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Brief Paper

Robust adaptive control of uncertain discrete-time systems[1]

Ying Zhang, Changyun Wen*, Yeng Chai Soh

School of Electrical and Electronics Engineering, Nanyang Technological University, Singapore 639798, Singapore

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Abstract

In this paper, a robust adaptive controller for a class of nonlinear uncertain discrete-time systems is developed by combining the backstepping procedures with a simple parameter estimator subject to parameter projection. It is shown that the proposed controller can ensure boundedness of all signals in the overall adaptive systems in the presence of unmodelled dynamics and disturbances. It can also guarantee that the tracking error is bounded by a function of the size of the unmodelled dynamics. In the ideal case when there are no unmodelled dynamics and disturbances, perfect tracking is ensured. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

In the last few years, adaptive control of nonlinear continuous-time systems have drawn extensive attention and many significant progresses have been made (Kanellakopoulos, 1995a; Kokotovic, 1992; Kanellakopoulos et al., 1992; Pomet and Praly, 1992; Krstic et al., 1995; Krstic and Kokotovic, 1995). To overcome some restrictions such as *matching conditions* and *overparameterisation* that the earlier established adaptive control methods suffer from, a promising backstepping technique was developed methodologically in Kanellakopoulos et al. (1992), Krstic et al. (1995) and Kanellakopoulos (1995a) where an adaptive controller for a large class of nonlinear systems can be designed in a systematic framework. In contrast with the conventional approaches based on certainty equivalence principle, the design of the control law and the parameter update law are carried out at the same time in the backstepping design method. This can provide better transient performance. However, the results obtained on continuous-time systems are not applicable to the discrete-time systems where the increment of Lyapunov function is no longer a linear function with respect to the increments of parameter estimates.

With the increasing applications of advanced computer technologies in industries, it is much more meaningful to implement adaptive control for nonlinear discrete-time systems. So far, however, only a few results have been reported on this topic (Kung and Womach, 1983; Agarwal and Sebory, 1987; Zhang and Lang, 1989; Lin and Yong, 1992; Song and Grizzle, 1993; Kanellakopoulos, 1995b; Yeh and Kokotovic, 1995). However, only in Yeh and Kokotovic (1995) an adaptive controller was designed by using the backstepping technique to achieve tracking of a reference signal for a class of nonlinear discrete-time systems. It was shown that under certain geometric conditions a large class of discrete-time nonlinear systems could be transformed into the *parametric-strict-feedback* form and the *parametric-pure-feedback* form for which the backstepping design approach could be applied. By using various update laws available in Goodwin and Sin (1984) and ultilizing their properties, the global boundedness and convergence were shown to be achieved without employing Lyapunov functions in the design. But the results of Yeh and Kokotovic (1995) were obtained only in the ideal case without considering nonparametric uncertainties such as unmodelled dynamics and external

* Corresponding author. Tel.: + 65-7994947; fax: + 65-7912687; e-mail: ecywen@ntu.edu.sg.

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disturbances which are usually inevitable in practical situations. The design of robust adaptive controllers for nonlinear discrete-time systems with uncertainties remains an unresolved problem. In this paper we will address this issue.

This paper presents a design approach of robust adaptive control for a class of nonlinear discrete-time systems with both parametric and nonparametric uncertainties. In our design, the backstepping procedure incorporating a simple parameter projection update law is employed to obtain the desired controller. An intermediate constant, which can be chosen without *a priori* knowledge of the unmodelled dynamics, is introduced in the normalising term of the parameter estimator to allow for the nonparametric uncertainties in the estimators properties. Using a similar stability analysis method developed in Wen (1989) and Wen and Hill (1992), it is shown that the proposed adaptive controller can ensure boundedness of all the signals in the closed-loop system even in the presence of unmodelled dynamics and disturbances. It can also ensure ε-small in the mean tracking error. When the disturbances and the unmodelled dynamics are removed, the ideal results obtained in Yeh and Kokotovic (1995) are still preserved. In particular, a perfect tracking is achieved.

The rest of the paper is organised as follows. Section 2 describes the class of nonlinear uncertain discrete-time systems to be controlled and Section 3 presents the design of the adaptive controllers in the presence of uncertainties. The stability of the adaptive system is analysed in Section 4. Finally, the paper is concluded in Section 5.

2. Problem formulation

The nonlinear discrete-time system under consideration is described by

$$\begin{aligned}
x_1^{t+1} &= x_2^t + \theta^T\alpha_1(x_1^t) + \eta_1(t),\\
x_2^{t+1} &= x_3^t + \theta^T\alpha_2(x_1^t, x_2^t) + \eta_2(t),\\
&\vdots \\
x_{n-1}^{t+1} &= x_n^t + \theta^T\alpha_{n-1}(x_1^t, x_2^t, \dots, x_{n-1}^t) + \eta_{n-1}(t),\\
x_n^{t+1} &= \theta^T\alpha_n(x_1^t, x_2^t, \dots, x_n^t) + \eta_n(t) + u(t),\\
y(t) &= x_1(t),
\end{aligned} \quad (1)$$

where $u(t)$ and $y(t)$ represent the system input and output, respectively, and θ is the unknown parameter vector in R^p. For each $1 \le i \le n$, $\alpha_i(x_1^t, \dots, x_i^t)$ are known nonlinear functions in $C(R^i, R^p)$ satisfying $\alpha_i(0) = 0$, and $\eta_i(t)$ are unknown functions. For simplicity of illustration, $\alpha_i(x_1^t, x_2^t, \dots, x_i^t)$ are denoted by $\alpha_i(t)$ for each $i = 1, 2, \dots, n$ in the remaining parts of this paper.

The discrete-time system described by Eq. (1) has two types of uncertainties. One is the parametric uncertainty denoted by the unknown parameter vector θ. Usually, the range of θ can be considered to be known a priori, which leads to the following assumption.

Assumption A.1. θ lies in a known compact set Θ, i.e. $\theta \in \Theta = \{\theta : \|\theta\| \le k_\theta; \|\theta - \theta'\| \le k_\theta, \forall \theta' \in \Theta\}$, where k_θ is a known constant.

