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Brief Paper

Robust adaptive control of nonlinear discrete-time systems by backstepping without overparameterization☆

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Abstract

This paper presents a robust backstepping adaptive controller to overcome the overparameterization problem in adaptive control of nonlinear discrete-time systems. It is shown that the proposed controller can guarantee the global boundedness of the states of the whole adaptive system in the presence of time-varying parametric and nonparametric uncertainties. It can also ensure that the tracking error falls within a compact set whose size is proportional to the magnitude of the uncertainties and disturbances. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Through the use of appropriate Lyapunov functions, much progress has been achieved in adaptive control of nonlinear continuous-time systems in the last few years (Kokotović, 1991; Kanellakopoulos, Kokotović, & Morse, 1991; Marino, Praly, & Kanellakopoulos, 1992; Pomet, & Praly, 1992; Krstic, Kanellakopoulos, & Kokotović, 1995; Kanellakopoulos, 1995). In contrast, very few results have been reported on nonlinear discrete-time systems (see Zhang, Wen, & Soh, 2000, and the references therein). This may be attributed to the difficulty to construct a discrete-time Lyapunov function whose increment is linear in the variations of its variables. As a preliminary study, the backstepping technique was employed in Yeh and Kokotović (1995) to design an adaptive controller for nonlinear discrete-time systems without using Lyapunov functions. In Zhao and Kanellakopoulos (1997a) and Zhao and Kanellakopoulos (1997b), another new recursive design scheme which is different from the standard backstepping approach was proposed for the adaptive control of nonlinear discrete-time systems. However, all these results in Yeh and Kokotović (1995); Zhao and Kanellakopoulos (1997a) and Zhao and Kanellakopoulos (1997b) were obtained in ideal cases where the systems are assumed to be modelled exactly. The design of an adaptive controller for uncertain nonlinear discrete-time systems remains unsolved. Again, the lack of suitable Lyapunov functions makes it a tough task to establish robust stability in the presence of model uncertainties.

Recently, the problem was addressed in Zhang et al. (1999) where local stability is guaranteed without using Lyapunov functions. Later, a similar backstepping adaptive controller was designed in Zhang, Wen, and Soh (2000) to achieve a global robust stability for a class of nonlinear discrete-time systems with uncertainties. However, the parameter estimation has to be performed in each backstepping step in Zhang et al. (2000) in order to obtain certain properties that are crucial to the robust stability analysis. This gives rise to the overparameterization problem as mentioned in Krstic et al. (1995). In the presence of unmodelled dynamics, this problem cannot be avoided in the same way as in Yeh and Kokotović (1995), where the parameter estimation is only carried out in the first step of backstepping design. Otherwise, only the unmodelled dynamics entering the system in the first system state equation can be considered in the properties of the parameter estimator. Also this problem

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cannot be easily solved by simply postponing the parameter estimation in Zhang et al. (2000) to the last backstepping step. This is because it is difficult to obtain those estimator properties in Zhang et al. (2000) which are crucial to the robust stability analysis as in Zhang et al. (2000).

In this paper, the overparameterization problem is addressed and an adaptive controller with only one parameter estimator is presented. As observed in Zhang et al. (1999, 2000), the parameter estimator needs to have similar properties as in Zhang et al. (2000) in order to establish robust stability in the same way as in Zhang et al. (2000). To this end, the parameter estimator in the proposed adaptive controller is not only postponed to the last step of the backstepping design but also constructed by using all the errors between the actual states and the virtual control variables in each backstepping step. In addition, all nonlinear system functions are employed in the normalization term of the parameter estimation. These make it possible to obtain those required properties by accounting for all the unmodelled dynamics no matter where they enter the system. It is shown that the proposed adaptive controller can guarantee the global stability of the adaptive system and a small-in-the-mean tracking error. Moreover, since only the lumped effect of the system nonlinearities are required in a sector, the restriction suffered by Zhang et al. (2000) are greatly relaxed, which greatly enlarges the class of systems considered in Zhang et al. (2000).

2. Problem formulation

Consider a class of uncertain nonlinear time-varying discrete-time systems described by

$$\begin{aligned}
x_1^{t+1} &= x_2^t + \theta^{\mathrm{T}}(t)\alpha_1(x_1^t) + \eta_1(t),\\
x_2^{t+1} &= x_3^t + \theta^{\mathrm{T}}(t)\alpha_2(x_1^t, x_2^t) + \eta_2(t),\\
&\vdots\\
x_{n-1}^{t+1} &= x_n^t + \theta^{\mathrm{T}}(t)\alpha_{n-1}(x_1^t, x_2^t, \dots, x_{n-1}^t) + \eta_{n-1}(t),\\
x_n^{t+1} &= \theta^{\mathrm{T}}(t)\alpha_n(x_1^t, x_2^t, \dots, x_n^t) + \eta_n(t) + u(t),\\
y(t) &= x_1(t),
\end{aligned} \tag{1}$$

where $u(t)$ and $y(t)$ represent the system input and output, respectively, and $\theta(t)$ is the unknown time-varying parameter vector in R^p. For each $1 \le i \le n$, $\alpha_i(x_1^t, \dots, x_i^t)$ is a known nonlinear function which is continuous and satisfies $\alpha_i(0) = 0$. For simplicity of illustration, $\alpha_i(x_1^t, x_2^t, \dots, x_i^t)$ are denoted by $\alpha_i(t)$ for each $i = 1, 2, \dots, n$ in the remaining parts of the paper.

In the discrete-time system described by (1), two kinds of uncertainties are considered. One is parametric uncertainty denoted by the unknown time-varying parameter vector $\theta(t)$. The other is the nonparametric uncertainty denoted by unknown functions $\eta_i(t)$, which may often be due to modelling errors and external disturbances. For these uncertainties, we usually have the following assumptions.

Assumption A.1. The unknown parameter $\theta(t)$ lies in a known convex compact set Θ, i.e.,

$$\theta(t) \in \Theta = \{\theta(t)\colon \|\theta(t)\| \le k_\theta; \|\theta(t) - \theta'(t)\| \le k_\theta, \forall \theta'(t) \in \Theta\}, \tag{2}$$

where k_θ is a positive constant.

