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Decentralized adaptive stabilization in the presence of unknown backlash-like hysteresis ☆

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Abstract

Due to the difficulty of handling both hysteresis and interactions between subsystems, there is still no result available on decentralized stabilization of unknown interconnected systems with hysteresis, even though the problem is practical and important. In this paper, we provide solutions to this challenging problem by proposing two new schemes to design decentralized output feedback adaptive controllers using backstepping approach. For each subsystem, a general transfer function with arbitrary relative degree is considered. The interactions between subsystems are allowed to satisfy a nonlinear bound with certain structural conditions. In the first scheme, no knowledge is assumed on the bounds of unknown system parameters. In case that the uncertain parameters are inside known compact sets, we propose an alternative scheme where a projection operation is employed in the adaptive laws. In both schemes, the effects of the hysteresis and the effects due to interactions are taken into consideration in devising local control laws. It is shown that the designed local adaptive controllers can ensure all the signals in the closed-loop system bounded. A root mean square type of bound is obtained for the system states as a function of design parameters. This implies that the transient system performance can be adjusted by choosing suitable design parameters. With Scheme II, the proposed control laws allow arbitrarily strong interactions provided their upper bounds are available. In the absence of hysteresis, perfect stabilization is ensured and the L_2 norm of the system states is also shown to be bounded by a function of design parameters when the second scheme is applied.

Keywords: Adaptive control; Backstepping; Backlash hysteresis; Decentralized regulation; Interconnected subsystem

1. Introduction

In the control of a large-scale system, one usually faces poor knowledge on the plant parameters and interactions between subsystems. Thus adaptive control technique in this case is an appropriate strategy to be employed. If some subsystems are distributed distantly, it is difficult for a centralized controller to gather feedback signals from these subsystems. Also the design and implementation of the centralized controller are complicated. Therefore decentralized controllers, designed independently for local subsystems and using local available signals for feedback, are proposed to overcome such problems. Such decentralized controllers, however, should be robust against the ignored interactions. In the context of decentralized adaptive control, only a limited number of results have been obtained, see for examples Datta and Ioannou (1991), Datta and Ioannou (1992), Gavel and Siljak (1989), Hill, Wen, and Goodwin (1988), Huseyin, Sezer, and Siljak (1982), Ioannou (1986), Shi and Singh (1992), Wen (1994), Wen and Hill (1992), Wen and Soh (1997), and Zhang, Wen, and Soh (2000).The scheme presented in Wen (1994) is the first result using backstepping technique to relax the requirement on the relative degree of subsystems. But the result is only applicable to interactions satisfying a first-order type of bound and transient performance is not established. In the case that the input of each loop is preceded by unknown backlash-like hysteresis, there is still no result available.

Hysteresis can be represented by both dynamic input–output and static constitutive relationships. It exists in a wide range

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of physical systems and materials, such as electro-magnetism (Mittal & Menq, 2000), piezoelectric actuators (Stepanenko & Su, 1998), brakes (Tao & Kokotovic, 1996), electronic circuits (Lamba, Grinfeld, McKee, & Simpson, 1997), motors (Adly, 1995), smart materials (Hong, Kim, Kim, & Jung, 2000), and so on (Brokate & Sprekels, 1996). When a plant is preceded by the hysteresis nonlinearity, the system usually exhibits undesirable inaccuracies or oscillations and even instability due to the combined effects of the non-differentiable and non-memoryless character of the hysteresis and the plant. Hysteresis nonlinearity is one of the key factors limiting both static and dynamic performance of feedback control systems. The development of control techniques to mitigate the effects of hysteresis is typically challenging and has recently attracted significant attention (Ahmad & Khorrami, 1999; Moheimani & Goodwin, 2001; Pare & How, 1998; Su, Stepanenko, Svoboda, & Leung, 2000; Sun, Zhang, & Jin, 1992; Tan & Baras, 2005; Tao & Kokotovic, 1995; Zhou, Wen, & Zhang, 2004). In Tan and Baras (2005), a model derivation for smart materials using physical principles leads to a hysteresis operator at the input end of a linear system. Adaptive recursive identification and inverse control are addressed. In Ahmad and Khorrami (1999), Sun et al. (1992), and Tao and Kokotovic (1995) an inverse hysteresis nonlinearity was constructed. An adaptive hysteresis inverse cascaded with the plant was employed to cancel the effects of hysteresis. In Su et al. (2000), a dynamic hysteresis model is used to pattern a backlash-like hysteresis rather than constructing an inverse model to mitigate the bounded effects of the hysteresis. In the paper, an adaptive state feedback control scheme is developed for a class of nonlinear systems. In the design, the term multiplying the control and the uncertain parameters of the system must be within a known compact set and a bound for the effects from hysteresis must also be available, in order to implement the projection operation in the estimator. If the hysteresis effect is not bounded by the given bound, system stability cannot be ensured. In Zhou et al. (2004), a state feedback control for a special structure of nonlinear systems with backlash-like hysteresis is developed using backstepping methodology. System stability was established and the tracking error was shown to converge to a residual.