Another kind of uncertainty appearing in system (1) is nonparametric. It is described by the unknown functions $\eta_i(t)$ which are due to unmodelled dynamics, external disturbances and time variations. As shown in Wen and Hill (1992), they are usually characterised by

$$\eta_i(t) < c_{\eta,i}\varepsilon \sup_{0 \le \tau \le t} \|[x_1^\tau, x_2^\tau, \dots, x_i^\tau]^T\| + d_i$$

for $i = 1, 2, \dots, n$,

where $c_{\eta,i}$ are known constants. Taking $c_\eta = \max_{1 \le i \le n}\{c_{\eta,i}\}$ and $d = \max_{1 \le i \le n}\{d_i\}$, we impose the following assumption about the nonparametric uncertainty.

Assumption A.2.

$$\eta_i(t) \le c_\eta \varepsilon \|[x_1^t, x_2^t, \dots, x_n^t]^T\| + d.$$

It will be shown later that knowledge of ε and d is not required to implement the adaptive controller.

The adaptive control problem is to obtain a control law for plant (1) such that all the signals in the resulting closed-loop system are bounded for arbitrary bounded reference set-point $y_m(t)$ and initial conditions. It is also desirable that for a certain known gain K, the tracking error $|y(t) - Ky_m(t)|$ is small in some sense. To solve the problem, an additional assumption on the nonlinear functions $\alpha_i(t)$ is required.

Assumption A.3. All the known nonlinear functions $\alpha_i(t)$ are Lipschitz functions, i.e.

$$\|\alpha_i(x_i(t)) - \alpha_i(x_i'(t))\| \le k_\alpha \|x_i(t) - x_i'(t)\|,$$

where $x_i(t), x_i'(t) \in C(R, R^i)$ and k_α is known a priori.

3. Adaptive control design using backstepping technique

Suppose that M_0 is an intermediate positive constant such that $\|x(0)\| \le M_0$, $\|y_m(t)\|_\infty \le M_0$, and $d/M_0 < \delta$ for a sufficiently small δ, where $x(0)$ denotes the initial conditions of the system. It is noted that for a given system, such an intermediate constant can always be found for any bounded initial conditions, set-point and disturbances. Then the desired adaptive controller can be obtained by performing the backstepping procedures as in

Table 1
Robust backstepping adaptive controller

Coordinate transformation:	
$z_1^t = x_1^t$	(2)
$z_2^t = x_2^t + \hat{\theta}_1(t)^{\mathrm{T}}\alpha_1(t)$	(3)
$z_3^t = x_3^t + \hat{\theta}_2(t)^{\mathrm{T}}\alpha_2(t) + \hat{\theta}_1(t)^{\mathrm{T}}\alpha_1(z_2^t)$	(4)
$z_{j+1}^t = x_{j+1}^t + \hat{\theta}_j(t)^{\mathrm{T}}\alpha_j(t) + \sum_{k=1}^{j-1}\hat{\theta}_k(t)^{\mathrm{T}}\bar{\alpha}_{k,j}(t), \quad 3 \le j \le n-1$	(5)
with	
$\alpha_1(t) = \alpha_1(z_1^t)$	(6)
$\chi_2(t+1) \triangleq \hat{\theta}_1(z_1^{t+1})^{\mathrm{T}}\alpha_1(t+1) - \hat{\theta}_1(t)^{\mathrm{T}}\alpha_1(z_2^t)$	(7)
$\chi_j(t+1) \triangleq \sum_{k=1}^{j-1}[\hat{\theta}_k(t+1)^{\mathrm{T}}\bar{\alpha}_{k,j-1}(t+1) - \hat{\theta}_k(t)^{\mathrm{T}}\bar{\alpha}_{k,j}(t)]$	(8)
$\chi_n(t+1) = \sum_{k=1}^{n-1}[\hat{\theta}_k(t+1)^{\mathrm{T}}\bar{\alpha}_{k,n-1}(t+1) - \hat{\theta}_k(t)^{\mathrm{T}}\bar{\alpha}_{k,n}(t)]$	(9)
$\bar{\alpha}_{i,j}(t) = \alpha_i\Big(z_{j-i+1}^t, z_{j-i+2}^t - \hat{\theta}_1(t)^{\mathrm{T}}\bar{\alpha}_{1,j-i+1}(t), \ldots, z_{j-i+k}^t - \sum_{l=1}^{k-1}\hat{\theta}_l(t)^{\mathrm{T}}\bar{\alpha}_{l,j-i+k-1}(t), \ldots, z_j^t - \sum_{l=1}^{i-1}\hat{\theta}_l(t)^{\mathrm{T}}\bar{\alpha}_{l,j-1}(t)\Big), \quad 1 \le i \le j-1$	(10)
Adaptive laws:	
$\hat{\theta}_j(t+1) = \wp\left\{\hat{\theta}_j(t) + \frac{\alpha_j(t)e_j(t+1)}{1 + M_0 + \lVert\alpha_j(t)\rVert^2}\right\}, \quad 1 \le j \le n$	(11)
with	
$e_1(t+1) \triangleq z_1^{t+1} - z_2^t$	(12)
$e_2(t+1) \triangleq z_2^{t+1} - z_3^t - \chi_2(t+1)$	(13)
$e_j(t+1) \triangleq z_j^{t+1} - z_{j+1}^t - \chi_j(t+1). \quad 3 \le j \le n$	(14)
$e_n(t+1) \triangleq z_n^{t+1} - y_m(t+n) - \chi_n(t+1) - \sum_{i=1}^{n} f_i z_i^t$	(15)
Control law:	
$u(t) = y_m(t+n) - \hat{\theta}_n(t)^{\mathrm{T}}\alpha_n(t) - \sum_{k=1}^{n-1}\hat{\theta}_k(t)^{\mathrm{T}}\bar{\alpha}_{k,n}(t)$	(16)

Yeh and Kokotovic (1995). For clearance, the obtained controller are summarised in Table 1.