Assumption A.2.

$$\sum_{t=t_0+1}^{t_0+N} \|\theta(t) - \theta(t-1)\| \le k_\varepsilon + \varepsilon_\theta N, \quad \forall t_0 \ge 0,\ N \ge 1, \tag{3}$$

where k_ε and ε_θ are constants.

Remark 2.1. As no smallness restriction is imposed on k_ε, this assumption not only allows for slowly time varying parameters in a uniform way as in Wen and Hill (1992) and Wen (1994), but also takes into account time-varying parameters with big jumps.

Assumption A.3. There exist constants ε and d such that

$$\eta_i(t) \le c_\eta \varepsilon \max_{0 \le \tau \le t-1} \|[x_1^\tau, x_2^\tau, \dots, x_n^\tau]^{\mathrm{T}}\| + d, \tag{4}$$

where c_η is a constant. It will be shown later that knowledge of ε and d is not required to implement the proposed adaptive controller.

Remark 2.2. From (4), it is noted that the modelling error $\eta_i(t)$ can have infinite memory as the function $\max_{0 \le \tau \le t-1} \|\cdot\|$ is included. However, this makes the stability analysis more difficult especially when the knowledge of ε and d are not available.

For nonlinear functions $\alpha_i(t)$, we have the following assumption.

Assumption A.4. All the known nonlinear functions $\alpha_i(t)$ satisfy the following two conditions:

$$k'_\alpha \|[x_1^t, x_2^t, \dots, x_n^t]^{\mathrm{T}}\| \le \sum_{i=1}^n \|\alpha_i(t)\| + \left\| \sum_{i=1}^n \alpha_i(t) \right\| \le k_\alpha \|[x_1^t, x_2^t, \dots, x_n^t]^{\mathrm{T}}\|, \tag{5}$$

$$\|\alpha_i(\xi(t)) - \alpha_i(\xi'(t))\| \le k_\alpha \|\xi(t) - \xi'(t)\|, \quad \forall \xi(t), \xi'(t) \in R^i, \tag{6}$$

where k'_α and k_α are arbitrary positive constants. All the norms in this paper are vector norms.

Remark 2.3. In comparison with the similar assumption employed in Zhang et al. (2000), (5) gives a weaker restriction on the nonlinearities as only the sum of $\|\alpha_i(t)\|$ is assumed to be bounded below by the norm of the states. This condition is easier to satisfy than that required in Zhang et al. (2000) where each of functions $\|\alpha_i(t)\|$ is required to be bounded below with a nonzero k'_α.

The adaptive control problem is to obtain a control law for plant (1) such that all the signals in the resulting closed-loop system are bounded for arbitrary bounded reference set-point $y_m(t)$ and initial conditions, and the tracking error $|y(t) - y_m(t)|$ is small in some sense.

3. Adaptive control design using backstepping technique

The desired controller can be obtained by performing the following backstepping procedures.

Step 1: Let

$$z_1^t = x_1^t, \tag{7}$$

$$z_2^t = x_2^t + \hat{\theta}(t)^{\mathrm{T}}\alpha_1(t). \tag{8}$$

Then

$$z_1^{t+1} = z_2^t + (\theta(t) - \hat{\theta}(t))^{\mathrm{T}}\alpha_1(t) + \eta_1(t). \tag{9}$$

The update law for $\hat{\theta}(t)$ will be obtained in the last step of the backstepping design.

Step j $(2 \le j \le n-1)$: To proceed, the following functions are needed:

$$\bar{\alpha}_{1,1}(t) \triangleq \alpha_1(z_1^t), \quad \bar{\alpha}_{1,2}(t) \triangleq \alpha_1(z_2^t), \tag{10}$$

$$\bar{\alpha}_{i,j}(t) \triangleq \alpha_i\Bigg(z_{j-i+1}^t, z_{j-i+2}^t - \hat{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{1,j-i+1}(t), \ldots, z_{j-i+l}^t - \sum_{k=1}^{l-1}\hat{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{k,j-i+l-1}(t), \ldots, z_j^t - \sum_{k=1}^{i-1}\hat{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{k,j-1}(t)\Bigg), \tag{11}$$

where $1 \le i \le j-1$.

Let

$$z_{j+1}^t = x_{j+1}^t + \hat{\theta}(t)^{\mathrm{T}}\alpha_j(t) + \sum_{k=1}^{j-1}\hat{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{k,j}(t). \tag{12}$$

Then

$$z_j^{t+1} = z_{j+1}^t + (\theta(t) - \hat{\theta}(t))^{\mathrm{T}}\alpha_j(t) + \chi_j(t+1) + \eta_j(t), \tag{13}$$

where

$$\chi_k(t+1) \triangleq \sum_{k=1}^{j-1}\hat{\theta}(t+1)^{\mathrm{T}}\bar{\alpha}_{k,j-1}(t+1) - \sum_{k=1}^{j-1}\hat{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{k,j}(t). \tag{14}$$

Step n: This is the last step of the backstepping design in which the control law and the adaptive law are given.

For system (1), the control law is taken as

$$u(t) = y_m(t+n) - \sum_{i=1}^{n} f_i z_i^t - \hat{\theta}(t)^{\mathrm{T}}\alpha_n(t) - \sum_{k=1}^{n-1}\hat{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{k,n}(t), \tag{15}$$

where

$$\bar{\alpha}_{i,n}(t) \triangleq \alpha_i\Bigg(z_{n-i+1}^t, z_{n-i+2}^t - \hat{\theta}(t)\bar{\alpha}_{1,n-i+1}(t), \ldots, z_j^t - \sum_{k=1}^{i-1}\hat{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{k,n-1}(t)\Bigg) \tag{16}$$

and f_i $(i = 1, 2, \ldots, n)$ are the coefficients of a strictly stable polynomial $F(q^{-1})$, i.e., $F(q^{-1}) = 1 + f_n q^{-1} + \cdots + f_1 q^{-n}$.

