PERGAMON

Automatica 36 (2000) 907–915

www.elsevier.com/locate/automatica

Brief Paper

Robust decentralized adaptive stabilization of interconnected systems with guaranteed transient performance☆

Ying Zhang[a], Changyun Wen[b,*], Yeng Chai Soh[b]

[a]*Automation Technology Division, Gintic Institute of Manufacturing Technology, 71 Nanyang Drive, Singapore 638075, Singapore*
[b]*School of Electrical and Electronics Engineering, Nanyang Technological University, Singapore 639798, Singapore*

Received 17 April 1998; revised 2 August 1999; received in final form 18 October 1999

Abstract

This paper presents a scheme for designing a totally decentralized adaptive stabilizers for a class of large-scale systems with subsystems having arbitrary relative degrees. In the control design, both strong static interactions and weak dynamic interactions are considered. It is shown that with the proposed controller global stability of the overall system and perfect regulation can be guaranteed in the presence of these interactions. The transient performance of the adaptive system with the proposed decentralized controller is also evaluated by both L_∞ and L_2 bounds of the tracking errors. It is shown that these bounds can be made arbitrarily small by properly choosing the control design parameters.

Keywords: Adaptive control; Stabilization; Decentralized control; Robustness; Backstepping

1. Introduction

In the control of a large-scale system, adaptive control strategy is an efficient and effective way to treat the parametric uncertainties in the system. In the situations where the centralized information and centralized computing capability are not available or not feasible, totally decentralized adaptive controllers are viable solutions. In this case it is required to design a local controller for each subsystem using only local information while guaranteeing the stability of the overall system. Also, it is desired to obtain a totally decentralized controller with improved transient performance. This problem has received increasing attention and a number of results have been obtained. According to the form of the interactions considered in the control design, these methods can be classified into two categories. One is to consider a kind of static interactions where the norm of states are usually bounded by a polynomial function (e.g., Ioannou, 1986; Gavel & Siljak, 1989; Fu, 1992; Shi & Singh, 1992; Wen, 1994a,b; Huseyin, Sezer & Siljak, 1982). The other considers the dynamic interactions (e.g., Ionannou & Kokotovic, 1985; Hill, Wen & Goodwin, 1988; Wen & Hill, 1992; Wen, 1994a,b; Ortega, 1996; Wen & Soh, 1997; Ortega & Herrera, 1993). Since dynamic interactions have infinite memory, it cannot be covered by static interactions and vice versa. It was shown that strong interactions between the subsystems can be allowed for the first case while only weak interactions are allowed in the second kind of decentralized controllers in order to establish the stability of the whole system. Recently in Jain and Khorrami (1997a,b), decentralized adaptive controllers were proposed for a class of large-scale nonlinear systems with static high-order interconnections. In these recent results, bounded disturbances with special gains are also considered. However, it is worthy to mention that so far the issue of transient performance was only considered in the partially decentralized adaptive control design (see Ho & Datta, 1996). For totally decentralized adaptive control design, it is still an open problem.

The major difficulty in the design of totally decentralized adaptive controllers is to establish the stability of the overall system. In the case of static interactions, it is easier to employ the second Lyapunov method to obtain

☆This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor R. Lozano under the direction of Editor F.L. Lewis.

* Corresponding author. Tel.: + 65-7994947; fax + 65-7912687.
E-mail address: ecywen@ntu.edu.sg (C. Wen)

0005-1098/00/$ - see front matter
PII: S0005-1098(99)00218-6

the stability results. But, it is harder for the case that the large-scale system has dynamic interactions and subsystems with relative degree greater than two. This difficulty was lately overcome in Wen and Hill (1992) and Wen (1994a,b) by using an inductive stability analysis method and in Ortega and Herrera (1996) by employing a high-order estimator, where only dynamic interactions were considered. Recently in Wen (1994a,b), the backstepping approach was employed to handle the decentralized control problem for large-scale systems which may have strong static interactions. The advantage of backstepping design technique is that the controller and the adaptive update laws can be designed at the same time, and this can improve the system transient performance. Also, the relative degree of the plant to be controlled is not an issue in the design. Although a backstepping technique was used to design decentralized adaptive controllers for systems with dynamic interconnections in Wen and Soh (1997), the adaptive law and controller design are separated and it is impossible to guarantee the transient performance by adjusting design parameters. In this paper, we use the backstepping technique to design totally decentralized adaptive controllers for large-scale systems with both strong static interactions and weak dynamic interactions. It is shown that decentralized adaptive stabilization can be achieved by the proposed controllers. The L_∞ and L_2 bounds of the tracking errors are given to evaluate the system transient performance, and the improved transient performance of the whole system can be achieved by choosing the control design parameters suitably. It can also be shown that the obtained results are still applicable to the case where the large-scale system is corrupted by the same type of disturbances as in Jain and Khorrami (1997a,b).

2. Problem formulation

Consider a large-scale system consisting of N interconnected subsystems, and the ith subsystem is modelled by

$$\begin{aligned} \dot{x}_{oi} &= A_{oi}x_{oi} + b_{oi}u_i + \sum_{j\neq i}^{N} \bar{f}_{ij}(t, y_j), \\ y_i &= c_{oi}^{\mathrm{T}}(1 + \mu_{ii}\Delta_{ii}(s))x_{oi} + \sum_{\substack{j=1\\ j\neq i}}^{N} \mu_{ij}\Delta_{ij}(s)y_j, \end{aligned} \tag{1}$$

where $x_{oi} \in R^{n_i}$, $y_i \in R$ and $u_i \in R$ are, respectively, the local states, output and input; A_{oi}, b_{oi}, c_{oi} are constant matrices of appropriate dimensions; $\bar{f}_{ij}(t, y_j) \in R^{n_i}$ and $\Delta_{ij}(s)y_j$ $(i \neq j)$ denote, respectively, the static interactions and the dynamic interactions from the jth subsystem to the ith subsystem; $\Delta_{ii}(s)$ is the unmodeled dynamics in the ith subsystem; and μ_{ij} are positive scalars specifying the magnitudes of dynamic interactions and unmodeled dynamics.

For system (1), we make the following assumptions.

Assumption 2.1. For each subsystem, the order n_i is known and the triple (A_{oi}, b_{oi}, c_{oi}) is completely controllable and observable. In the transfer function

$$\begin{aligned} G_i(s) &= c_{oi}^{\mathrm{T}}(sI_{n_i} - A_{oi})^{-1}b_{oi} \\ &= \frac{B_i(s)}{A_i(s)} = \frac{b_i^{m_i}s^{m_i} + \cdots + b_i^1 s + b_i^0}{s^{n_i} + a_i^{n_i-1}s^{n_i-1} + \cdots + a_i^0}, \end{aligned} \tag{2}$$

$B_i(s)$ is stable. The relative degree $\rho_i = n_i - m_i$ and the sign of $b_i^{m_i}$ are known.

Assumption 2.2. For the nonlinear interactions $\bar{f}_{ij}(t, y_j)$, we have $\|\bar{f}_{ij}(t, y_j)\| \le \bar{\gamma}_{ij}|y_j|$, where $\bar{\gamma}_{ij}$'s are positive numbers denoting the strengths of interactions.