Summarising the above steps, the resulting closed-loop system is expressed by

$$z(t+1) = Fz(t) + by_m(t+n) + \Psi(t+1) + \boldsymbol{e}(t+1), \quad y(t) = c^{\mathrm{T}}z(t), \tag{17}$$

$$\hat{\theta}_i(t+1) = \wp\left\{\hat{\theta}_i(t) + \frac{(z_i^{t+1} - z_i^t - \chi_i(t))\alpha_i(t)}{1 + M_0 + \lVert\alpha_i(t)\rVert^2}\right\}, \quad 1 \le i \le n-1, \tag{18}$$

$$\hat{\theta}_n(t+1) = \wp\left\{\hat{\theta}_n(t) + \frac{(z_n^{t+1} - y_m(t+n) - \chi_n(t))\alpha_n(t)}{1 + M_0 + \lVert\alpha_n(t)\rVert^2}\right\}, \tag{19}$$

where

$$z(t) = \begin{bmatrix} z_1^t \\ z_2^t \\ \vdots \\ z_n^t \end{bmatrix}, \quad F = \left[\begin{array}{c|c} \mathbf{0} & I_{(n-1)\times(n-1)} \\ 0 & \mathbf{0} \end{array}\right] \in R^{n\times n},$$

$$b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in R^n, \qquad c = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in R^n, \tag{20}$$

$$\boldsymbol{e}(t+1) \triangleq [e_1(t+1), e_2(t+1), \ldots, e_n(t+1)]^{\mathrm{T}}, \tag{21}$$

$$\Psi(t+1) \triangleq [\chi_1(t+1), \chi_2(t+1), \ldots, \chi_n(t+1)]^{\mathrm{T}}, \tag{22}$$

with

$$\chi_1(t+1) = 0. \tag{23}$$

From the definitions of z_j^t, it is trivial to show that the relationship between the new state variables $z(t)$ and the original states $x(t) \triangleq [x_1^t, x_2^t, \ldots, x_n^t]^{\mathrm{T}} \in R^n$ can be specified by the following lemma.

Lemma 1. *For $z(t)$ obtained by Eqs.* (2)–(5), *we have*

$$b_l\lVert x(t)\rVert \le \lVert z(t)\rVert \le b_u\lVert x(t)\rVert, \tag{24}$$

where b_l and b_u are constants which depend on k_α and k_θ.

The properties of estimator (18) and (19) are summarised in the following lemma, which will be used in the next section to set up the robust stability of the closed-loop system.

Lemma 2. *Assume that*

$$\lVert x(t_0 - 1)\rVert \le M_0, \quad \lVert x(\tau)\rVert > M_0, \tau = t_0, t_0+1, \ldots, t-1$$

and $\lVert x(\tau_1)\rVert < M_1$, $\tau_1 = t_0, t_0+1, \ldots, t-1$ where M_1 is a constant such that $M_1^2 = k_1M_0^2 + k_2 > M_0^2$ where k_1 and k_2 are constants which will be determined in the later discussion. Then

(1)

$$|e_i(t_0)| \le (k_\alpha k_\theta + a_1)M_0 + a_1, \quad \forall i, \tag{25}$$

$$|\tilde{e}_i(t+1)| \le k_\theta + a_1, \quad \forall t \ge t_0, \forall i, \tag{26}$$

where

$$\tilde{e}_i(t+1) \triangleq \frac{e_i(t+1)}{(1 + M_0 + \lVert\alpha_i(t)\rVert^2)^{1/2}}, \tag{27}$$

$$a_1 = c_\eta \varepsilon (k_1^{1/2} + k_1^{1/2}) + \delta. \tag{28}$$

(2)

$$\|\hat{\theta}_i(t+1) - \hat{\theta}_i(t)\| \le |\tilde{e}_i(t+1)|, \quad \forall i, \tag{29}$$

(3)

$$\sum_{\tau=t_0}^{t+1} |\tilde{e}_i(\tau)|^2 \le k_\theta^2 + (a_2 + a_3)(t - t_0), \quad \forall i, \tag{30}$$

where

$$a_2 = 2(k_\theta(k_1^{1/2} + k_2^{1/2}) + 2c_\eta\varepsilon(k_1 + k_2))c_\eta\varepsilon,$$
$$a_3 = 2\delta(2\delta + k_\theta). \tag{31}$$

(4)

$$\|\chi_i(t+1)\| \le c_1 \left\| \begin{bmatrix} e_1(t+1) \\ e_2(t+1) \\ \vdots \\ e_{i-1}(t+1) \end{bmatrix} \right\| + c_2 \left\| \begin{bmatrix} z_1^t \\ z_2^t \\ \vdots \\ z_i^t \end{bmatrix} \right\| \left\| \begin{bmatrix} \tilde{e}_1(t+1) \\ \tilde{e}_2(t+1) \\ \vdots \\ \tilde{e}_{i-1}(t+1) \end{bmatrix} \right\|, \quad \forall i, \tag{32}$$

$$\|\chi_i(t_0)\| \le (c_3 + c_4 a_1) M_0 + c_5 a_1 + c_6, \quad \forall i, \tag{33}$$

where $c_j, (j = 1, 2, \dots, 6)$ *are constants depending on* k_α *and* k_θ.

Proof. (1) From the definitions of $e_i(t+1)$, we have

$$e_i(t+1) \triangleq z_i^{t+1} - z_{i+1}^t - \chi_i(t+1) = (\theta - \hat{\theta}_i(t))^{\mathrm{T}} \alpha_i(t) + \eta_i(t)$$
$$= -\tilde{\theta}_i(t)^{\mathrm{T}} \alpha_i(t) + \eta_i(t) \tag{34}$$

with $\tilde{\theta}_i(t) \triangleq \hat{\theta}_i(t) - \theta$.

Applying Assumptions A.1–A.3 gives

$$|e_i(t+1)| \le k_\theta \|\alpha_i(t)\| + c_\eta \varepsilon \sup_{0 < \tau \le t} \|x(\tau)\| + d$$
$$\le k_\theta k_\alpha \|[x_1^t, x_2^t, \dots, x_n^t]^{\mathrm{T}}\|$$
$$+ c_\eta \varepsilon (k_1 M_0^2 + k_2)^{1/2} + d, \tag{35}$$

where $M_1^2 = k_1 M_0^2 + k_2$ is used.

