_2(t), \ldots, \rho_m(t)\}. \tag{52}$$

Clearly, the result is proved if $\rho(t)$ is bounded. It can be noted that $\rho(t)$ is continuous and thus, starting with $\tau_0 = 0$ and $k = 1, 2, \ldots$, we can divide the time axis $[0, \infty)$ into the following two subsequences:

$$\mathbb{R}_k^- = [\tau_{k-1}, s_k] \quad \mathbb{R}_k^+ = (s_k, \tau_k),$$

where

$$\mathbb{R}_k^- = \{t | \rho(t) \le M_0\} \quad \text{and} \quad \mathbb{R}_k^+ = \{t | \rho(t) > M_0\}, \tag{53}$$

i.e.,

$$[0, \infty) = \left(\cup_{k=1}^{\infty} \mathbb{R}_k^-\right) \cup \left(\cup_{i=1}^{\infty} \mathbb{R}_i^+\right). \tag{54}$$

$\rho(t)$ can be ensured to be bounded if it is bounded in $\mathbb{R}_k^+, \forall k \ge 1$. This can be shown through induction. Thus we consider $t \in \mathbb{R}_1^+$ first. From the continuity of $\rho(t)$, $\exists t_1 \in \mathbb{R}_1^+$ and an $i \in \{1, 2, \ldots, m\}$ such that $\sup_{0\le\tau\le t}\rho(\tau) = \rho(t)$ and $\rho(t) = \rho_i(t)$ for all $t \le t_1$ and $t \in \mathbb{R}_1^+$. Thus $\sup_{0\le\tau\le t}\rho_i(\tau) = \rho_i(t)$ and $\sup_{0\le\tau\le t}\rho_j(\tau) \le \sup_{0\le\tau\le t}\rho_j(\tau)$ $\forall j \ne i$ for all $t \le t_1$ and $t \in \mathbb{R}_1^+$. Therefore, the conditions of Lemmas 3.1, 4.1 and 4.2 are satisfied for all $t \le t_1$ and $t \in \mathbb{R}_1^+$. Then using Lemma 4.2 and noting that $\rho_i(s_1) = \rho(s_1) = M_0$, we can show that for $t \le t_1$ and all $\varepsilon \le \varepsilon_1^*$,

$$\sup_{0\le\tau\le t} \rho(\tau) \le M, \tag{55}$$

i.e.,

$$\sup_{0\le\tau\le t} \rho_i(\tau) \le M \quad \forall i = 1, 2, \ldots, m. \tag{56}$$

If the conditions of Lemma 3.1 are violated for $t \ge t_1$ and $t \in \mathbb{R}_1^+$, the following two possibilities may occur to $\rho(t)$.

Case 1: $\sup_{0\le\tau\le t}\rho(\tau) = \rho(t)$ but $\rho(t) = \rho_j(t)$, $j \in \{1, 2, \ldots, m\} \setminus i$ for all $t > t_1$.

In this case, the condition that $\sup_{0\le\tau\le t}\rho_j(\tau) \le \sup_{0\le\tau\le t}\rho_i(\tau)$, $\forall j \ne i$ cannot be satisfied. Thus Lemma 4.2 cannot be applied for $t > t_1$. However, we now consider $\rho_j(t)$. Clearly, there exists a t_j^1 such that $\rho_j(t_j^1) = M_0$ and $\rho_j(t) > M_0$ for all $t \in [t_j^1, t_1] \subset \mathbb{R}_1^+$. Also in this case, we have

$$\sup_{0\le\tau\le t} \rho_j(\tau) = \rho_j(t) \quad \text{and} \quad \sup_{0\le\tau\le t} \rho_i(\tau) \le \sup_{0\le\tau\le t} \rho_j(\tau)$$
$$\forall i \ne j \quad \text{and} \quad t > t_1. \tag{57}$$

Thus from (56) and (57), Lemma 4.3 can be applied to $\rho_j(t)$ for $t \ge t_j^1$. Then following the same steps as in the proof of Lemma 4.2 and applying Lemma 4.3 with 'initial condition' $\rho_j(t_j^1) = M_0$, we shall obtain (55) or (56) for $t \ge t_1$ and all $\varepsilon \le \varepsilon^*$, where

$$\varepsilon^* = \frac{\varepsilon_1^*}{\sqrt{c_1} + \sqrt{c_2}}. \tag{58}$$

Case 2: $\sup_{0\le\tau\le t}\rho(\tau) \ne \rho(t)$ for $t \in [t_1, t_2] \subset \mathbb{R}_1^+$ and $\sup_{0\le\tau\le t}\rho(\tau) = \rho(t)$ for $t > t_2$.