Assumption 2.3. $\Delta_{ij}(s)$ $(i = 1, \dots, N; j = 1, \dots, N)$ are stable and strictly proper with unity high-frequency gains.

Remark 1. In system (1), both the static and the dynamic interactions, which are denoted by $\bar{f}_{ij}(t, y_j)$ and $\Delta_{ij}(s)y_j$, are included. Besides, each subsystem is allowed to have unmodeled dynamics denoted by $\Delta_{ii}(s)$.

Remark 2. Assumptions 2.1 and 2.2 are similar to those made in Wen (1994a,b). They are also usually used in nonadaptive decentralized control design schemes (e.g., Huseyin et al., 1982). Since unmodeled dynamics in each subsystem are considered, an additional Assumption 2.3 is made. It is evident that the unmodeled dynamics satisfying such an assumption can often be found in many practical situations such as large-scale power systems. Thus, this assumption is reasonable in practice.

The control objective is to design a local adaptive controller for each subsystem given by (1) using only local information such that the overall interconnected system is stable and all the outputs y_i are regulated to zeros.

3. Design of decentralized controllers

In this section, the backstepping technique is employed to design the desired controllers. To this end, system (1) is firstly transformed into a form for which the backstepping design can be performed.

As well known, there exists a nonsingular matrix T_i for ith subsystem (1), by which the ith subsystem can be transformed into the following form:

$$\begin{aligned} \dot{x}_i &= A_{0i}x_i - a_i x_i^1 + \begin{bmatrix} 0_{(\rho_i-1)\times 1} \\ b_i \end{bmatrix} u_i + f_i, \\ y_i &= x_i^1 + \mu_{ii}\Delta_{ii}(s)x_i^1 + \sum_{\substack{j=1\\ j\neq i}}^{N} \mu_{ij}\Delta_{ij}(s)y_j, \end{aligned} \tag{3}$$

where

$$A_{0i} = \left[\begin{array}{c|c} \mathbf{0}_{(n_i-1)\times 1} & I_{n_i-1} \\ \hline 0 & \mathbf{0}_{1\times(n_i-1)} \end{array}\right]$$

$$a_i = [a_i^{n_i-1}, \dots, a_i^1, a_i^0]^{\mathrm{T}},$$

$$b_i = [b_i^{m_i}, \dots, b_i^1, b_i^0]^{\mathrm{T}},$$

$$x_i = T_i^{-1} x_{oi}, \quad f_i = \sum_{j\neq i}^{N} T_i^{-1} \bar{f}_{ij}(t, y_j).$$

For clarity of illustration, the superscript k of a vector will be used to denote the kth element of this vector in the remaining parts of this paper.

Let $L_i(s)$ be a stable polynomial given by $L_i(s) = s^{n_i} + l_i^1 s^{n_i-1} + \cdots + l_i^{n_i-1}s + l_i^{n_i}$. Then the estimate of x_i can be obtained by

$$\hat{x}_i = -\xi_{i,n_i} - \sum_{k=0}^{n_i-1} a_i^k \xi_{i,k} + \sum_{k=0}^{m_i} b_i^k v_{i,k}, \tag{4}$$

where

$$\dot{\eta}_i = A_{1i}\eta_i + e_{i,n_i} y_i, \quad \xi_{i,k} = (A_{1i})^k \eta_i \quad \text{for } 1 \le k \le n_i,$$

$$\dot{\lambda}_i = A_{1i}\lambda_i + e_{i,n_i} u_i, \quad v_{i,k} = (A_{1i})^k \lambda_i \quad \text{for } 1 \le k \le m_i$$

with $A_{1i} = A_{0i} - [l_i^1, l_i^2, \dots, l_i^{n_i}]^{\mathrm{T}}(e_{i,1})^{\mathrm{T}}$ and $e_{i,k}$ denoting the kth coordinate vector in R^{n_i}. From (3) and (4), it can be shown that the estimation error $\varepsilon_i = x_i - \hat{x}_i$ satisfies

$$\dot{\varepsilon}_i = A_{1i}\varepsilon_i + (a_i - l_i)\left(\mu_{ii}\Delta_{ii}(s)x_i^1 + \sum_{\substack{j=1\\ j\neq i}}^{N} \mu_{ij}\Delta_{ij}(s)y_j\right) + f_i, \tag{5}$$

where $l_i \triangleq [l_i^1, l_i^2, \dots, l_i^{n_i}]^{\mathrm{T}}$.

Introduce the following notations:

$$\theta_i = [b_i^{\mathrm{T}}, a_i^{\mathrm{T}}]^{\mathrm{T}}, \tag{6}$$

$$\omega_i = [v_{i,m_i}^2, v_{i,m_i-1}^2, \dots, v_{i,0}^2, \Xi_{i,(2)} - y_i(e_{i,1})^{\mathrm{T}}]^{\mathrm{T}}, \tag{7}$$

$$\bar{\omega}_i = [0, v_{i,m_i-1}^2, \dots, v_{i,0}^2, \Xi_{i,(2)} - y_i(e_{i,1})^{\mathrm{T}}]^{\mathrm{T}} \tag{8}$$

with $\Xi_{i,(2)} = -(e_{i,2})^{\mathrm{T}}[\xi_{i,n_i-1}, \dots, \xi_{i,1}, \xi_{i,0}]$, and define

$$z_i^1 = y_i, \tag{9}$$

$$z_i^k = v_{i,m_i}^k - \alpha_{i,k-1} \quad \text{for } 2 \le k \le \rho_i, \tag{10}$$

where $\alpha_{i,k-1}$ is the virtual control in the $(k-1)$th step. Then, the ith local adaptive controller can be obtained by applying the similar control design procedures as in Krstic, Kanellakopoulos and Kokotović (1995) to each subsystem. These controllers are summarized as below.

Adaptive laws:

$$\dot{\hat{\theta}}_i = \Gamma_i \tau_{i,\rho_i} \tag{11}$$

$$\dot{\hat{\varrho}}_i = -\gamma_i sgn(b_i^{m_i})\bar{\alpha}_{i,1} z_i^1, \tag{12}$$

where Γ_i is positive-definite matrix with appropriate dimensions, and τ_{i,ρ_i} is obtained through the following recursive procedures:

$$\tau_{i,1} = (\omega_i - \hat{\varrho}_i \bar{\alpha}_{i,1} e_{i,1}) z_i^1, \tag{13}$$

$$\tau_{i,k} = \tau_{i,k-1} - \frac{\partial \alpha_{i,k-1}}{\partial y_i}\omega_i z_i^k, \quad i = 2,3,\dots,\rho_i. \tag{14}$$