Due to difficulties in considering the effects of interconnections, extension of single-loop results to multi-loop interconnecting systems is challenging, which is why the number of available results is still limited, especially for the case when the relative degree of each subsystem is greater than two. In the presence of hysteresis in unknown interconnected systems, there is no result available for decentralized stabilization so far. In this paper, we develop two output feedback decentralized backstepping adaptive stabilizers for a class of interconnected systems with arbitrary subsystem relative degrees and with the input of each subsystem preceded by unknown backlash-like hysteresis modelled by a differential equation as in Brokate and Sprekels (1996), Stepanenko and Su (1998), and Su et al. (2000). The interactions between subsystems are allowed to satisfy a nonlinear bound. The effects of both hysteresis and interactions are taken into consideration in the development of local control laws. For each subsystem, we consider a general transfer function. In Scheme I, the term multiplying the control and the system parameters are not assumed to be within known intervals. Compared with conventional backstepping approaches, two new terms are added in the parameter updating laws in order to ensure boundedness of estimates. In Scheme II, we assume uncertain parameters are inside some known bounded intervals, which is a priori information available. Thus we use projection operation in the adaptive laws. It is established that the designed local controllers with both schemes can ensure all the signals in the closed-loop system bounded. Besides stability, a root mean square type of bound is also obtained for system states as a function of design parameters. This implies that the transient system performance can be adjusted by choosing suitable design parameters. With Scheme II, arbitrarily strong interactions can be accommodated provided their upper bounds are available. In the absence of hysteresis, perfect stabilization is ensured and the L_2 norm of the system states is also shown to be bounded by a function of design parameters when Scheme II is used.

The paper is organized as follows: the problem of this paper is formulated in Section 2. In Section 3, filters are designed to estimate system states. In Section 4, two adaptive control design schemes based on the backstepping technique are proposed. In the derivation of the control law and the estimator, only the details of those steps different from the standard backstepping design are presented. In Section 5, system stability is established. Simulation results are presented to illustrate the effectiveness of our proposed schemes in Section 6. Finally, the paper is concluded in Section 7.

2. Problem formulation

A system consisting of N interconnected subsystems of order n_i modelled below is considered as

$$\dot{x}_{0i} = A_{0i}x_{0i} + b_{0i}u_i + \sum_{j=1}^{N} \bar{f}_{ij}(t, y_j), \tag{1}$$

$$y_j = c_{0i}^{\mathrm{T}} x_{0i} \quad \text{for } i = 1, \ldots, N, \tag{2}$$

where $x_{0i} \in R^{n_i}$, $u_i \in R^1$ and $y_i \in R^1$ are the states, input and output of the ith subsystem, respectively, $\bar{f}_{ij}(t, y_j) \in R^{n_i}$ denotes the nonlinear interactions from the jth subsystem to the ith subsystem for $j \neq i$, or a nonlinear un-modelled part of the ith subsystem for $j = i$. The matrices and vectors in (1) and (2) have appropriate dimensions, and their elements are constant but unknown.