Since $\|x(t_0 - 1)\| \le M_0$, it follows immediately that

$$|e_i(t_0)| \le k_\alpha k_\theta M_0 + c_\eta \varepsilon (k_1 M_0^2 + k_2)^{1/2} + d$$
$$\le (k_\alpha k_\theta + a_1) M_0 + a_1. \tag{36}$$

From Eqs. (27) and (35), we have

$$|\tilde{e}_i(\tau+1)| = \frac{|e_1(\tau+1)|}{(1 + M_0 + \|\alpha_i(\tau)\|^2)^{1/2}}$$
$$\le k_\theta + \frac{c_\eta \varepsilon (k_1 M_0^2 + k_2)^{1/2} + d}{(1 + M_0 + \|\alpha_i(\tau)\|^2)^{1/2}}$$
$$\le k_\theta + \frac{d}{(1 + M_0^2)^{1/2}} + \frac{c_\eta \varepsilon (k_1^1/2 M_0 + k_2^{1/2})}{(1 + M_0^2)^{1/2}}$$
$$\le k_\theta + \delta + c_\eta \varepsilon (k_1^{1/2} + k_2^{1/2}) \le k_\theta + a_1. \tag{37}$$

(2) Let $\hat{\theta}_{ip}(\tau)$ denote a parameter estimate before applying the projector $\wp$, i.e.

$$\tilde{\theta}_{ip}(\tau+1) - \tilde{\theta}_i(\tau) = \frac{\alpha_i(\tau) e_i(\tau+1)}{1 + M_0 + \|\alpha_i(\tau)\|^2}.$$

Then

$$\|\hat{\theta}_i(\tau+1) - \hat{\theta}_i(\tau)\| \le \|\theta_{ip}(\tau+1) - \hat{\theta}_i(\tau)\|$$
$$= \left\| \frac{\alpha_i(\tau) e_i(\tau+1)}{(1 + M_0 + \|\alpha_i(\tau)\|^2)} \right\|$$
$$\le |\tilde{e}_i(\tau+1)|, \quad \forall \tau. \tag{38}$$

(3) Introducing $v(t+1) = \tilde{\theta}_i^{\mathrm{T}}(t+1)\tilde{\theta}_i(t+1)$, we get

$$v(\tau+1) - v(\tau)$$
$$\le \tilde{\theta}_{ip}(\tau+1)^{\mathrm{T}} \tilde{\theta}_{ip}(\tau+1) - \tilde{\theta}(\tau)^{\mathrm{T}} \tilde{\theta}(\tau)$$
$$\le [\tilde{\theta}_{ip}(\tau+1) - \tilde{\theta}_i(\tau)]^{\mathrm{T}} [\tilde{\theta}_{ip}(\tau+1) - \tilde{\theta}_i(\tau) + 2\tilde{\theta}_i(\tau)] \tag{39}$$
$$= \frac{\|\alpha_i(\tau)\|^2 e_i(\tau+1)^2}{(1 + M_0 + \|\alpha_i(\tau)\|^2)^2} + \frac{2\alpha_i(\tau)^T \tilde{\theta}_i(\tau) e_i(\tau+1)}{1 + M_0 + \|\alpha_i(\tau)\|^2}$$
$$\le \frac{e_i(\tau+1)^2}{1 + M_0 + \|\alpha_i(\tau)\|^2} + \frac{2\alpha_i(\tau)^{\mathrm{T}} \tilde{\theta}_i(\tau) e_i(\tau+1)}{1 + M_0 + \|\alpha_i(\tau)\|^2}. \tag{40}$$

From Eq. (34), we have

$$2\alpha_i(\tau)^{\mathrm{T}} \tilde{\theta}_i(\tau) e_i(\tau+1)$$
$$= 2(\eta_i(\tau) - e_i(\tau+1)) e_i(\tau+1)$$
$$\le -2e_i(\tau+1)^2 + 2|\eta_i(\tau)|(k_\theta \|\alpha_i(\tau)\| + |\eta_i(\tau)|)$$
$$\le -2e_i(\tau+1)^2 + 2k_\theta \|\alpha_i(\tau)\| |\eta_i(\tau)| + 2|\eta_i(\tau)|^2. \tag{41}$$

Substituting Eq. (41) into Eq. (40) yields

$$v(\tau+1) - v(\tau)$$
$$\le \frac{-e_i^2(\tau+1)}{1 + M_0 + \|\alpha_i(\tau)\|^2} + 2\frac{k_\theta \|\alpha_i(\tau)\| |\eta_i(\tau)|}{1 + M_0 + \|\alpha_i(\tau)\|^2}$$
$$+ 2\frac{|\eta_i(\tau)|^2}{1 + M_0 + \|\alpha_i(\tau)\|^2}$$

$$\leq -|\tilde{e}_i(\tau+1)|^2 + 2k_\theta \frac{c_\eta \varepsilon (k_1 M_0 + k_2)^{1/2} + d}{1 + M_0 + \|\alpha_i(\tau)\|^2} \|\alpha_i(\tau)\|$$

$$+ 4\frac{c_\eta^2 \varepsilon^2 (k_1 M_0^2 + k_2) + d^2}{1 + M_0 + \|\alpha_i(\tau)\|^2}$$

$$\leq -\tilde{e}_i(\tau+1)^2 + 2k_\theta \frac{c_\eta \varepsilon (k_1 M_0^2 + k_2)^{1/2}}{1 + M_0 + \|\alpha_i(\tau)\|^2} \|\alpha_i(\tau)\|$$

$$+ \frac{4c_\eta^2 \varepsilon^2 (k_1 M_0^2 + k_2)}{1 + M_0 + \|\alpha_i(\tau)\|^2} + \frac{4d^2 + 2dk_\theta \|\alpha_i(\tau)\|}{1 + M_0 + \|\alpha_i(\tau)\|^2}$$

$$\leq -\tilde{e}_i(\tau+1)^2 + 2k_\theta c_\eta \varepsilon \frac{k_1^{1/2} M_0 + k_2^{1/2}}{(1 + M_0^2)^{1/2}}$$

$$+ 4c_\eta^2 \varepsilon^2 \frac{k_1 M_0^2 + k_2}{1 + M_0^2} + \frac{4\delta^2 M_0^2 + 2k_\theta \delta M_0 \|\alpha_i(\tau)\|}{1 + M_0 + \|\alpha_i(\tau)\|^2}$$

$$\leq -\tilde{e}_i(\tau+1)^2 + 2k_\theta c_\eta \varepsilon (k_2^{1/2} + k_2^{1/2})$$

$$+ 4c_\eta^2 \varepsilon^2 (k_1 + k_2) + 4\delta^2 + 2k_\theta \delta. \tag{42}$$

Therefore,

$$\tilde{e}_i(\tau+1)^2 \leq v(\tau) - v(\tau+1) + a_2 + a_3. \tag{43}$$

Summing both sides of Eq. (43) gives

$$\sum_{\tau=t_0+1}^{t} |\tilde{e}_i(\tau)|^2 \leq \|\tilde{\theta}_i(t_0)\|^2 - \|\tilde{\theta}_i(t)\|^2 + (a_{12} + a_{13})(t - t_0), \tag{44}$$

which results in Eq. (30) by applying Assumption A.3.