Control law:

$$u_i = \alpha_{i,\rho_i} - v_{i,m_i}^{\rho_i+1}, \tag{15}$$

where

$$\alpha_{i,1} = \hat{\varrho}_i \bar{\alpha}_{i,1},$$

$$\bar{\alpha}_{i,1} = -(c_i^1 + 2d_i^1)z_i^1 - \xi_{i,n_i}^2 - \bar{\omega}_i^{\mathrm{T}}\hat{\theta}_i$$

$$\alpha_{i,2} = -\hat{b}_i^{m_i} z_i^1 - \left[c_i^2 + 4d_i^2\left(\frac{\partial \alpha_{i,1}}{\partial y_i}\right)^2\right] z_i^2 + \beta_{i,2},$$

$$\alpha_{i,k} = -z_i^{k-1} - \left[c_i^k + 4d_i^k\left(\frac{\partial \alpha_{i,k-1}}{\partial y_i}\right)^2\right] z_i^k + \beta_{i,k},$$

$$\begin{aligned}\beta_{i,k} = {} & \frac{\partial \alpha_{i,k-1}}{\partial y_i}(\xi_{i,n_i}^2 + \omega_i^{\mathrm{T}}\hat{\theta}_i) \\ & + \frac{\partial \alpha_{i,k-1}}{\partial \eta}(A_i\eta_i + e_{i,n_i} y_i) + l_i^k v_{i,m_i}^1 \\ & + \sum_{j=1}^{m+k-1} \frac{\partial \alpha_{i,k-1}}{\partial \lambda_i^j}(-l_i^j \lambda_i^1 + \lambda_i^{j+1}).\end{aligned}$$

So far we have designed the local adaptive controllers for each of the subsystems. The stability of the overall closed-loop system consisting of the interconnected subsystems and these decentralized controllers will be established in the next section.

4. Stability analysis

The purpose of this section is to prove that there exists a positive number μ^* such that the closed-loop system with the controller given by (15) is asymptotically stable for all $\mu_{ij} \in [0, \mu^*)$. To this end, the model for each local closed-loop system is given first.

By applying the similarity transformation as used in Krstic et al. (1995), each subsystem given by (3) can be represented by

$$\dot{x}_i^1 = x_i^2 - a_i^{n_i-1}x_i^1 + f_i^1,$$

$$\vdots$$

$$\dot{x}_i^{\rho_i} = (c_{bi})^{\mathrm{T}}\bar{x}_i - a_i^0 x_i^1 + b_i^{m_i} u_i + f_i^{\rho_i},$$

$$\dot{\zeta}_i = A_{bi}\zeta_i + b_{bi}x_i^1 + \bar{f}_i,$$

$$y_i = x_i^1 + \mu_{ii}\Delta_{ii}(x)x_i^1 + \sum_{\substack{j=1\\ j\neq i}}^{N} \mu_{ij}\Delta_{ij}(s)y_j, \tag{16}$$

where $\bar{x}_i = [x_i^1, x_i^2, \dots, x_i^{\rho_i}, \zeta_i^{\mathrm{T}}]^{\mathrm{T}}$, $c_{bi} \in R^{n_i}$, $\zeta_i \in R^{m_i}$ is the zero dynamics of the ith subsystem, A_{bi} is a matrix in

$R^{m_i \times m_i}$ and its eigenvalues are exactly the zeros of $B_i(s)$, while $b_{bi} \in R^{m_i}$ and $\bar{f}_i \in R^{m_i}$ denote the effects of the transformed interactions.

Then the error system subject to controller (15) is characterized by

$$\dot{z}_i = A_{zi}z_i + W_{\varepsilon i}\varepsilon_i^2 + (W_{\theta i})^{\mathrm{T}}\tilde{\theta}_i - b_i^{m_i}\bar{\alpha}_{i,1}\tilde{\varrho}_i e_{i,1} + W_{\varepsilon i}\left[(s + a_i^{n_i - 1})\left(\mu_{ii}\Delta_{ii}(s)x_i^1 + \sum_{\substack{j=1\\ j\neq i}}^{N} \mu_{ij}\Delta_{ij}(s)y_j\right) + f_i^1\right],$$

$$\dot{\varepsilon}_i = A_{1i}\varepsilon_i + (a_i - l_i)\left(\mu_{ii}\Delta_{ii}(s)x_i^1 + \sum_{\substack{j=1\\ j\neq i}}^{N} \mu_{ij}\Delta_{ij}(s)y_j\right) + f_i,$$

$$\dot{\tilde{\zeta}}_i = A_{bi}\tilde{\zeta}_i + b_{bi}x_i^1 + \bar{f}_i,$$

$$\dot{\tilde{\eta}}_i = A_{1i}\tilde{\eta}_i + e_{i,n_i}z_i^1,$$

$$\dot{\tilde{\theta}}_i = -\Gamma_i W_{\theta i} z_i,$$

$$\dot{\hat{\varrho}}_i = -\gamma_i sgn(b_i^{m_i})\bar{\alpha}_{i,1}e_{i,1}^{\mathrm{T}} z_i, \tag{17}$$

where

$$\tilde{\zeta}_i \triangleq \zeta_i - \zeta_i^r, \quad \dot{\zeta}_i^r = A_{bi}\zeta_i^r, \quad \zeta_i^r(0) = 0,$$

$$\tilde{\eta}_i \triangleq \eta_i - \eta_i^r, \quad \dot{\eta}_i^r = A_{1i}\eta_i^r, \quad \eta_i^r(0) = 0$$

and $A_{zi}, W_{\varepsilon i}$, and $W_{\theta i}$ are defined as in Krstic et al. (1995).

Remark 3. Comparing with the corresponding error system in Krstic et al. (1995), three new items appear in the error equations on z_i, ε_i and $\tilde{\zeta}_i$, respectively, due to the presence of unmodeled dynamics and interactions.

In order to take the unmodeled dynamics into account in the stability analysis, we let v_{ij} be the states associated ith $\Delta_{ij}(s)$ and they are given by

$$\dot{v}_{ii} = A_{vii}v_{ii} + b_{vii}x_i^1,$$

$$\Delta_{ii}(s)x_i^1 = (1, 0, \ldots, 0)v_{ii},$$

$$\dot{v}_{ij} = A_{vij}v_{ij} + b_{vij}y_j \quad \text{for } j \neq i,$$

$$\Delta_{ij}(s)y_j = (1, 0, \ldots, 0)v_{ij}$$

where A_{vij} are Hurwitz.

From the stability of $\Delta_{ij}(s)$, it is obvious that

$$\|\Delta_{ii}(s)x_i^1\|^2 \leq \|\chi\|^2, \tag{18}$$

where

$$\chi_i \triangleq [z_i^{\mathrm{T}}, \varepsilon_i^{\mathrm{T}}, \tilde{\eta}_i^{\mathrm{T}}, \tilde{\zeta}_i^{\mathrm{T}}, v_{i1}^{\mathrm{T}}, \ldots, v_{iN}^{\mathrm{T}}]^{\mathrm{T}} \quad \text{and} \quad \chi \triangleq [\chi_1^{\mathrm{T}}, \chi_2^{\mathrm{T}}, \ldots, \chi_N^{\mathrm{T}}]^{\mathrm{T}}.$$

Since $\Delta_{ij}(s)$ is strictly proper, we have $\|\Delta_{ii}(s)(s + a_i^{n_i - 1})x_i^1\|^2 \leq k_{i,1}|x_i^1|^2 + k_{i,2}\|\chi\|^2$ and $\|\sum_{j=1, j\neq i}^{N} \mu_{ij}\Delta_{ij}(s)y_j\|^2 \leq k_{i,0}\max_{1\leq i,j\leq N}\{\mu_{ij}\}\|\chi\|^2$ where $k_{i,0}$, $k_{i,1}$ and $k_{i,2}$ are constants.