The update law for $\hat{\theta}(t)$ is given by

$$\hat{\theta}(t+1) = \wp\left\{\hat{\theta}(t) + \frac{\sum_{i=1}^{n}\alpha_i(t)\sum_{i=1}^{n}e_n(t+1)}{1 + \|\sum_{i=1}^{n}\alpha_i(t)\|^2 + \sum_{i=1}^{n}\|\alpha_i(t)\|^2}\right\}, \tag{17}$$

where $\wp\{\cdot\}$ denotes a projection operator and

$$e_1(t+1) \triangleq z_1^{t+1} - z_2^t, \tag{18}$$

$$e_j(t+1) \triangleq z_j^{t+1} - z_{j+1}^t - \chi_j(t+1), \tag{19}$$

$$e_n(t+1) \triangleq z_n^{t+1} + \sum_{i=1}^{n} f_i z_i^t - y_m(t+n) - \chi_n(t+1). \tag{20}$$

Compared with (Zhang et al., 2000), the parameter estimation is only carried out once in the last step. It is also noted that the parameter estimator given in (17) uses all the predicted errors in the backstepping steps, which will be found in the next section, it brings much convenience to the robust stability analysis.

Substituting (15) into (1), we have

$$z_n^{t+1} = y_m(t+n) - \sum_{i=1}^{n} f_i z_i^t + (\theta(t) - \hat{\theta}(t))^{\mathrm{T}}\alpha_n(t) + \chi_n(t+1) + \eta_n(t), \tag{21}$$

where

$$\chi_n(t+1) = \sum_{k=1}^{n-1}\hat{\theta}(t+1)^{\mathrm{T}}\bar{\alpha}_{k,n-1}(t+1) - \sum_{k=1}^{n-1}\hat{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{k,n}(t+1). \tag{22}$$

The resulting closed-loop system is expressed by

$$z(t+1) = Fz(t) + by_m(t+n) + \Psi(t+1) + \mathbf{e}(t+1),$$
$$y(t) = c^{\mathrm{T}}z(t), \tag{23}$$

$$\hat{\theta}(t+1) = \wp\left\{\hat{\theta}(t) + \frac{\sum_{i=1}^{n}e_i(t+1)\sum_{i=1}^{n}\alpha_i(t)}{1 + \|\sum_{i=1}^{n}\alpha_i(t)\|^2 + \sum_{i=1}^{n}\|\alpha_i(t)\|^2}\right\}, \tag{24}$$

where

$$F=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&1\\-f_1&-f_2&-f_3&\cdots&-f_n\end{bmatrix},\quad z(t)=\begin{bmatrix}z_1^t\\z_2^t\\\vdots\\z_n^t\end{bmatrix},$$

$$b=\begin{bmatrix}0\\\vdots\\0\\1\end{bmatrix}\in R^n,\quad c=\begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix}\in R^n, \tag{25}$$

$$\mathbf{e}(t+1)\triangleq[e_1(t+1),e_2(t+1),\ldots,e_n(t+1)]^{\mathrm{T}}, \tag{26}$$

$$\Psi(t+1)\triangleq[\chi_1(t+1),\chi_2(t+1),\ldots,\chi_n(t+1)]^{\mathrm{T}} \tag{27}$$

with

$$\chi_1(t+1)\triangleq 0. \tag{28}$$

4. Stability analysis

In this section, we will show that there exist small constants ε^* and ε_θ^* such that for each $\varepsilon\in[0,\varepsilon^*]$ and $\varepsilon_\theta\in[0,\varepsilon_\theta^*]$, all the signals in the closed-loop system (23) are bounded for any bounded initial conditions, bounded tracking reference signal and external disturbances. As observed in Zhang et al. (1999, 2000) the robust stability can be obtained using an inductive approach if the parameter estimator satisfies some appropriate properties as those in Zhang et al. (1999, 2000). Thus, the properties of the parameter estimator (24) are firstly investigated and summarised in the following lemma. The proof of the lemma is given in the Appendix.

Lemma 1. *Suppose M_0 is a positive constant satisfying that $\|x(0)\|\le M_0$, $\|y_m(t)\|_\infty<M_0$, and $d/(k'_\alpha M_0)<\delta$ for $0<\delta<1$, where $x(0)$ denotes the initial conditions of the system. Assume that there exist constants M_1, k_1 and k_2 such that*

$$\|x(t_0-1)\|\le M_0,\quad \|x(\tau)\|>M_0,$$

$$\tau=t_0,t_0+1,\ldots,t-1,$$

$$\|x(\tau_1)\|<M_1,\ \tau_1=0,1,\ldots,t-1,$$

$$M_1^2=k_1(k'_\alpha M_0)^2+k_2>M_0.$$

Then (i)

$$|e_i(t_0)|\le(k_\alpha k_\theta+a_1)M_0+a_1, \tag{29}$$

$$|\tilde{e}(t+1)|\le\frac{nk_\alpha}{k'_\alpha}k_\theta+a'_1,\quad \forall t\ge t_0, \tag{30}$$

where

$$\tilde{e}(t+1)=\frac{\sum_{i=1}^n e_i(t+1)}{(1+\|\sum_{i=1}^n\alpha_i(t)\|^2+\sum_{i=1}^n\|\alpha_i(t)\|^2)^{1/2}}, \tag{31}$$

$$a_1=c_\eta\varepsilon(k'_\alpha k_1^{1/2}+k_2^{1/2})+k'_\alpha\delta,$$

$$a'_1=n(c_\eta\varepsilon(k_1^{1/2}+k_2^{1/2})+\delta). \tag{32}$$

(ii)

$$\|\hat{\theta}(t+1)-\hat{\theta}(t)\|\le|\tilde{e}_i(t+1)|. \tag{33}$$

(iii)

$$\sum_{\tau=t_0}^{t-1}|\tilde{e}(\tau)|^2\le\overline{k_\theta^2}+(a_2+a_3)(t-t_0), \tag{34}$$

where

$$a_2=2(k_\theta(k_1^{1/2}+k_2^{1/2})+2c_\eta\varepsilon(k_1+k_2))nc_\eta\varepsilon+2\varepsilon_\theta\left(\left(\frac{3}{2}+\frac{k_\alpha}{k'_\alpha}\right)k_\theta+a'_1+\frac{1}{2}n\varepsilon_\theta\right),$$

$$a_3=2\delta(2\delta+k_\theta)n,\quad \overline{k_\theta^2}=nk_\theta^2+2k_\varepsilon\left(\left(\frac{3}{2}+\frac{k_\alpha}{k'_\alpha}\right)k_\theta+a'_1\right).$$