(4) To show the final result, the following inequality is useful.

$$|\tilde{\theta}_i(t+1)^T \bar{\alpha}_{i,k}(t+1) - \tilde{\theta}_i(t)^{\mathrm{T}} \bar{\alpha}_{i,k+1}(t)|$$

$$\leq c_{1,i} \left\| \begin{bmatrix} e_{k-i+1}(t+1) \\ e_{k-i+2}(t+1) \\ \vdots \\ e_k(t+1) \end{bmatrix} \right\| + c_{2,i} \left\| \begin{bmatrix} \chi_{k-i+1}(t+1) \\ \chi_{k-i+2}(t+1) \\ \vdots \\ \chi_k(t+1) \end{bmatrix} \right\|$$

$$+ c_{3,i} \left\| \begin{bmatrix} z_{k-i+2}^t \\ z_{k-i+3}^t \\ \vdots \\ z_{k+1}^t \end{bmatrix} \right\| \left\| \begin{bmatrix} \tilde{e}_1(t+1) \\ \tilde{e}_2(t+1) \\ \vdots \\ \tilde{e}_i(t+1) \end{bmatrix} \right\|, \tag{45}$$

where $c_{1,i}, c_{2,i}, c_{3,i}$ are constants depending on k_α and k_θ only.

An inductive strategy is adopted to examine the validation of this inequality. Firstly, consider $i = 1$. From the definitions of $\bar{\alpha}_{1,k}(t)$ and $e_k(t+1)$, Assumptions A.1 and A.3, and Eq. (29), we have

$$|\hat{\theta}_1(t+1)^{\mathrm{T}} \bar{\alpha}_{1,k}(t+1) - \hat{\theta}_1(t)^{\mathrm{T}} \bar{\alpha}_{1,k+1}(t)|$$

$$\leq |\hat{\theta}_1(t+1)^{\mathrm{T}} \bar{\alpha}_{1,k}(t+1) - \hat{\theta}_1(t+1)^{\mathrm{T}} \bar{\alpha}_{1,k+1}(t)|$$

$$+ |\hat{\theta}_1(t+1)^{\mathrm{T}} \bar{\alpha}_{1,k+1}(t) - \hat{\theta}_1(t)^{\mathrm{T}} \bar{\alpha}_{1,k+1}(t)|$$

$$\leq k_\theta k_\alpha |z_k^{t+1} - z_{k+1}^t| + k_\alpha |z_{k+1}^t| \, \|\hat{\theta}_1(t+1) - \hat{\theta}_1(t)\|$$

$$\leq k_\theta k_\alpha (|e_k(t+1)| + |\chi_k(t+1)|) + k_\alpha |z_{k+1}^t \| \tilde{e}_1(t+1)|, \tag{46}$$

which obviously supports the inequality (45). In particular, if $k = 1$, we obtain

$$|\chi_2(t+1)| \leq k_\theta k_\alpha |e_1(t+1)| + k_\alpha |z_2^t \| \tilde{e}_1(t+1)|, \tag{47}$$

where $\chi_1(t+1) = 0$ is used. This actually verifies Eq. (32) for $i = 1$.

Then consider $i = 2$, we have

$$|\hat{\theta}_2(t+1)^{\mathrm{T}} \bar{\alpha}_{2,k}(t+1) - \hat{\theta}_2(t)^{\mathrm{T}} \bar{\alpha}_{2,k+1}(t)|$$

$$\leq |\hat{\theta}_2(t+1)^{\mathrm{T}} \bar{\alpha}_{2,k}(t+1) - \hat{\theta}_2(t+1)^{\mathrm{T}} \bar{\alpha}_{2,k+1}(t)|$$

$$+ |\hat{\theta}_2(t+1)^T \bar{\alpha}_{2,k+1}(t) - \hat{\theta}_2(t)^{\mathrm{T}} \bar{\alpha}_{2,k+1}(t)|$$

$$\leq k_\theta k_\alpha \Bigg(\left\| \begin{bmatrix} z_{k-1}^{t+1} - z_k^t \\ z_k^{t+1} - z_{k+1}^t \end{bmatrix} \right\|$$

$$+ (|\hat{\theta}_1(t+1)^{\mathrm{T}} \bar{\alpha}_{1,k-1}(t+1) - \hat{\theta}_1(t)^{\mathrm{T}} \bar{\alpha}_{1,k}(t)|) \Bigg)$$

$$+ c \left\| \begin{bmatrix} z_k^t \\ z_{k+1}^t \end{bmatrix} \right\| \|\hat{\theta}_2(t+1) - \hat{\theta}_2(t)\|, \tag{48}$$

where $c = \max\{k_\alpha, k_\theta k_\alpha\}$.

Substituting Eq. (46) into Eq. (48) and using the definition of $e_k(t+1)$ gives

$$|\hat{\theta}_2(t+1)^{\mathrm{T}} \bar{\alpha}_{2,k}(t+1) - \hat{\theta}_2(t)^{\mathrm{T}} \bar{\alpha}_{2,k+1}(t)|$$

$$\leq c_{1,2} \left\| \begin{bmatrix} e_{k-1}(t+1) \\ e_k(t+1) \end{bmatrix} \right\| + c_{2,2} \left\| \begin{bmatrix} \chi_{k-1}(t+1) \\ \chi_k(t+1) \end{bmatrix} \right\|$$

$$+ c_{3,2} \left\| \begin{bmatrix} z_k^t \\ z_{k+1}^t \end{bmatrix} \right\| \left\| \begin{bmatrix} \tilde{e}_1(t+1) \\ \tilde{e}_2(t+1) \end{bmatrix} \right\|, \tag{49}$$

where $c_{1,2}, c_{2,2}, c_{3,2}$ are constants combining k_θ and k_α. Thus Eq. (45) holds for $i = 2$.