It is clear from (3) and (9) that

$$x_i^1 = z_i^1 - \left(\mu_{ii}\Delta_{ii}(s)x_i^1 + \sum_{\substack{j=1\\ j\neq i}}^{N} \mu_{ij}\Delta_{ij}(s)y_j\right). \tag{19}$$

Thus,

$$\|\Delta_{ii}(s)(s + a_i^{n_i - 1})x_i^1\|^2 \leq \left(k_{i,3} + 2k_{i,4}\mu_{ii}^2 + 2k_{i,1}k_{i,0}\max_{\substack{1\leq j\leq N\\ j\neq i}}\{\mu_{ij}\}\right)\|\chi\|^2, \tag{20}$$

where $k_{i,3} = k_{i,2} + 2k_{i,1}$ and $k_{i,4} = 2k_{i,1}$. They are all constants which are independent of μ_{ij}.

We are now in the position to present the stability result of system (17).

Theorem 1. *Consider the adaptive system consisting of plant* (3), *estimators* (11)–(12) *and the controller law* (15). *Under Assumptions* A.1–A.3, *there exists a constant* μ^* *such that for all* $\mu_{ij} \leq \mu^*$, *all the signals of the closed-loop system are globally uniformly bounded, and*

$$\lim_{t\to\infty} |y_i(t)| = 0$$

for arbitrary initial $x_i(0)$.

Proof. Introduce the augmented Lyapunov function as

$$V_i = \frac{1}{2}\sum_{j=1}^{\rho_i}(z_i^j)^2 + \sum_{j=1}^{\rho_i}\frac{1}{d_i^j}\varepsilon_i^{\mathrm{T}}P_i\varepsilon_i + \frac{1}{2\gamma_i}(\varrho_i - \hat{\varrho}_i)^2 + \frac{1}{2}\tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\theta}_i + \frac{1}{k_i^{\eta_i}}\tilde{\eta}_i^{\mathrm{T}}P_i\tilde{\eta}_i + \frac{1}{K_i^{\zeta_i}}\tilde{\zeta}_i^{\mathrm{T}}P_{bi}\tilde{\zeta}_i + \sum_{j=1}^{N} q_{ij}v_{ij}^{\mathrm{T}}P_{vij}v_{ij}, \tag{21}$$

where P_i, P_{bi} and P_{vij} are the matrices satisfying the following Lyapunov equations:

$$(A_{1i})^{\mathrm{T}}P_i + P_iA_{1i} = -I_{n_i},$$

$$(A_{bi})^{\mathrm{T}}P_{bi} + P_{bi}A_{bi} = -I_{n_i - \rho_i},$$

$$(A_{vij})^{\mathrm{T}}P_{vij} + P_{vij}A_{vij} = -I_{v_{ij}}.$$

Using (17) and the relation $y_i = z_i^1 = x_i^1 + \mu_{ii}\Delta_{ii}(s)x_i^1 + \sum_{j=1, j\neq i}^{N}\mu_{ij}\Delta_{ij}(s)y_j$, applying the inequality $2xy \leq (\|x\|^2 + \|y\|^2)$, and taking $K_i^{\eta_i}, K_i^{\zeta_i}$ and q_{ij} such that $K_i^{\eta_i} \geq 16\|P_ie_{i,n_i}\|^2/c_i^1$, $K_i^{\zeta_i} \geq 32\|P_{bi}b_{bi}\|^2/c_i^1$, and $q_{ij} \leq c_i^1/32\|P_{vij}b_{vij}\|$, $\forall 1 \leq j \leq N$, we obtain

$$\dot{V}_i \leq -\alpha_i\|\chi_i\|^2 + \mu^2\left[k_{i,5}\left((s + a_i^{n_i - 1})\left(\Delta_{ii}(s)x_i^1 + \sum_{\substack{j=1\\ j\neq i}}^{N}\Delta_{ij}(s)y_j\right)\right)^2 + k_{i,6}\left(\Delta_{ii}(s)x_i^1 + \sum_{\substack{j=1\\ j\neq i}}^{N}\Delta_{ij}(s)y_j\right)^2\right] - \frac{c_i^1}{8}(z_i^1)^2 + \|f_i\|^2\|P_i\|^2\sum_{j=1}^{\rho_i}\left(\frac{8}{d_i^j}\right) + \frac{1}{c_i^1}\|f_i^1\|^2 + \sum_{j=1}^{\rho_i}\frac{1}{4d_i^j}\|f_i^1\|^2 + \frac{4}{K_i^{\zeta_i}}\|P_{bi}\|^2\|\bar{f}_i\|^2, \tag{22}$$

where

$$\alpha_i = \min\left\{\frac{c_i^1}{8}, \frac{1}{2}c_i^2, \ldots, \frac{1}{2}c_i^{\rho_i}, \frac{1}{2}\sum_{j=2}^{\rho_i}\frac{1}{d_i^j}, \frac{1}{2K_i^{\eta_i}}, \frac{1}{4K_i^{\zeta_i}}, \min_{1\le j\ne i\le N}\frac{q_{ij}}{2}\right\},$$

$$k_{i,5} = \frac{2}{c_i^1} + \sum_{j=2}^{\rho_i}\frac{1}{4d_i^j},$$

$$k_{i,6} = \|a_i - l_i\|^2\|P_i\|^2\sum_{j=1}^{\rho_i}\frac{8}{d_i^j} + \frac{4}{K_i^{\zeta_i}}\|P_{bi}b_{bi}\|^2$$

$$+ 4q_{ii}\|P_{vii}b_{vii}\|^2, \mu = \max_{1\le i,j\le N}\{\mu_{ij}\}.$$

From Assumption 2.2, we can show that

$$\|f_i\|^2\|P_i\|^2\sum_{j=1}^{\rho_i}\left(\frac{8}{d_i^j}\right) + \frac{1}{c_i^1}\|f_i^1\|^2 + \sum_{j=1}^{\rho_i}\frac{1}{4d_i^j}\|f_i^1\|^2$$

$$+ \frac{4}{K_i^{\zeta_i}}\|P_{bi}\|^2\|\bar{f}_i\|^2 \le \sum_{k\ne i} d_i\gamma_{ik}(z_i^1)^2. \tag{23}$$

Combining inequalities (18), (20), (22) and (23) gives

$$\dot{V}_i \le -(\alpha_i - (k_{i,6} + k_{i,3}k_{i,5})\mu^2 - k_{i,4}k_{i,5}\mu^4)\|\chi\|^2$$

$$+ \left(-\frac{c_i^1}{8}(z_i^1)^2 + \sum_{k\ne i} d_i\gamma_{ik}|z_i^1|\right). \tag{24}$$

Now define a Lyapunov function of the overall system as $V = \sum_{i=1}^N V_i$. We have

$$\dot{V} \le -\sum_{i=1}^{N}(\alpha_i - (k_{i,6} + k_{i,3}k_{i,5})\mu^2$$

$$- k_{i,4}k_{i,5}\mu^4)\|\chi\|^2 - Z^{\mathrm{T}}SZ, \tag{25}$$

where

$$Z^{\mathrm{T}} = [z_1^1, z_2^1, \ldots, z_N^1],$$

$$S = (s_{ik})_{N\times N} = \begin{cases} \dfrac{c_i^1}{8}, & i = j, \\ -(d_i\gamma_{ik} + d_k\gamma_{ki}), & i \ne k. \end{cases} \tag{26}$$