(iv)

$$\|\chi_i(t+1)\|\le c_1\|\mathbf{e}_{[1,i-1]}(t+1)\|+c_2\|\mathbf{z}_{[1,i]}(t)\||\tilde{e}(t+1)|, \tag{35}$$

$$\|\chi_i(t_0)\|\le(c_3+c_4a_1)M_0+c_5a_1, \tag{36}$$

where

$$\mathbf{z}_{[i,j]}(t)\triangleq[z_i^t,z_{i+1}^t,\ldots,z_j^t]^{\mathrm{T}}\in R^{(j-i+1)}, \tag{37}$$

$$\mathbf{e}_{[i,j]}(t)\triangleq[e_i(t),e_{i+1}(t),\ldots,e_j(t)]^{\mathrm{T}}\in R^{(j-i+1)}, \tag{38}$$

$$\chi_{[i,j]}(t)\triangleq[\chi_i(t),\chi_{i+1}(t),\ldots,\chi_j(t)]^{\mathrm{T}}\in R^{(j-i+1)} \tag{39}$$

and c_j, $(j=1,2,\ldots,5)$ are constants depending on k'_α, k_α and k_θ.

From this lemma, it can be seen that the proposed parameter estimator has similar properties as in Zhang et al. (1999, 2000). It is noted from the proof of the lemma that these properties are obtained through the use of all prediction errors in the parameter estimator rather than simply postponing the parameter estimation to the last backstepping step.

Remark 4.1. Note that M_0 is not a design parameter and it is only used for stability analysis. For any bounded $x(0)$ and $y_m(t)$, such a constant M_0 always exists.

With these properties, the stability together with a tracking property of the closed-loop system can be established by following a similar method as in Wen

(1994) and Zhang et al. (1999, 2000) where an inductive strategy is adopt. The details are omitted here due to space limit.

Theorem 1. *Consider the adaptive system consisting of plant* (1), *update law* (17) *and controller* (15). *Under Assumptions* A.1–A.4, *there exist constants* ε^* *and* ε_θ^* *such that for each* $\varepsilon \in [0, \varepsilon^*]$ *and* $\varepsilon_\theta \in [0, \varepsilon_\theta^*]$, $\|z(t)\|$ *is bounded for all bounded initial conditions and setpoints. In addition, the tracking error satisfies*

$$\sum_{\tau=t_0}^{t-1} \left| y(\tau) - \frac{1}{K} y_m(\tau) \right| \le \beta_1 + \beta_2 O(\varepsilon, \varepsilon_\theta)(t - t_0), \tag{40}$$

where $K = 1 + \sum_{i=1}^n |f_i|$, β_1 *and* β_2 *are constants, and* $O(\varepsilon, \varepsilon_\theta)$ *is a function such that* $\lim_{\varepsilon \to 0, \varepsilon_\theta \to 0} O(\varepsilon, \varepsilon_\theta) = 0$.

As a special case, it can be concluded that if there is no nonparametric uncertainity and the system parameters are constants, i.e., $\varepsilon = 0$, $\varepsilon_\theta = 0$ and $\delta = 0$, $\|\hat{\theta}(t+1) - \hat{\theta}(t)\| \to 0$, $|e_i(t+1)| \to 0$, and $|y(t) - (1/K) y_m(t)| \to 0$, which implies perfect tracking is achieved.

Remark 4.2. In the above theorem, ε^* and ε_θ^* play a role like the stability margin for the overall system. The existence of such constants show certain degree of robustness of the proposed controller.

ε and ε_θ depend on the magnitude of the ignored unmodelled dynamics and variations rate of the nominal parameters. Clearly, small magnitude will give small ε and ε_θ. In the ideal case, $\varepsilon = 0$ and $\varepsilon_\theta = 0$. Similar to all the results on robustness (adaptive or nonadaptive) with respect to unmodelled dynamics, it is not necessary to specify the values of ε and ε_θ as they are not design parameters (Narendra & Annaswamy, 1989).

5. Conclusion

This paper presents a scheme of designing adaptive controller for a class of nonlinear uncertain discrete-time systems. The proposed adaptive controller overcomes the overparameterization problem in adaptive control of nonlinear discrete-time systems by backstepping design. The success is attributed to employing all the prediction errors of all the backstepping steps in the parameter adaptive law. It is different from the results in Yeh and Kokotović (1995) and Zhang et al. (1999, 2000) where the overparameterization problem in the presence of unmodelled dynamics is usually very difficult to handle by simply postponing the parameter estimation to the last step. With the proposed controller, the global boundedness of the adaptive closed-loop system is guaranteed for any bounded initial conditions, set-point signals and external disturbances. Moreover, a small-in-the-mean tracking error can be achieved.

A.1 Appendix Proof of Lemma 1

In order to achieve the conclusions in Lemma 1, the following lemma is useful.

Lemma A.1. *Using the same denotations, we have the following inequality.*

$$\begin{aligned} &|\tilde{\theta}(t+1)^{\mathrm{T}} \bar{\alpha}_{i,k}(t+1) - \tilde{\theta}(t)^{\mathrm{T}} \bar{\alpha}_{i,k+1}(t)| \\ &\quad \le c_{1,i} \|\mathbf{e}_{[k-i+1,k]}(t+1)\| + c_{2,i} \|\chi_{[k-i+1,k]}(t+1)\| \\ &\qquad + c_{3,i} \|\mathbf{z}_{[k-i+2,k+1]}(t)\| \, |\tilde{e}(t+1)|. \end{aligned} \tag{A.1}$$