Usually each loop has a backlash-like hysteresis nonlinearity and u_i is the output of such hysteresis described by

$$u_i(t) = BH_i(w_i(t)), \tag{3}$$

where $w_i(t)$ is the input of the hysteresis, $BH_i(\cdot)$ is the backlash hysteresis operator.

In this paper, we consider a hysteresis proposed in Stepanenko and Su (1998), Su et al. (2000), and Zhou et al.

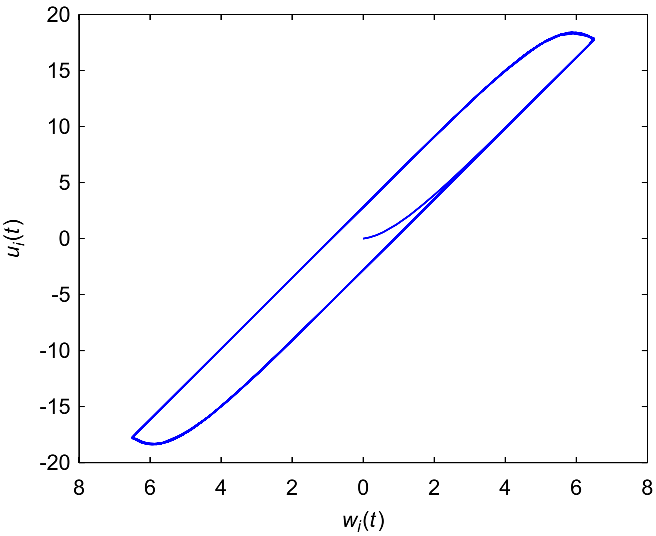

Fig. 1. Hysteresis curves.

(2004) and described by a continuous-time dynamic model

$$\frac{\mathrm{d}u_i}{\mathrm{d}t} = \alpha_i' \left|\frac{\mathrm{d}w_i}{\mathrm{d}t}\right| (c_i' w_i - u_i) + h_i \frac{\mathrm{d}w_i}{\mathrm{d}t}, \tag{4}$$

where α_i', c_i' and h_i are constants, $c_i' > 0$ is the slope of the lines satisfying $c_i' > h_i$.

Based on the analysis in Su et al. (2000), this equation can be solved explicitly

$$u_i(t) = c_i' w_i(t) + \bar{\bar{d}}_i(t), \tag{5}$$

$$\bar{\bar{d}}_i(t) = [u_i(0) - c_i' w_i(0)]\mathrm{e}^{-\alpha_i'(w_i - w_i(0))\operatorname{sgn}\dot{w}_i} + \mathrm{e}^{-\alpha_i' w_i \operatorname{sgn}\dot{w}_i} \int_{w_i(0)}^{w_i} [h_i - c_i']\mathrm{e}^{\alpha_i' \xi(\operatorname{sgn}\dot{w}_i)}\,\mathrm{d}\xi. \tag{6}$$

The solution indicates that dynamic equation (4) can be used to model a class of backlash-like hysteresis as shown in Fig. 1, where $\alpha_i' = 1$, $c_i' = 3.1635$, $h_i = 0.345$, the input signal $w_i(t) = 6.5\sin(2.3t)$ and the initial condition $u_i(0) = 0$. For $\bar{\bar{d}}_i(t)$, it is bounded clearly from Fig. 1 and the bound is unknown.

Remark 1. A number of different methods of modelling hysteresis are available in literature (Brokate & Sprekels, 1996; Hong et al., 2000; Macki, Nistri, & Zecca, 1993). The hysteresis model of this paper focuses on the fact that the output can only change its characteristics when the input changes direction. This model uses a phenomenological approach, postulating an integral operator or differential equation to model the relation. The works in Coleman and Hodgdon (1986), Hodgdon (1988a, 1988b) show that such a model is useful in applied electro-magnetics because the functions and parameters can be fine-tuned to match experimental results in a given situation. This hysteresis nonlinearity is the key factor limiting both static and dynamic performance of feedback control systems.