It is clear that $Z^{\mathrm{T}}SZ$ can be guaranteed to be positive by suitably choosing the controller gains c_i^1. Therefore, the conclusion of the theorem can be confirmed by taking μ^* as

$$\mu^* = \min_{1\le i\le N}$$

$$\times\left\{\sqrt{\frac{\sqrt{(k_{i,3}k_{i,5} + k_{i,6})^2 + 4k_{i,4}k_{i,5}\alpha_i} + (k_{i,3}k_{i,5} + k_{i,6})}{2k_{i,4}k_{i,5}}}\right\}. \quad \square \tag{27}$$

Remark 4. This theorem only guarantees the stability of the composite system in the presence of *sufficiently weak* dynamic interactions. This is understandable because the interconnected system considered here is with a general model description. This general system description leads to the conservativeness on the strength of dynamic interactions in the same way as in Ionannou and Kokotovic (1985), Ionannou (1986), Wen and Hill (1992), Wen (1994a,b), and Wen and Soh (1997). Such a conservativeness can be greatly relaxed by allowing for certain information exchange between subsystems in control design as in Hill et al. (1988) and Ho and Datta (1996). However, this apparantly sacrifies the total decentralization. It is also noted that there is no conservativeness imposed on the *static* interactions in our results. This is similiar to the results in Gavel and Siljak (1989), Shi and Singh (1992), Wen (1994a,b) and Jain and Khorrami (1997a,b).

From the proof of Theorem 1, it can be readily shown that the following corollary is valid in the presence of unmodeled dynamics.

Corollary 1. *The error system* (17) *has a globally uniformly stable equilibrium at the origin. Moreover, its* $(4n_i + m_i + 2)$*-dimensional state converges to the* $(n_i + m_i + 2)$*-dimensional manifold*

$$M_i = \{z_i = 0, \varepsilon_i = 0, \tilde{\zeta}_i = 0, \tilde{\eta}_i = 0\}. \tag{28}$$

5. Transient performance of the adaptive system

In this section, the transient performance of the overall large-scale system under the proposed decentralized controllers is characterized by giving the L_2 and L_∞ bounds of the tracking errors. To this end, the following lemmas are useful.

Lemma 1. *Let $h_{\Delta_{ij}}$ be the impulse response of $\Delta_{ij}(s)$. Then,*

$$\|x_i^1\|_2 \le \frac{1}{1-\mu_{ii}\|h_{\Delta_{ii}}\|_1}\left(\|z_i^1\|_2 + \sum_{\substack{j=1\\ j\ne i}}^{N}\mu_{ij}\|h_{\Delta_{ij}}\|_1\|z_j^1\|_2\right),$$

$$\|x_i^1\|_\infty \le \frac{1}{1-\mu_{ii}\|h_{\Delta_{ii}}\|_1}\left(\|z_i^1\|_\infty + \sum_{\substack{j=1\\ j\ne i}}^{N}\mu_{ij}\|h_{\Delta_{ij}}\|_1\|z_j^1\|_\infty\right).$$

Proof. The conclusion follows immediately from (9), (19) and Theorem B.2 in Krstic et al. (1995). $\square$

Lemma 2. *Consider the interactions $\bar{f}_{ij}(t, y_j)$ satisfying Assumption* 2. *Then,*

$$\|f_i^1\|_p \le \|f_i\|_p \le \gamma_{ij}\|Z\|_p, \quad p = 2, \infty. \tag{29}$$

Proof. The conclusion follows directly from the definition of p-norm and Assumption A.2. $\square$

Lemma 3. *Suppose* $\max_{1\le i,\, j\le N}\{\mu_{ij}\}\le\mu^*$. *The states of the adaptive system subject to the decentralized controllers are bounded by*

$$\|z_i(t)\|\le\|z_i(0)\|e^{-c_i^0 t}+\frac{1}{2\sqrt{c_i^0 d_i^0}}\Bigg(\|\tilde{\theta}_i\|_\infty\|h_{\omega i}\|_1\|z_i^1\|_p + \kappa_{\omega i}\|\tilde{\theta}_i\|_\infty + \mu_{ii}M_1(i,j)\|z_i^1\|_p + \sum_{\substack{j=1\\ j\ne i}}^{N}\mu_{ij}M_2(i,j)\|z_j^1\|_p+\gamma_{ij}M_3(i,j)\|Z\|_p\Bigg),$$
$$p=2,\infty, \tag{30}$$

where

$$M_1(i,j)=\frac{\|h_{\varepsilon i}\|_1\|h_{\Delta_{ii}}\|_1+\|h''_{\Delta_{ii}}\|_1+\|\tilde{\theta}_i\|_\infty\|h'_{\Delta_{ii}}\|_1}{1-\mu_{ii}\|h_{\Delta_{ii}}\|_1}, \tag{31}$$

$$M_2(i,j)=\frac{\|h_{\varepsilon i}\|_1\|h_{\Delta_{ij}}\|_1+\|h''_{\Delta_{ii}}\|_1+\|\tilde{\theta}_i\|_\infty\|h'_{\Delta_{ij}}\|_1}{1-\mu_{ii}\|h_{\Delta_{ii}}\|_1}\|h_{\Delta_{ij}}\|_1 + \|h'_{\Delta_{ij}}\|_1(\|\tilde{\theta}_i\|_\infty+1), \tag{32}$$

$$M_3(i,j)=\|h'_{\Delta_{\varepsilon i}}\|_1+1 \tag{33}$$

and all constants $h_{\Delta_{ij}}, h'_{\Delta_{ij}}, h''_{\Delta_{ij}}, h_{\omega i}, h_{\varepsilon i}, h'_{\varepsilon i}$ *and* $\kappa_{\omega i}$ *are defined in the proof of the lemma.*

Proof. Define $V_{i\rho_i}=\frac{1}{2}z_i^{\mathrm{T}}z_i$. Differentiating $V_{i\rho_i}$ along (17), we have