Proof. Here an inductive strategy is adopted to verify (A.1). First, consider $i = 1$. From the definitions of $\bar{\alpha}_{1,k}(t)$ and $e_k(t+1)$, (2), (6) and (33), we have

$$\begin{aligned} &|\hat{\theta}(t+1)^{\mathrm{T}} \bar{\alpha}_{1,k}(t+1) - \hat{\theta}(t)^{\mathrm{T}} \bar{\alpha}_{1,k+1}(t)| \\ &\quad \le |\hat{\theta}(t+1)^{\mathrm{T}} \bar{\alpha}_{1,k}(t+1) - \hat{\theta}(t+1)^{\mathrm{T}} \bar{\alpha}_{1,k+1}(t)| \\ &\qquad + |\hat{\theta}(t+1)^{\mathrm{T}} \bar{\alpha}_{1,k+1}(t+1) - \hat{\theta}(t)^{\mathrm{T}} \bar{\alpha}_{1,k+1}(t)| \\ &\quad \le k_\theta k_\alpha |z_k^t - z_{k+1}^t| + k_\alpha |z_{k+1}^t| \|\hat{\theta}(t+1) - \hat{\theta}(t)\| \\ &\quad \le k_\theta k_\alpha (|e_k(t+1)| + |\chi_k(t+1)|) + k_\alpha |z_{k+1}^t| |\tilde{e}(t+1)|, \end{aligned} \tag{A.2}$$

which obviously supports inequality (A.1). Particularly, if $k = 1$,

$$\chi_2(t+1) \le k_\theta k_\alpha |e_1(t+1)| + k_\alpha |z_2^t| |\tilde{e}(t+1)|, \tag{A.3}$$

where $\chi_1(t+1) = 0$ is used. This actually verifies (35) for $i = 1$.

Then consider $i = 2$, we have

$$\begin{aligned} &|\hat{\theta}(t+1)^{\mathrm{T}} \bar{\alpha}_{2,k}(t+1) - \hat{\theta}(t)^{\mathrm{T}} \bar{\alpha}_{2,k+1}(t)| \\ &\quad \le |\hat{\theta}(t+1)^{\mathrm{T}} \bar{\alpha}_{2,k}(t+1) - \hat{\theta}(t+1)^{\mathrm{T}} \bar{\alpha}_{2,k+1}(t)| \\ &\qquad + |\hat{\theta}(t+1)^{\mathrm{T}} \bar{\alpha}_{2,k+1}(t+1) - \hat{\theta}(t)^{\mathrm{T}} \bar{\alpha}_{2,k+1}(t)| \\ &\quad \le k_\theta k_\alpha \left(\left\| \begin{bmatrix} z_{k-1}^t - z_k^t \\ z_k^t - z_{k+1}^t \end{bmatrix} \right\| + (|\hat{\theta}(t+1)^{\mathrm{T}} \bar{\alpha}_{1,k}(t+1) \right. \\ &\qquad \left. - \hat{\theta}(t)^{\mathrm{T}} \bar{\alpha}_{1,k+1}(t)|) \right) \\ &\qquad + k_\alpha \left\| \begin{bmatrix} z_k^t \\ z_{k+1}^t \end{bmatrix} \right\| \|\hat{\theta}(t+1) - \hat{\theta}(t)\|. \end{aligned} \tag{A.4}$$

Substituting (A.2) into (A.4) and using the definition of $e_k(t+1)$ gives

$$\begin{aligned} &|\hat{\theta}(t+1)^{\mathrm{T}} \bar{\alpha}_{2,k}(t+1) - \hat{\theta}(t)^{\mathrm{T}} \bar{\alpha}_{2,k+1}(t)| \\ &\quad \le c_{1,2} \left\| \begin{bmatrix} e_{k-1}(t+1) \\ e_k(t+1) \end{bmatrix} \right\| + c_{2,2} \left\| \begin{bmatrix} \chi_{k-1}(t+1) \\ \chi_k(t+1) \end{bmatrix} \right\| \\ &\qquad + c_{3,2} \left\| \begin{bmatrix} z_k^t \\ z_{k+1}^t \end{bmatrix} \right\| |\tilde{e}(t+1)|, \end{aligned} \tag{A.5}$$

where $c_{1,2}, c_{2,2}$, and $c_{3,2}$ are constants combining k_θ and k_α. Thus (A.1) holds for $i = 2$.

Finally, assume (A.1) holds for all $1 \le p \le i-1$, i.e.,

$$|\tilde{\theta}(t+1)^{\mathrm{T}}\bar{\alpha}_{p,k}(t+1) - \tilde{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{p,k+1}(t)| \le c_{1,p}\|\mathbf{e}_{[k-p+1,k]}(t+1)\| + c_{2,p}\|\chi_{[k-p+1,k]}(t+1)\| + c_{3,p}\|\mathbf{z}_{[k-p+2,k+1]}(t)\|\|\tilde{e}(t+1)|, \tag{A.6}$$

where $c_{1,p}$, $c_{2,p}$ and $c_{3,p}$ are constants depending upon k_α and k_θ. Then we show that (A.1) is also true for $p = i$.

From the definitions of $\bar{\alpha}_{i,k}(t)$, it follows that

$$\begin{aligned}
&|\hat{\theta}(t+1)^{\mathrm{T}}\bar{\alpha}_{i,k}(t+1) - \hat{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{i,k+1}(t)| \\
&\le |\hat{\theta}(t+1)^{\mathrm{T}}\bar{\alpha}_{i,k}(t+1) - \hat{\theta}(t+1)^{\mathrm{T}}\bar{\alpha}_{i,k+1}(t)| \\
&\quad + |\hat{\theta}(t+1)^{\mathrm{T}}\bar{\alpha}_{i,k+1}(t) - \hat{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{i,k+1}(t)| \\
&\le k_\theta|\alpha_i(z_{k-i+1}^{t+1}, z_{k-i+2}^{t+1} - \hat{\theta}^{\mathrm{T}}(t+1)\alpha_{1,k-i+1}(t+1), \ldots, z_k^{t+1} \\
&\quad - \sum_{l=1}^{i-1}\hat{\theta}(t+1)^{\mathrm{T}}\bar{\alpha}_{l,k-1}(t+1)) \\
&\quad - \alpha_i(z_{k-i+2}^t, z_{k-i+3}^t - \hat{\theta}^{\mathrm{T}}(t+1)\alpha_{1,k-i+2}(t), \ldots, z_{k+1}^t \\
&\quad - \sum_{l=1}^{i-1}\hat{\theta}(t+1)^{\mathrm{T}}\bar{\alpha}_{l,k}(t+1))| \\
&\quad + \|\bar{\alpha}_{i,k+1}(t)\|\|\hat{\theta}(t+1) - \hat{\theta}(t)\|.
\end{aligned} \tag{A.7}$$