In this case, the condition that $\sup_{0\le\tau\le t}\rho_i(\tau) = \rho_i(\tau)$ cannot be satisfied for $t \ge t_1$. However, (55) or (56) automatically holds for $t \in [t_1, t_2]$. If t_2 is infinite, the result is proved. For a finite t_2 and when $t > t_2$, (55) or (56) can be shown under the condition (58) by using Lemma 4.3 and following the same argument as in Case 1.

In this way, the boundedness of ρ is established over $\mathbb{R}_1^+$.

Now assuming (55) or (56) holds $\forall t \in \mathbb{R}_k^+$, it can be shown that, by following the proof of Lemma 4.2 and the above argument, (55) is also true $\forall t \in \mathbb{R}_{k+1}^+$ from Lemma 4.3 with the 'initial condition' $\|\rho_i(t_i^{p+1})\| = M_0$ with $p \in \{1, 2, \ldots, \}$ and $t_i^{p+1} \in \mathbb{R}_{k+1}^+$.

After establishing the boundedness of $\rho_i(t), \forall i = 1, 2, \ldots, m$, we can have $\omega_i(t), \xi_i(t), v_i(t), \eta_i(t), e_i(t), y_i(t)$ and $u_i(t)$ bounded.

(ii) Now note that

$$e_{i,y_i} = \hat{b}_i^{m_i} z_i^1 + \eta_i + \omega_i^{\mathrm{T}}\tilde{\theta}_i. \tag{59}$$

Once the boundedness of all the signals is established, then for all $t_i^0 > 0$, Lemma 3.1 hold, and z_i^1, η_i and $\omega_i^{\mathrm{T}}\tilde{\theta}_i$ have the same properties as $\omega_i^{\mathrm{T}}\tilde{\theta}_i/m_{s_i}$. Thus (51) can be obtained. □

Remark 4.2. (i) Two of the key points which enable us to prove the stability result are the use of the intermediate number M_0 and the division of the time interval into $\mathbb{R}_-$ and $\mathbb{R}_+$. This ensures that the 'initial condition' $\rho_i(t_i^{p+1}) = M_0$ for all $p = 1, 2, \ldots$ and $i = 1, 2, \ldots, m$.

(ii) If the subsystem i is decoupled from the others and it has no modeling error, then $e_{i,y_i} \in L_2$. From (59), $\dot{y}_i$ is shown to be bounded as $\dot{y}_i^*$ is bounded. It follows that $e_{i,y_i} \to 0$ as $t \to \infty$.

(iii) In Wen (1994), the strength of the unmodeled interactions can be allowed arbitrarily large. However, the interactions should satisfy certain structural condition. In this paper, such a structural condition has been relaxed. But the interactions should satisfy a more conservative condition (58), which implies the interactions should be sufficiently weak.

5. *Conclusions*

In this paper, a scheme for designing decentralized adaptive controllers using the techniques of integrator backstepping is presented. With the local normalization signals introduced in this paper, the structural conditions on subsystem interactions of earlier schemes using similar techniques can be removed. The tracking problem and the effects of bounded external disturbances are also considered. It has been shown that global stability of the overall adaptive feedback system can be ensured provided the strength of the interactions and subsystem unmodeled dynamics is sufficiently weak. For each subsystem, the effect of the modeling error, including interactions from other subsystems, can be allowed to have infinite memory. Despite the modeling error, we have shown that small in the mean tracking error can be achieved. If a subsystem is decoupled from the rest and has no local unmodeled dynamics and disturbances, perfect tracking of a reference trajectory in that subsystem is ensured.

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