$$\begin{aligned}\dot{V}_{i\rho_i} = & -\sum_{j=1}^{\rho_i}c_i^j(z_i^j)^2-2d_i^1(z_i^1)^2-4\sum_{j=2}^{\rho_i}d_i^j\left(\frac{\partial\alpha_{j-1}}{\partial y_i}\right)^2(z_i^j)^2\\ & -\sum_{j=2}^{\rho_i}z_i^j\frac{\partial\alpha_{j-1}}{\partial y_i}(\omega_i^{\mathrm{T}}\tilde{\theta}_i+\varepsilon_i^2+(s+a_i^{n_i-1})\\ & \times\Bigg(\mu_{ii}\Delta_{ii}(s)x_i^1+\sum_{\substack{j=1\\ j\ne i}}^{N}\mu_{ij}\Delta_{ij}(s)y_j\Bigg)+f_i^1\Bigg)\\ & +z_i^1(\omega_i^{\mathrm{T}}\tilde{\theta}_i+\varepsilon_i^2+(s+a_i^{n_i-1})\\ & \times\Bigg(\mu_{ii}\Delta_{ii}(s)x_i^1+\sum_{\substack{j=1\\ j\ne i}}^{N}\mu_{ij}\Delta_{ij}(s)y_j\Bigg)+f_i^1\Bigg)\\ \le & -\sum_{j=1}^{\rho_i}c_i^j(z_i^j)^2+\sum_{j=2}^{\rho_i}\frac{1}{4d_i^j}(\theta_i^{\mathrm{T}}\omega_i+\varepsilon_i^2+(s+a_i^{n_i-1})\\ & \times\Bigg(\mu_{ii}\Delta_{ii}(s)x_i^1+\sum_{\substack{j=1\\ j\ne i}}^{N}\mu_{ij}\Delta_{ij}(s)y_j\Bigg)+f_i^1\Bigg)^2\\ \le & -c_i^0z_i^{\mathrm{T}}z_i+\frac{1}{4d_i^0}(\omega_i^{\mathrm{T}}\tilde{\theta}_i+\varepsilon_i^2+(s+a_i^{n_i-1})\\ & \times\Bigg(\mu_{ii}\Delta_{ii}(s)x_i^1+\sum_{\substack{j=1\\ j\ne i}}^{N}\mu_{ij}\Delta_{ij}(s)y_j\Bigg)+f_i^1\Bigg)^2,\end{aligned}$$

where $c_i^0\triangleq\max_{1\le j\le\rho_i}\{c_i^j\}$ and $d_i^0\triangleq\sum_{j=1}^{\rho_i}\{d_i^j\}$.

Applying Lemma B.5 in Krstic et al. (1995), we have

$$\|z_i(t)\|^2\le\|z_i(0)\|^2e^{-2c_i^0t} + \frac{1}{4c_i^0d_i^0}\Bigg\|\omega_i^{\mathrm{T}}\tilde{\theta}_i+\varepsilon_i^2+(s+a_i^{n_i-1}) \times\Bigg(\mu_{ii}\Delta_{ii}(s)x_i^1+\sum_{\substack{j=1\\ j\ne i}}^{N}\mu_{ij}\Delta_{ij}(s)y_j\Bigg)+f_i^1\Bigg\|_p^2,$$
$$p=2,\infty.$$

Thus,

$$\|z_i(t)\|\le\|z_i(0)\|e^{-c_i^0t}+\frac{1}{2\sqrt{c_i^0d_i^0}}\Bigg(\|\omega_i^{\mathrm{T}}\tilde{\theta}_i\|_p+\|\varepsilon_i^2\|_p + \Bigg\|(s+a_i^{n_i-1})\Bigg(\mu_{ii}\Delta_{ii}(s)x_i^1+\sum_{\substack{j=1\\ j\ne i}}^{N}\mu_{ij}\Delta_{ij}(s)y_j\Bigg)\Bigg\|_p + \|f_i^1\|_p^2\Bigg). \tag{34}$$

Next, we evaluate the four p norms on the right-hand side of (34), respectively.

First, it is known from (5) that

$$\dot{\varepsilon}_i=A_{1i}\varepsilon_i+(a_i-l_i)\Bigg(\mu_{ii}\Delta_{ii}(s)x_i^1+\sum_{\substack{j=1\\ j\ne i}}^{N}\mu_{ij}\Delta_{ij}(s)y_j\Bigg)+f_i. \tag{35}$$

Alternatively, ε_i^2 can be expressed as

$$\varepsilon_i^2=H_{\varepsilon i}\Bigg(\mu_{ii}\Delta_{ii}(s)x_i^1+\sum_{\substack{j=1\\ j\ne i}}^{N}\mu_{ij}\Delta_{ij}(s)y_j\Bigg)+H'_{\varepsilon i}f_i,$$

where $H_{\varepsilon i}$ and $H'_{\varepsilon i}$ are the transfer functions of system (35) considering $\mu_{ii}\Delta_{ii}(s)x_i^1+\sum_{j=1,j\ne i}^{N}\mu_{ij}\Delta_{ij}(s)y_j$ and f_i as the input respectively.

Since A_{1i} is stable and strictly proper, then $H_{\varepsilon i}$ is stable and proper, and $H'_{\varepsilon i}$ is stable and strictly proper. Thus,

$$\begin{aligned}\|\varepsilon_i^2\|_p\le & \|h_{\varepsilon i}\|_1\Bigg\|\mu_{ii}\Delta_{ii}(s)x_i^1+\sum_{\substack{j=1\\ j\ne i}}^{N}\mu_{ij}\Delta_{ij}(s)y_j\Bigg\|_p + \|h'_{\varepsilon i}\|_1\|f_i\|_p\\ \le & \|h_{\varepsilon i}\|_1\Bigg(\mu_{ii}\|h_{\Delta_{ii}}\|_1\|x_i^1\|_p+\sum_{\substack{j=1\\ j\ne i}}^{N}\mu_{ij}\|h_{\Delta_{ij}}\|_1\|z_j^1\|_p\Bigg) + \|h'_{\varepsilon i}\|_1\|f_i\|_p,\end{aligned}$$

where $h_{\varepsilon i}$ and $h_{\Delta_{ij}}$ are impulse responses of $H_{\varepsilon i}(s)$ and $\Delta_{ij}(s)$, respectively.

Using Lemmas 1 and 2, we obtain

$$\|\varepsilon_i^2\|_p \le \|h_{\varepsilon i}\|_1\Bigg[\frac{\mu_{ii}\|h'_{\Delta_{ii}}\|_1}{1-\mu_{ii}\|h_{\Delta_{ii}}\|_1}\|z_i^1\|_p + \frac{\|h_{\Delta'_{ii}}\|_1}{1-\mu_{ii}\|h_{\Delta_{ii}}\|_1}\sum_{\substack{j=1\\j\neq i}}^{N}\mu_{ij}\|h_{\Delta_{ij}}\|_1\|z_j^1\|_p + \sum_{\substack{j=1\\j\neq i}}^{N}\mu_{ij}\|h'_{\Delta_{ij}}\|_1\|z_j^1\|_p\Bigg] + \gamma_{ij}\|h'_{\varepsilon i}\|_1\|Z\|_p. \quad (36)$$

Next, we evaluate $\|\omega_i^{\mathrm{T}}\tilde{\theta}_i\|_p$. It follows from the definition of ω_i that

$$\omega_i = \frac{s+l_i}{L_i(s)}\Bigg[[s^{m_i-1},\ldots,s,1]\frac{A_i(s)}{B_i(s)}y_i,[s^{n_i-1},\ldots,s,1]\times\Bigg(y_i-\Bigg(\mu_{ii}\Delta_{ii}(s)x_i^1+\sum_{\substack{j=1\\j\neq i}}^{N}\mu_{ij}\Delta_{ij}(s)y_j\Bigg)\Bigg)\Bigg]+\omega_i^0(t)$$

$$= H_{\omega i}y_i + H_{\Delta_{ii}}x_i^1 + \sum_{\substack{j=1\\j\neq i}}^{N}H_{\Delta_{ij}}y_j + \omega_i^0(t),$$

where

$$H_{\omega i} \triangleq \frac{s+l_i}{L_i(s)}\Bigg[[s^{m_i-1},\ldots,s,1]\frac{A_i(s)}{B_i(s)}y_i,[s^{n_i-1},\ldots,s,1]y_i\Bigg],$$

$$H_{\Delta_{ij}} \triangleq \frac{s+l_i}{L_i(s)}[0,0,\ldots,0,-[s^{n_i-1},\ldots,s,1]\mu_{ij}\Delta_{ij}]$$

and $\omega_i^0(t)\triangleq\kappa_{\omega i}\mathrm{e}^{-\lambda_{\omega i}t}$ which is an exponentially decaying term due to the initial value of ω_i. Note that $\kappa_{\omega i}$ and $\lambda_{\omega i}$ depend only on the plant and filter parameters instead of c_i, d_i and Γ_i.