Now assume Eq. (45) holds for all $1 \le p \le i-1$, i.e.

$$|\hat{\theta}_p(t+1)^{\mathrm{T}}\bar{\alpha}_{p,k}(t+1) - \hat{\theta}_p(t)^{\mathrm{T}}\bar{\alpha}_{p,k+1}(t)| \le c_{1,p}\left\|\begin{bmatrix} e_{k-p+1}(t+1) \\ e_{k-p+2}(t+1) \\ \vdots \\ e_k(t+1)\end{bmatrix}\right\| + c_{2,p}\left\|\begin{bmatrix} \chi_{k-p+1}(t+1) \\ \chi_{k-p+2}(t+1) \\ \vdots \\ \chi_k(t+1)\end{bmatrix}\right\| + c_{3,p}\left\|\begin{bmatrix} z^t_{k-p+2} \\ z^t_{k-p+3} \\ \vdots \\ z^t_{k+1}\end{bmatrix}\right\| \left\|\begin{bmatrix} \tilde{e}_1(t+1) \\ \tilde{e}_2(t+1) \\ \vdots \\ \tilde{e}_p(t+1)\end{bmatrix}\right\|, \quad (50)$$

where $c_{1,p}, c_{2,p}$ and $c_{3,p}$ are constants depending upon k_α and k_θ. Then we show that Eq. (45) also holds for $p = i$.

From the definitions of $\bar{\alpha}_{i,k}(t)$, it follows that

$$\begin{aligned} &|\hat{\theta}_i(t+1)^{\mathrm{T}}\bar{\alpha}_{i,k}(t+1) - \hat{\theta}_i(t)^{\mathrm{T}}\bar{\alpha}_{i,k+1}(t)| \\ &\le |\hat{\theta}_i(t+1)^{\mathrm{T}}\bar{\alpha}_{i,k}(t+1) - \hat{\theta}_i(t+1)^{\mathrm{T}}\bar{\alpha}_{i,k+1}(t)| \\ &\quad + |\hat{\theta}_i(t+1)^{\mathrm{T}}\bar{\alpha}_{i,k+1}(t) - \hat{\theta}_i(t)^{\mathrm{T}}\bar{\alpha}_{i,k+1}(t)| \\ &\le k_\theta\Bigg|\alpha_i\Bigg(z^{t+1}_{k-i+1}, z^{t+1}_{k-i+2} - \hat{\theta}_1^{\mathrm{T}}\alpha_{1,k-i+1}(t+1), \ldots, z^{t+1}_k \\ &\quad - \sum_{l=1}^{i-1}\hat{\theta}_l(t+1)^{\mathrm{T}}\bar{\alpha}_{l,k-1}(t+1)\Bigg) \\ &\quad - \alpha_i\Bigg(z^t_{k-i+2}, z^t_{k-i+3} - \hat{\theta}_1^{\mathrm{T}}\alpha_{1,k-i+2}(t), \ldots, z^t_{k+1} \\ &\quad - \sum_{l=1}^{i-1}\hat{\theta}_l(t+1)^{\mathrm{T}}\bar{\alpha}_{l,k}(t+1)\Bigg)\Bigg| \\ &\quad + \|\bar{\alpha}_{i,k+1}(t)\|\,\|\hat{\theta}_i(t+1) - \hat{\theta}_i(t)\|. \end{aligned} \quad (51)$$

Using Assumption A.3 and Eq. (29), and noting that $\bar{\alpha}_{i,k+1}(t)$ is a function of $z^t_{k+1}, z^t_k, \ldots, z^t_{k-i+2}$, we have

$$\begin{aligned} &|\hat{\theta}_i(t+1)^{\mathrm{T}}\bar{\alpha}_{i,k}(t+1) - \hat{\theta}_i(t)^{\mathrm{T}}\bar{\alpha}_{i,k+1}(t)| \\ &\le k_\theta k_\alpha\left\|\begin{bmatrix} z^{t+1}_{k-i+1} - z^t_{k-i+2} \\ z^{t+1}_{k-i+2} - z^t_{k-i+3} \\ \vdots \\ z^{t+1}_k - z^t_{k+1}\end{bmatrix}\right\| \\ &\quad + K'\sum_{l=1}^{i-1}|\hat{\theta}_l(t+1)^{\mathrm{T}}\bar{\alpha}_{l,k-1}(t+1) - \hat{\theta}_l(t)^{\mathrm{T}}\bar{\alpha}_{l,k}(t)| \\ &\quad + K''\left\|\begin{bmatrix} z^t_{k-i+2} \\ z^t_{k-i+3} \\ \vdots \\ z^t_{k+1}\end{bmatrix}\right\| |\tilde{e}_i(t+1)|, \end{aligned} \quad (52)$$

where K' and K'' are constants depending on k_θ and k_α only.

Substituting Eqs. (14) and (50) into Eq. (52) gives

$$|\hat{\theta}_i(t+1)^{\mathrm{T}}\bar{\alpha}_{i,k}(t+1) - \hat{\theta}_i(t)^{\mathrm{T}}\bar{\alpha}_{i,k+1}(t)| \le c_{1,i}\left\|\begin{bmatrix} e_{k-i+1}(t+1) \\ e_{k-i+2}(t+1) \\ \vdots \\ e_k(t+1)\end{bmatrix}\right\| + c_{2,i}\left\|\begin{bmatrix} \chi_{k-i+1}(t+1) \\ \chi_{k-i+2}(t+1) \\ \vdots \\ \chi_k(t+1)\end{bmatrix}\right\| + c_{3,i}\left\|\begin{bmatrix} z^t_{k-i+2} \\ z^t_{k-i+3} \\ \vdots \\ z^t_{k+1}\end{bmatrix}\right\| \left\|\begin{bmatrix} \tilde{e}_1(t+1) \\ \tilde{e}_2(t+1) \\ \vdots \\ \tilde{e}_i(t+1)\end{bmatrix}\right\|, \quad (53)$$

where $c_{m,i}, (m = 1,2,3)$ are constants combining $c_{m,p}$, $(m = 1,2,3;\ 1 \le p \le i-1)$, k_α and k_θ. Thus, $c_{m,i}$, $(m = 1,2,3)$ depend on k_θ and k_α only. So far we have proved inequality (45).

Using Eq. (45), it follows immediately from the definition of $\chi_i(t+1)$ that:

$$|\chi_i(t+1)| \le c'_{1,i}\left\|\begin{bmatrix} e_1(t+1) \\ e_2(t+1) \\ \vdots \\ e_{i-1}(t+1)\end{bmatrix}\right\| + c'_{2,i}\left\|\begin{bmatrix} \chi_1(t+1) \\ \chi_2(t+1) \\ \vdots \\ \chi_{i-1}(t+1)\end{bmatrix}\right\| + c'_{3,i}\left\|\begin{bmatrix} z^t_1 \\ z^t_2 \\ \vdots \\ z^t_i\end{bmatrix}\right\| \left\|\begin{bmatrix} \tilde{e}_1(t+1) \\ \tilde{e}_2(t+1) \\ \vdots \\ \tilde{e}_{i-1}(t+1)\end{bmatrix}\right\|, \quad (54)$$

where $c'_{m,i}$, $(m = 1, 2, 3)$ are constants.