Using (6), (33) and noting that $\bar{\alpha}_{i,k+1}(t)$ is a function of $z_{k+1}^t, z_k^t, \ldots, z_{k-i+2}^t$, we have

$$\begin{aligned}
&|\tilde{\theta}(t+1)^{\mathrm{T}}\bar{\alpha}_{i,k}(t+1) - \tilde{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{i,k+1}(t)| \\
&\le k_\theta k_\alpha\|\mathbf{z}_{[k-i+1,k]}(t+1) - \mathbf{z}_{[k-i+2,k+1]}(t)\| \\
&\quad + K'\sum_{l=1}^{i-1}|\hat{\theta}(t+1)^{\mathrm{T}}\bar{\alpha}_{l,k-l}(t+1) - \hat{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{l,k-l+1}(t)| \\
&\quad + k_\alpha\|\mathbf{z}_{[k-i+2,k+1]}(t)\|\|\tilde{e}(t+1)|,
\end{aligned} \tag{A.8}$$

where K' is a constant depending on k_θ and k_α.

Substituting (19) and (A.6) into (A.8) gives

$$\begin{aligned}
&|\tilde{\theta}(t+1)^{\mathrm{T}}\bar{\alpha}_{i,k}(t+1) - \tilde{\theta}(t)^{\mathrm{T}}\bar{\alpha}_{i,k+1}(t)| \\
&\le c_{1,i}\|\mathbf{e}_{[k-i+1,k]}(t+1)\| + c_{2,i}\|\chi_{[k-i+1,k]}(t+1)\| \\
&\quad + c_{3,i}\|\mathbf{z}_{[k-i+2,k+1]}(t)\|\|\tilde{e}(t+1)|,
\end{aligned} \tag{A.9}$$

where $c_{m,i}$ $(m = 1,2,3)$ are constants combining $c_{m,p}$ $(m = 1,2,3; 1 \le p \le i-1)$, k_α and k_θ. Thus $c_{m,i}$ $(m = 1,2,3)$ are dependent on k_θ and k_α only. So, we have proved inequality (A.1).

Proof of Lemma 1. (i) From the definitions of $e_i(t+1)$, we have

$$\begin{aligned}
e_i(t+1) &\triangleq z_i^{t+1} - z_{i+1}^t - \chi_i(t+1) \\
&= (\theta - \hat{\theta}(t))^{\mathrm{T}}\alpha_i(t) + \eta_i(t) = -\tilde{\theta}(t)^{\mathrm{T}}\alpha_i(t) + \eta_i(t).
\end{aligned} \tag{A.10}$$

Applying Assumptions A.1, A.3 and A.4 gives

$$|e_i(t+1)| \le k_\theta\|\alpha_i(t)\| + c_\eta\varepsilon \max_{0<\tau\le t-1}\|x(\tau)\| + d \tag{A.11}$$

$$\le k_\theta k_\alpha\|[x_1^t, x_2^t, \ldots, x_i^t]^{\mathrm{T}}\| + c_\eta\varepsilon(k_1(k'_\alpha M_0)^2 + k_2)^{1/2} + d, \tag{A.12}$$

where $M_1^2 = k_1(k'_\alpha M_0)^2 + k_2$ is used.

Since $\|x(t_0-1)\| \le M_0$, it follows immediately that

$$\begin{aligned}
|e_i(t_0)| &\le k_\alpha k_\theta M_0 + c_\eta\varepsilon(k_1(k'_\alpha M_0)^2 + k_2)^{1/2} + d \\
&\le k_\alpha k_\theta M_0 + c_\eta\varepsilon(k'_\alpha k_1^{1/2} + k_2^{1/2})(M_0 + 1) + d \\
&\le k_\alpha k_\theta M_0 + c_\eta\varepsilon(k'_\alpha k_1^{1/2} + k_2^{1/2})M_0 + \delta k'_\alpha M_0 \\
&\quad + c_\eta\varepsilon(k'_\alpha k_1^{1/2} + k_2^{1/2}) + \delta k'_\alpha \\
&\le (k_\alpha k_\theta + a_1)M_0 + a_1.
\end{aligned} \tag{A.13}$$

From (31), (A.12) and Assumption A.4, we have

$$\begin{aligned}
|\tilde{e}(\tau+1)| &= \frac{\sum_{i=1}^n|e_i(\tau+1)|}{(1 + \|\sum_{i=1}^n\alpha_i(\tau)\|^2 + \sum_{i=1}^n\|\alpha_i(\tau)\|^2)^{1/2}} \\
&\le n\frac{k_\alpha}{k'_\alpha}k_\theta + n\frac{c_\eta\varepsilon(k_1(k'_\alpha M_0)^2 + k_2)^{1/2} + d}{(1 + \|\sum_{i=1}^n\alpha_i(\tau)\|^2 + \sum_{i=1}^n\|\alpha_i(\tau)\|^2)^{1/2}} \\
&\le n\frac{k_\alpha}{k'_\alpha}k_\theta + n\frac{c_\eta\varepsilon(k_1(k'_\alpha M_0)^2 + k_2)^{1/2} + d}{(1 + {k'_\alpha}^2 M_0^2)^{1/2}} \\
&\qquad \textit{for } \tau \ge t_0 \\
&\le n\frac{k_\alpha}{k'_\alpha}k_\theta + n\delta + nc_\eta\varepsilon(k_1^{1/2} + k_2^{1/2}) \le n\frac{k_\alpha}{k'_\alpha}k_\theta + a'_1.
\end{aligned} \tag{A.14}$$

(ii) Let $\hat{\theta}(\tau)$ denote a parameter estimate before applying a projector $\wp$, i.e.,

$$\hat{\theta}_p(\tau+1) - \hat{\theta}(\tau) = \frac{\sum_{i=1}^n\alpha_i(\tau)\sum_{i=1}^n e_i(\tau+1)}{1 + \|\sum_{i=1}^n\alpha_i(\tau)\|^2 + \sum_{i=1}^n\|\alpha_i(\tau)\|^2}.$$