Thus,

$$\|\omega_i\|_p \le \|H_{\omega i}y_i\|_p + \|H_{\Delta_{ii}}x_i^1\|_p + \sum_{\substack{j=1\\j\neq i}}^{N}\|H_{\Delta_{ij}}y_j\|_p + \kappa_{\omega i}$$

$$\le \|h_{\omega i}\|_1\|z_i^1\|_p + \|h'_{\Delta_{ii}}\|_1\|x_i^1\|_p + \sum_{\substack{j=1\\j\neq i}}^{N}\|h'_{\Delta_{ij}}\|_1\|z_j^1\|_p + \kappa_{\omega i},$$

where $h_{\omega i}$ and $h'_{\Delta_{ij}}$ are impulse responses of $H_{\omega i}(s)$ and $H_{\Delta_{ij}}(s)$, respectively.

Again using Lemmas 1 and 3, we get

$$\|\omega_i^{\mathrm{T}}\tilde{\theta}_i\|_p \le \|\tilde{\theta}_i\|_\infty(\|h_{\omega i}\|_1\|z_i^1\|_p+\kappa_{\omega i}) + \|\tilde{\theta}_i\|_\infty\Bigg(\frac{\mu_{ii}\|h'_{\Delta_{ii}}\|_1}{1-\mu_{ii}\|h_{\Delta_{ii}}\|_1}\|z_i^1\|_p + \frac{\|h_{\Delta'_{ii}}\|_1}{1-\mu_{ii}\|h_{\Delta_{ii}}\|_1}\times\sum_{\substack{j=1\\j\neq i}}^{N}\mu_{ij}\|h_{\Delta_{ij}}\|_1\|z_j^1\|_p + \sum_{\substack{j=1\\j\neq i}}^{N}\mu_{ij}\|h'_{\Delta_{ij}}\|_1\|z_j^1\|_p\Bigg). \quad (37)$$

Finally, we calculate $\|(s+a_i^{n_i-1})(\mu_{ii}\Delta_{ii}(s)x_i^1+\sum_{j=1,j\neq i}^{N}\mu_{ij}\Delta_{ij}(s)y_j)\|_p$.

Since $\Delta_{ij}(s)$ are all stable and strictly proper, we have

$$\Bigg\|(s+a_i^{n_i-1})\Bigg(\mu_{ii}\Delta_{ii}(s)x_i^1+\sum_{\substack{j=1\\j\neq i}}^{N}\mu_{ij}\Delta_{ij}(s)y_j\Bigg)\Bigg\|_p$$

$$\le \sum_{\substack{j=1\\j\neq i}}^{N}\mu_{ij}\|h''_{\Delta_{ij}}\|_1\|z_j^1\|_p + \frac{\mu_{ii}\|h''_{\Delta_{ii}}\|_1}{1-\mu_{ii}\|h_{\Delta_{ii}}\|_1}\times\Bigg(\|z_i^1\|_p+\sum_{\substack{j=1\\j\neq i}}^{N}\mu_{ij}\|h_{\Delta_{ij}}\|_1\|z_j^1\|_p\Bigg)$$

$$= \frac{\mu_{ii}\|h''_{\Delta_{ii}}\|_1}{1-\mu_{ii}\|h_{\Delta_{ii}}\|_1}\|z_i^1\|_p + \sum_{\substack{j=1\\j\neq i}}^{N}\mu_{ij}\Bigg(\|h''_{\Delta_{ij}}\|_1+\frac{\mu_{ii}\|h''_{\Delta_{ii}}\|_1}{1-\mu_{ii}\|h_{\Delta_{ii}}\|_1}\|h_{\Delta_{ij}}\|_1\Bigg)\|z_j^1\|_p, \quad (38)$$

where $h''_{\Delta_{ij}}$ are impulse responses of $(s+a_i^{n_i-1})\Delta_{ij}(s)$.

Substituting (36)–(38) and (29) into (34) gives (30), which confirms the lemma. □

Theorem 2. *Consider the case of zero initial values, i.e.,* $z_i(0)=0$, $\varepsilon_i(0)=0$, $v_{ij}(0)=0$, $\eta(0)=0$, $\zeta(0)=0$. *If* $\Gamma_i=\gamma_iI_i$, *then the* L_∞ *norm and* L_2 *norm of* $z_i(t)$ *are given, respectively, by*

$$\|z_i(t)\|_\infty \le \frac{1}{2\sqrt{c_i^0d_i^0}}\frac{\|\tilde{\theta}_i(0)\|}{\sqrt{\gamma_i}}\Bigg\{\|\tilde{\theta}_i(0)\|\,\|h_{\omega i}\|_1 + \mu_{ii}\bar{M}'_1(i,j)+\sum_{\substack{j=1\\j\neq i}}^{N}\mu_{ij}\bar{M}'_2(i,j) + \gamma_{ij}M_3(i,j)\Bigg\} + \frac{\kappa_{\omega i}}{2\sqrt{c_i^0d_i^0}}\|\tilde{\theta}_i(0)\|, \quad (39)$$

$$\|z_i(t)\|_2 \le \frac{1}{2c_i^0\sqrt{d_i^0}}\frac{\|\tilde{\theta}_i(0)\|}{\sqrt{2\gamma_i}}\Bigg\{\|\tilde{\theta}_i(0)\|\,\|h_{\omega i}\|_1 + \mu_{ii}\bar{M}'_1(i,j)+\sum_{\substack{j=1\\j\neq i}}^{N}\mu_{ij}\bar{M}'_2(i,j)+\gamma_{ij}M_3(i,j)\Bigg\} + \frac{\kappa_{\omega i}}{2\sqrt{c_i^0d_i^0}}\|\tilde{\theta}_i(0)\|, \quad (40)$$

where

$$\bar{M}'_1(i,j) = \frac{\|h_{\varepsilon i}\|_1\|h_{\Delta_{ii}}\|_1+\|h''_{\Delta_{ii}}\|_1+\|\tilde{\theta}_i(0)\|\,\|h'_{\Delta_{ii}}\|_1}{1-\mu_{ii}\|h_{\Delta_{ii}}\|_1}, \quad (41)$$

$$\bar{M}'_2(i,j) = \frac{\|h_{\varepsilon i}\|_1 \|h_{\Delta_{ij}}\|_1 + \|h''_{\Delta_{ii}}\|_1 + \|\tilde{\theta}_i(0)\| \, \|h'_{\Delta_{ij}}\|_1}{1 - \mu_{ii}\|h_{\Delta_{ii}}\|_1} \|h_{\Delta_{ij}}\|_1 + \|h'_{\Delta_{ij}}\|_1 (\|\tilde{\theta}_i(0)\| + 1). \tag{42}$$

Proof. First, we prove (39). It follows from the definition of $V(t)$ that

$$\|z_i(t)\|_\infty \leq \sqrt{2V(0)}, \tag{43}$$

$$\|\tilde{\theta}_i\|_\infty \leq \sqrt{\lambda_{\max}(\Gamma_i)}\sqrt{2V(0)} \tag{44}$$

because $V(t)$ is a nonincreasing function.