Since $\chi_1(t+1) = 0$ and $\chi_2(t+1) \le k_\theta k_\alpha |e_1(t+1)| + k_\alpha |z^t_2| \|\tilde{e}_1(t+1)|$, it can be shown from Eq. (54) that

$$|\chi_i(t+1)| \le c''_{1,i}\left\|\begin{bmatrix} e_1(t+1) \\ e_2(t+1) \\ \vdots \\ e_{i-1}(t+1)\end{bmatrix}\right\| + c''_{2,i}\left\|\begin{bmatrix} z^t_1 \\ z^t_2 \\ \vdots \\ z^t_i\end{bmatrix}\right\| \left\|\begin{bmatrix} \tilde{e}_1(t+1) \\ \tilde{e}_2(t+1) \\ \vdots \\ \tilde{e}_{i-1}(t+1)\end{bmatrix}\right\|, \quad (55)$$

where $c''_{1,i}, c''_{2,i}$ are constants combining k_θ and k_α.

Taking $c_1 = \max_{1 \le i \le n}\{c''_{1,i}\}$ and $c_2 = \max_{1 \le i \le n}\{c''_{2,i}\}$, Eq. (32) follows.

Using Eq. (25) and inequality

$$\begin{aligned} |e_i(t_0)| &\le (1 + M_0 + \|\alpha_i(t_0 - 1)\|)|\tilde{e}_i(t_0)| \\ &\le (1 + M_0 + k_\alpha\|x(t_0 - 1)\|)|\tilde{e}_i(t_0)|, \end{aligned}$$

we get (33). □

It is noted that the presented update law has the same properties as those given in Yeh and Kokotovic (1995) if the nonparametric uncertainty is removed, i.e. ε and δ are identically zeros. Moreover, the constants a_1 and a_2 in Lemma 2 are functions of ε and can be made sufficiently small by reducing ε.

4. Stability analysis

In this section we show that there exists a small constant ε^* such that for each $\varepsilon \in [0, \varepsilon^*]$, all the signals in closed-loop system (17) are bounded for any bounded initial conditions, bounded tracking reference signal and external disturbances. The stability analysis method used in Wen (1989) is adapted to derive the conclusion.

The stability of the closed-loop system can be established by the following theorem.

Theorem 1. *Consider the adaptive system consisting of plant* (1), *update law* (11) *and controller* (16). *Under Assumptions* A.1–A.3, *there exists constant* ε^* *such that for each* $\varepsilon \in [0, \varepsilon^*]$, $\|z(t)\|$ *is bounded for all bounded initial conditions, setpoints and disturbances. In addition, the tracking error satisfies that*

$$\sum_{\tau=t_{0+1}}^{t} |y(\tau) - y_m(\tau)| \le \beta_1 + \beta_2 0(\varepsilon, \delta)(t - t_0), \tag{56}$$

where β_1 *and* β_2 *are constants, and* $0(\varepsilon)$ *is a function such that* $\lim_{\substack{\varepsilon\to 0\\ \delta\to 0}} 0(\varepsilon, \delta) = 0$.

Proof. By the same idea as in Wen and Hill (1992), the time interval is divided into two subsequences

$$N_1 \triangleq \{t \in Z_+ |\ \|x(t)\| > M_0\}, \tag{57}$$

$$N_2 \triangleq \{t \in Z_+ |\ \|x(t)\| \le M_0\}, \tag{58}$$

where Z_+ denotes all positive integers.

Clearly, it is sufficient to show that $\|z(t)\|$ is bounded for $t \in N_1$ to obtain the boundedness of $z(t)$ in the whole time interval $[0, \infty)$. To this end, we choose time instant t_0 such that $t_0 - 1 \in N_2$ and $[t_0, t-1] \in N_1$. The inductive strategy is used to prove the result. Suppose that M is a positive constant such that $M_1 < M/b_u$. It is known from Lemma 1 that $\|z(t_0-1)\| \le b_u\|x(t_0-1)\| \le b_u M_1 \le M$. Now assume that $\|z(\tau)\| < M$ for $\tau = t_0, t_0+1, \ldots, t-1$. Then we show that $\|z(t)\| < M$.

The solution of system (23) is

$$z(t) = \Phi(t, t_0)z(t_0) + \sum_{\tau=t_0}^{t-1} \Phi(t, \tau+1) \times [by_m(\tau+n) + \Psi(\tau+1) + \boldsymbol{e}(\tau+1)] \tag{59}$$

$$= \Phi(t, t_0)[Fz(t_0-1) + by_m(t_0+n-1) + \Psi(t_0) + \boldsymbol{e}(t_0)] + \sum_{\tau=t_0}^{t-1} \Phi(t, \tau+1)[by_m(\tau+n) + \Psi(\tau+1) + \boldsymbol{e}(\tau+1)], \tag{60}$$

where $\Phi(t, \tau)$ is the transition function of the system $z(t+1) = Fz(t)$, i.e.

$$\Phi(t, \tau) = \begin{cases} F^{t-\tau}, & 0 \le t-\tau < n, \\ 0, & t-\tau \ge n. \end{cases}$$

Since F is a strictly stable matrix, the transition matrix $\Phi(t, \tau)$ satisfies

$$\|\Phi(t, \tau)\| \le C\sigma^{t-\tau},$$

where C and σ are constants, and $\sigma < 1$.

Using Eqs. (25), (26), (32), (33) and (59) gives

$$\|z(t)\| \le C\sigma^{t-t_0}[(C_1 + C_2 a_1)M_0 + C_3 a_1 + C_0] + \sum_{\tau=t_0}^{t-1} C\sigma^{t-\tau-1}[C_4\|z(\tau)\|\,\|\tilde{\boldsymbol{e}}(\tau+1)\| + C_5\|\tilde{\boldsymbol{e}}(\tau+1)\| + C_6 M_0], \tag{61}$$

where $\tilde{\boldsymbol{e}}(t+1) = [\tilde{e}_1(t+1), \tilde{e}_2(t+1), \ldots, \tilde{e}_n(t+1)]^T$, and $C_i, (i = 0, 1, \ldots, 6)$ are constants depending only upon k_θ and k_α.