Then

$$\begin{aligned}
\|\hat{\theta}(\tau+1) - \hat{\theta}(\tau)\| &\le \|\theta_p(\tau+1) - \hat{\theta}(\tau)\| \\
&= \frac{\|\sum_{i=1}^n\alpha_i(\tau)\|\|\sum_{i=1}^n e_i(\tau+1)|}{1 + \|\sum_{i=1}^n\alpha_i(\tau)\|^2 + \sum_{i=1}^n\alpha_i(\tau)\|^2} \le |\tilde{e}(\tau+1)|, \quad \forall\tau.
\end{aligned} \tag{A.15}$$

(iii) Introducing $v(t+1) = \tilde{\theta}^{\mathrm{T}}(t+1)\tilde{\theta}(t+1)$, we get

$$\begin{aligned}
v(\tau+1) - v(\tau) &\le \tilde{\theta}_p(\tau+1)^{\mathrm{T}}\tilde{\theta}_p(\tau+1) - \tilde{\theta}(\tau)^{\mathrm{T}}\tilde{\theta}(\tau) \\
&\le [\tilde{\theta}_p(\tau+1) - \tilde{\theta}(\tau)]^{\mathrm{T}}[\tilde{\theta}_p(\tau+1) - \tilde{\theta}(\tau) + 2\tilde{\theta}(\tau)]
\end{aligned} \tag{A.16}$$

$$
\begin{aligned}
&= \frac{(\sum_{i=1}^{n}\alpha_i(\tau))^2(\sum_{i=1}^{n}e_i(\tau+1))^2}{(1+\|\sum_{i=1}^{n}\alpha_i(\tau)\|^2+\sum_{i=1}^{n}\alpha_i(\tau)\|^2)^2}\\
&\quad + \frac{2\sum_{i=1}^{n}\alpha_i(\tau)^{\mathrm{T}}\tilde{\theta}(\tau)\sum_{i=1}^{n}e_i(\tau+1)}{1+\|\sum_{i=1}^{n}\alpha_i(\tau)\|^2+\sum_{i=1}^{n}\alpha_i(\tau)\|^2}\\
&\quad - 2[\theta(\tau)-\theta(\tau-1)]^{\mathrm{T}}\Bigg[\tilde{\theta}(\tau-1)\\
&\quad + \frac{\sum_{i=1}^{n}\alpha_i(\tau)\sum_{i=1}^{n}e_i(\tau+1)}{1+\|\sum_{i=1}^{n}\alpha_i(\tau)\|^2+\sum_{i=1}^{n}\alpha_i(\tau)\|^2}\\
&\quad - \frac{1}{2}(\theta(\tau)-\theta(\tau-1))\Bigg]\\
&\le \frac{(\sum_{i=1}^{n}e_i(\tau+1))^2}{1+\|\sum_{i=1}^{n}\alpha_i(\tau)\|^2+\sum_{i=1}^{n}\alpha_i(\tau)\|^2}\\
&\quad + \frac{2\sum_{i=1}^{n}\alpha_i(\tau)^{\mathrm{T}}\tilde{\theta}(\tau)\sum_{i=1}^{n}e_i(\tau+1)}{1+\|\sum_{i=1}^{n}\alpha_i(\tau)\|^2+\sum_{i=1}^{n}\alpha_i(\tau)\|^2}\\
&\quad - 2[\theta(\tau)-\theta(\tau-1)]^{\mathrm{T}}\Bigg[\tilde{\theta}(\tau-1)\\
&\quad + \frac{\sum_{i=1}^{n}\alpha_i(\tau)\sum_{i=1}^{n}e_i(\tau+1)}{1+\|\sum_{i=1}^{n}\alpha_i(\tau)\|^2+\sum_{i=1}^{n}\alpha_i(\tau)\|^2}\\
&\quad - \frac{1}{2}(\theta(\tau)-\theta(\tau-1))\Bigg].
\end{aligned}
\tag{A.17}
$$

From (A.10), we have

$$
\begin{aligned}
&2\sum_{i=1}^{n}\alpha_i(\tau)^{\mathrm{T}}\tilde{\theta}(\tau)\sum_{i=1}^{n}e_i(\tau+1)\\
&= 2\sum_{i=1}^{n}(\eta_i(\tau)-e_i(\tau+1))\sum_{i=1}^{n}e_i(\tau+1)\\
&\le -2\left(\sum_{i=1}^{n}e_i(\tau+1)\right)^2 + 2k_\theta\sum_{i=1}^{n}\|\alpha_i(\tau)\|\\
&\quad \times \sum_{i=1}^{n}|\eta_i(\tau)| + 2\left(\sum_{i=1}^{n}|\eta_i(\tau)|\right)^2.
\end{aligned}
\tag{A.18}
$$