On the other hand, it is known from the definition of $V(t)$ that $V(0) = (1/2\gamma_i)\|\tilde{\theta}_i(0)\|^2$ under the conditions of the theorem. Substituting this $V(0)$ and (44) into (30) gives (39).

Next, we prove (40). It follows from (25) that $\dot{V} \leq -c_i^0 \|z_i^{\mathrm{T}}(t) z_i(t)\|$. Thus, $\|z_i\|_2^2 = \int_0^t |z_i^{\mathrm{T}}(\tau) z_i(\tau)| \, \mathrm{d}\tau \leq (1/c_i^0)(V(0) - V(\infty)) \leq (1/c_i^0)V(0)$. Again using $V(0) = (1/2\gamma_i)\|\tilde{\theta}_i(0)\|^2$, we have (40). $\square$

Remark 5. This theorem shows that the zero-state transient performance of the adaptive system subject to the proposed decentralized controller can be made arbitrarily small by increasing the control design parameters c_i^0 and d_i^0.

Remark 6. It is shown by (30) that in the case of the nonzero state $z_i(0)$, the influence of $z_i(0)$ decays exponentially.

Remark 7. From the analysis in the previous sections, it can be seen that the obtained results can be extended straightforwardly to allow for bounded disturbances with the same gains as in Jain and Khorrami (1997a,b).

6. Conclusions

This paper studies the problem of decentralized adaptive control of a large-scale system with both strong static interactions and weak dynamic interactions between subsystems. By using the adaptive backstepping technique, totally decentralized regulators are obtained without any restrictions on subsystem relative degrees. It has been shown that the proposed decentralized regulators can ensure the global stability of the whole system even in the presence of ignored interactions and unmodeled dynamics in each subsystem. The L_2 and L_∞ bounds of the tracking error are given to evaluate the transient performance of the adaptive system with the proposed controller. It is shown that the tracking transient performance can be improved to any prespecified level by choosing proper controller design parameters.

References

Ionannou, P., & Kokotovic, P. V. (1985). Decentralized adaptive control of interconnected systems with reduced-order models. *Automatica*, *21*, 401–412.

Ionannou, P. (1986). Decentralized adaptive control of interconnected systems. *IEEE Transactions on Automatic Control*, *31*, 291–298.

Gavel, D. T., & Siljak, D. D. (1989). Decentralized adaptive control: structural conditions for stability. *IEEE Transactions on Automatic Control*, *34*, 413–426.

Hill, D. J., Wen, C., & Goodwin, G. C. (1988). Stability analysis of decentralized robust adaptive control. *System & Control Letters*, *11*, 277–284.

Wen, C., & Hill, D. J. (1992). Global boundedness of discrete-time adaptive control by parameter projection. *Automatica*, *28*, 1143–1157.

Fu, L. C. (1992). Robust adaptive decentralized control of robot manipulators. *IEEE Transactions on Automatic Control*, *37*, 106–110.

Shi, L., & Singh, S. K. (1992). Decentralized adaptive controller design of a large-scale systems with high-order interconnections. *IEEE Transactions on Automatic Control*, *37*, 1106–1118.

Wen, C. (1994a). Indirect robust totally decentralized adaptive control of continuous-time interconnected systems. *IEEE Transactions on Automatic Control*, *39*, 953–959.

Ortega, R., & Herrera, A. (1993). A solution to the decentralized stabilization problem. *Systems & Control Letters*, *20*, 299–306.

Wen, C. (1994b). Decentralized adaptive regulation. *IEEE Transactions on Automatic Control*, *39*, 2163–2166.

Wen, C., & Soh, Y. C. (1997). Decentralized adaptive control using integrator backstepping. *Automatica*, *33*, 1719–1724.

Ortega, R. (1996). An energy amplification condition for decentralized adaptive stabilization. *IEEE Transactions on Automatic Control*, *41*, 285–289.

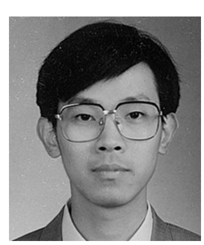
Jain, S., & Khorrami, F. (1997a). Decentralized adaptive control of a class of large-scale nonlinear systems. *IEEE Transactions on Automatic Control*, *42*, 136–154.

Jain, S., & Khorrami, F. (1997b). Decentralized adaptive output feedback design for large-scale nonlinear systems. *IEEE Transactions on Automatic Control*, *42*, 729–735.

Huseyin, O., Sezer, M. E., & Siljak, D. D. (1982). Robust decentralized control using output feedback. *IEE Proceeding D: Control Theory and Application*, *129*, 310–314.

Krstic, M., Kanellakopoulos, I., & Kokotović, P. V. (1995). *Nonlinear and adaptive control design.* New York, Wiley.

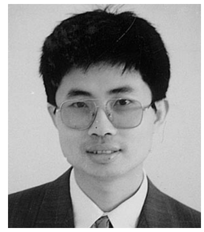
Ho, M., & Datta, A. (1996). Partially decentralized adaptive control of interconnected systems with improved performance. *International Journal of Control*, *64*, 529–561.



Ying Zhang received the BS, MS and Ph.D. degrees in Electrical Engineering in 1989, 1992 and 1995 respectively, all from the Southeast University, Nanjing, P.R. China. From 1996 to 1997, he was a post doctoral fellowin School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. Since 1998, he has been with Gintic Institute of Manufacturing Technology, Singapore. His research interests are in the areas of system identification, adaptive control, signal processing and computer vision.

Yen Chai Soh received the BEng (Hons. I) degree in electrical and electronic engineering from the University of Canterbury, New Zealand, in 1983, and the Ph.D. degree in electrical engineering from the University of Newcastle, Australia, in 1987.

From 1986 to 1987, he was a research assistant in the Department of Electrical and Computer Engineering, University of Newcastle. He joined the Nanyang Technological Universtiy, Singapore, in 1987 where he is a associate professor and the head of the Control and Instrumention Division, School of Electrical and Electronic Engineering. His current research interestes are in the areas of adaptive control, robust system theory and applications, estimation and filtering, and model reduction.



Changyum Wen received his BEng from Xian Jaiotong University in 1983 and Ph.D. from the University of Newcastle in Australian. From August 1989 to August 1991, he was a Postdoctoral Fellow at University of Adelaide, Australia. Since 1991, he has been with the School of Electrical and Electronic Engineering at Nanyang Technological University where he is currently an Associate Professor. He is an *Associate Editor of IEEE Transaction on Automatic Control*. His major research interests are in areas of adaptive control, learning control, robust control, signal processing and their applications.