Performing the same procedures as in Wen and Hill (1992), which involves squaring both sides of Eq. (61), applying Schwarz inequality and Grown-wall lemma as well as using the theorem about the arithmetic and geometric mean of a sequence, we have

$$\|z(t)\|^2 \le [C_7 + C_8 c_\eta^2 \varepsilon^2 (k_1 + k_2)]M_0^2 + C_9, \tag{62}$$

where C_9 is a constant combining k_α and k_θ.

Let

$$k_2 = C_9/b_u^2, \tag{63}$$

$$k_1 = \frac{1}{b_u^2}\max\left\{1, \frac{C_7 + C_8 c_\eta^2(\bar{\bar{\varepsilon}}^*)^2 C_9}{1 - C_8 c_\eta^2(\bar{\bar{\varepsilon}}^*)^2}\right\}, \tag{64}$$

where $\bar{\bar{\varepsilon}}^*$ is a constant satisfying $C_8 c_\eta^2(\bar{\bar{\varepsilon}}^*)^2 \le 1$.

Then it follows from Eq. (62) that

$$\|z(t)\|^2 \le b_u^2(k_1 M_0^2 + k_2) = b_u^2 M_1^2 \le M^2. \tag{65}$$

Therefore, taking $\varepsilon^* = \max\{\bar{\varepsilon}^*, \bar{\bar{\varepsilon}}^*\}$ confirms the first part of theorem.

Since the boundedness of all the states in the closed-loop system has been established, it follows immediately from the definitions of $\tilde{e}_i(t+1)$ and Eq. (30) that

$$\sum_{\tau=t_0+1}^{t} \|e(t)\| \le \beta_3 + \beta_4 0(\varepsilon, \delta)(t - t_0), \tag{66}$$

where β_3, β_4 are constants.

Using Eq. (32), we have

$$\sum_{\tau=t_0}^{t} \|\Psi(\tau+1)\| \le \beta_5 + \beta_6 0(\varepsilon, \delta)(t - t_0), \tag{67}$$

where β_5 and β_6 are constants. Applying Eqs. (66) and (67) to system (17), (56) follows. □

Remark 3.1. It is noted that if there is no nonparametric uncertainty, i.e. $\varepsilon = 0$ and $\delta = 0$, the stability of the adaptive system can also be ensured by Theorem 1. In such case, $\|\hat{\theta}_i(t+1) - \hat{\theta}_i(t)\| \to 0$, $|e_i(t+1)| \to 0$, and $|y(t) - y_m(t)| \to 0$, which means perfect tracking is achieved.

Remark 3.2. The adaptive controller in Table 1 was obtained by employing an updating law in each backstepping step. This gives rise to overparameterisation. To avoid this problem, an identifier-based indirect adaptive controller can be designed by following the same steps as in Table 1 but postponing the determination of the update law. Replacing all $\hat{\theta}_i$ in Table 1 with a common $\hat{\theta}$, the adaptive controller without overparameterisation is thus obtained by

$$u(t) = y_m(t+n) - \hat{\theta}(t)^{\mathrm{T}}\alpha_n(t) - \sum_{k=1}^{n-1} \hat{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{k,n}(t), \tag{68}$$

$$\hat{\theta}(t+1) = \wp\left\{\hat{\theta}(t) + \frac{\Phi(t)^{\mathrm{T}}\zeta(t+1)}{1 + M_0 + trace(\Phi(k)\Phi(k)^{\mathrm{T}})}\right\}, \tag{69}$$

where

$$\zeta(t+1) \triangleq z(t+1) - Fz(t) - by_m(t+n) - \Psi(t+1), \tag{70}$$

$$\Phi(t) = [\alpha_1(t), \alpha_2(t), \ldots, \alpha_n(t)]^{\mathrm{T}}, \tag{71}$$

$$\eta(t+1) = [\eta_1(t), \eta_2(t), \ldots, \eta_n(t)]^{\mathrm{T}}. \tag{72}$$

For this adaptive controller, we can also obtain the same robust stability results as stated in Lemma 1 and Theorem 1.

5. Conclusion

This paper studies the problem of adaptive control for a class of nonlinear discrete-time systems with unmodelled dynamics and external disturbances. Since the modelling errors are considered, the class of the nonlinear discrete-time systems for which the adaptive control can be applied has been enlarged. By combining the backstepping technique with parameter projection, a design scheme of robust adaptive controller is obtained. With this scheme, the boundedness of the whole adaptive closed-loop system is guaranteed for any bounded initial conditions, set-point signals and external disturbances. A small mean tracking error is also achieved. It is also clear that the stability and convergence results obtained in the ideal case are still preserved if there are no unmodelled dynamics and disturbances. Particularly in this ideal case, perfect tracking of a reference trajectory is ensured.

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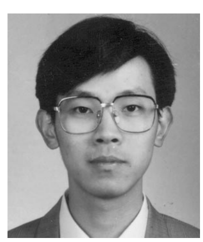
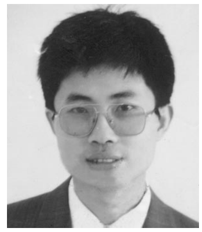


Dr Ying Zhang received his BE, ME and PhD degrees in Electrical Engineering in 1989, 1992 and 1995, respectively, all from Southeast University, Nanjing, People's Republic of China. From 1996 to 1998, he was a post doctoral fellow and later a research fellow in the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. Currently, he is with Gintic Institute of Manufacturing Technology. His research interests are in the areas of system identification, adaptive control and signal procssing.

Yeng Chai Soh received the BEng (Hons. I) degree in electrical and electronic engineering from the University of Canterbury, New Zealand, in 1983, and the PhD degree in electrical engineering from the University of Newcastle, Australia, in 1987.

From 1986 to 1987, he was a research assistant in the Department of Electrical and Computer Engineering, University of Newcstle. He joined the Nanyang Technological University, Singapore, in 1987 where he is currently an associate professor and the Head of the Control and Instrumentation Division, School of Electrical and Electronic Engineering. His current research interests are in the areas of adaptive control, robust system theory and applications, estimation and filtering, and model reduction.



Changyun Wen received his BEng from Xian Jiaotong University in 1983 and PhD from the University of Newcastle in Australia. From August 1989 to August 1991, he was a Postdoctoral Fellow at the University of Adelaide, Australia. Since August 1991, he has been with the School of Electrical and Electronic Engineering at Nantang Technological University where he is currently a Senior Lecturer. His major research interests are in the areas of adaptive control, learning control, robust control, signal processing and their applications.