Combining (4), (30), (31), (A.10), (A.17) and (A.8), we have

$$
\begin{aligned}
&v(\tau+1)-v(\tau)\\
&\le \frac{-(\sum_{i=1}^{n}e_i(\tau+1))^2}{1+\sum_{i=1}^{n}\|\alpha_i(\tau)\|^2}\\
&\quad + 2\frac{k_\theta\sum_{i=1}^{n}\|\alpha_i(\tau)\|\sum_{i=1}^{n}|\eta_i(\tau)|}{1+\|\sum_{i=1}^{n}\alpha_i(\tau)\|^2+\sum_{i=1}^{n}\alpha_i(\tau)\|^2}\\
&\quad + 2\frac{(\sum_{i=1}^{n}|\eta_i(\tau)|)^2}{1+\|\sum_{i=1}^{n}\alpha_i(\tau)\|^2+\sum_{i=1}^{n}\alpha_i(\tau)\|^2}\\
&\quad - 2\Bigg[\theta(\tau)-\theta(\tau-1)]^{\mathrm{T}}[\tilde{\theta}(\tau-1)\\
&\quad + \frac{\sum_{i=1}^{n}\alpha_i(\tau)\sum_{i=1}^{n}e_i(\tau+1)}{1+\|\sum_{i=1}^{n}\alpha_i(\tau)\|^2+\sum_{i=1}^{n}\alpha_i(\tau)\|^2}\\
&\quad - \frac{1}{2}(\theta(\tau)-\theta(\tau-1))\Bigg]\\
&\quad + 2\|\theta(\tau)-\theta(\tau-1)\|\left(\left(\frac{3}{2}+\frac{k_\alpha}{k'_\alpha}\right)k_\theta + a'_1\right)\\
&\le -\tilde{e}(\tau+1)^2 + 2nk_\theta\frac{c_\eta\varepsilon(k_1(k'_\alpha M_0)^2+k_2)^{1/2}}{1+\|\sum_{i=1}^{n}\alpha_i(\tau)\|^2+\sum_{i=1}^{n}\alpha_i(\tau)\|^2},\\
&\quad \times\sum_{i=1}^{n}\|\alpha_i(\tau)\| + n\frac{4c_\eta^2\varepsilon^2(k_1(k'_\alpha M_0)^2+k_2)}{1+\sum_{i=1}^{n}\|\alpha_i(\tau)\|^2}\\
&\quad + n\frac{4d^2+2dk_\theta\|\alpha_i(\tau)\|}{1+\sum_{i=1}^{n}\|\alpha_i(\tau)\|^2}\\
&\quad + 2\|\theta(\tau)-\theta(\tau-1)\|\left(\left(\frac{3}{2}+\frac{k_\alpha}{k'_\alpha}\right)k_\theta + a'_1\right)\\
&\le -\tilde{e}_i(\tau+1)^2 + 2nk_\theta c_\eta\varepsilon(k_1^{1/2}+k_2^{1/2})\\
&\quad + 4nc_\eta^2\varepsilon^2(k_1+k_2) + 4n\delta^2 + 2nk_\theta\delta\\
&\quad + 2\|\theta(\tau)-\theta(\tau-1)\|\left(\left(\frac{3}{2}+\frac{k_\alpha}{k'_\alpha}\right)k_\theta + a'_1\right), \forall\tau\ge t_0.
\end{aligned}
\tag{A.19}
$$

Therefore,

$$
\begin{aligned}
\tilde{e}(\tau+1)^2 &\le v(\tau)-v(\tau+1) + 2nk_\theta c_\eta\varepsilon(k_1^{1/2}+k_2^{1/2})\\
&\quad + 4nc_\eta^2\varepsilon^2(k_1+k_2) + 4n\delta^2 + 2nk_\theta\delta\\
&\quad + 2\|\theta(\tau)-\theta(\tau-1)\|\left(\left(\frac{3}{2}+\frac{k_\alpha}{k'_\alpha}\right)k_\theta + a'_1\right).
\end{aligned}
\tag{A.20}
$$

Summing both sides of (A.20) gives

$$
\begin{aligned}
\sum_{\tau=t_0}^{t-1}|\tilde{e}(\tau)|^2 &\le \|\tilde{\theta}(t_0)\|^2 - \|\tilde{\theta}(t)\|^2\\
&\quad + 2k_\varepsilon\left(\left(\frac{3}{2}+\frac{k_\alpha}{k'_\alpha}\right)k_\theta + a'_1\right) + (a_2+a_3)(t-t_0),
\end{aligned}
\tag{A.21}
$$

which confirms (34) by applying Assumption A.2.

(iv) Using Lemma A.1, it follows immediately from the definition of $\chi_i(t+1)$ that

$$
\begin{aligned}
|\chi_i(t+1)| &\le c'_{1,i}\|\mathbf{e}_{[1,i-1]}(t+1)\| + c'_{2,i}\|\boldsymbol{\chi}_{[1,i-1]}(t+1)\|\\
&\quad + c'_{3,i}\|\mathbf{z}_{[1,i]}(t)\|\|\tilde{e}(t+1)\|,
\end{aligned}
\tag{A.22}
$$

where $c'_{m,i}$ $(m=1,2,3)$ are constants.

Since $\chi_1(t+1)=0$ and $\chi_2(t+1) \le k_\theta k_\alpha |e_1(t+1)| + k_\alpha |z_2^t| \|\tilde{e}(t+1)|$, it can be shown from (A.22) that

$$|\chi_i(t+1)| \le c''_{1,i} \|\mathbf{e}_{[1,i-1]}(t+1)\| + c''_{2,i} \|\mathbf{z}_{[1,i]}(t)\| |\tilde{e}(t+1)|, \tag{A.24}$$

where $c''_{1,i}, c''_{2,i}$ are constants combining k_θ and k_α.

Taking $c_1 = \max_{1 \le i \le n}\{c''_{1,i}\}$ and $c_2 = \max_{1 \le i \le n}\{c''_{2,i}\}$, (35) follows.

Using (29) and inequality

$$\begin{aligned} &\|z(t_0-1)\| |\tilde{e}(t_0)| \\ &\le \frac{\|z(t_0-1)\| |\sum_{i=1}^n e_i(t_0)|}{(1+\|\sum_{i=1}^n \alpha_i(t_0-1)\|^2 + \sum_{i=1}^n \|\alpha_i(t_0-1)\|^2)^{1/2}} \\ &\le \frac{b_u \|x(t_0-1)\| |\sum_{i=1}^n e_i(t_0)|}{(1+k'_\alpha \|x(t_0-1)\|^2)^{1/2}} \\ &\le \frac{b_u}{k'_\alpha}((k_\theta k_\alpha + a_1)M_0 + a_1), \end{aligned} \tag{A.25}$$

Eq. (36) follows.

Remark A.1. Note that the constants a_1, a'_1 and a_2 are functions of ε and ε_θ which depends on the magnitude of the unmodelled dynamics, external disturbances and variations rate of the nominal parameters. In the ideal case that the system has no nonparameteric uncertainties and all the nominal system parameters are constants, they are equal to zeros and the update law has the same properties as those given in Yeh and Kokotović (1995).

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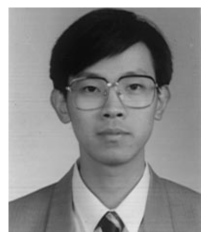

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