Now substituting (5) to (1) gives

$$\dot{x}_{0i} = A_{0i} x_{0i} + \bar{b}_{0i} w_i + \sum_{j=1}^{N} \bar{f}_{ij}(t, y_j) + \bar{d}_i(t), \tag{7}$$

$$y_j = c_{0i}^{\mathrm{T}} x_{0i}, \tag{8}$$

where $\bar{b}_{0i} = b_{0i} c_i'$ and $\bar{d}_i(t) = b_{0i}\bar{\bar{d}}_i(t)$. For each local system, we make the following assumptions.

Assumption 1. n_i is known.

Assumption 2. The triple $(A_{0i}, \bar{b}_{0i}, c_{0i})$ are completely controllable and observable.

Assumption 3. Inthe transfer function

$$G_i(s) = c_{0i}^{\mathrm{T}}(sI - A_{0i})^{-1}\bar{b}_{0i} = \frac{N_i(s)}{D_i(s)} = \frac{b_i^{m_i} s^{m_i} + \cdots + b_i^1 s + b_i^0}{s^{n_i} + a_i^{n_i - 1} s^{n_i - 1} + \cdots + a_i^1 s + a_i^0}, \tag{9}$$

where $N_i(s)$ is a Hurwitz polynomial. The sign of $b_i^{m_i}$ and the relative degree $\rho_i (= n_i - m_i)$ of $G_i(s)$ are known.

Assumption 4. The nonlinear interaction terms satisfy

$$\|\bar{f}_{ij}(t, y_j)\| \leqslant \bar{\gamma}_{ij} |y_j \psi_j(y_j)|, \tag{10}$$

where $\|\cdot\|$ denotes the Euclidean norm, $\bar{\gamma}_{ij}$ are constants denoting the strength of the interaction, and $\psi_j(y_j)$, $j = 1, 2, \ldots, N$ are known nonlinear functions and differentiable at least ρ_i times.

Remark 2. Assumption 4 means that the effects of the nonlinear interactions to a local subsystem from other subsystems or its unmodelled part is bounded by a function of the output of this subsystem. With this condition, it is possible for the designed local controllers to stabilize the interconnected systems with strong interactions. In fact, this assumption is much more relaxed version of the linear bounding conditions used in Huseyin et al. (1982), Sezer and Siljak (1981a, 1981b), and Wen (1994).

The control objective is to design totally decentralized adaptive controllers for system (1) and (4) satisfying Assumptions 1–4 so that the closed-loop system is stable and the system performance in certain sense is adjustable by design parameters.

3. Local state estimation filters

In this section, a filter using only local input and output will be designed to estimate the states of each unknown local system in the presence of both interaction and hysteresis. To achieve this, each local system model given in (1) is transformed to a more suitable form. From Assumption 2, there exists a non-singular matrix T_i, such that under transformation $x_{0i} = T_i x_i$, (7) and (8) can be transformed to

$$\dot{x}_i = A_i x_i + a_i y_i + \begin{bmatrix} 0 \\ b_i \end{bmatrix} w_i + f_i + d_i, \tag{11}$$

$$y_i = (e^1_{n_i})^{\mathrm{T}} x_i \quad \text{for } i = 1, \ldots, N, \tag{12}$$

where

$$A_i = \begin{bmatrix} 0 & & \\ \vdots & I_{n_i-1} & \\ 0 & \cdots & 0 \end{bmatrix}, \quad a_i = \begin{bmatrix} -a_i^{n_i-1} \\ \vdots \\ -a_i^0 \end{bmatrix}, \quad b_i = \begin{bmatrix} b_i^{m_i} \\ \vdots \\ b_i^0 \end{bmatrix}, \tag{13}$$

$$f_i = \sum_{j=1}^{N} T_i^{-1} \bar{f}_{ij}, \quad d_i = T_i^{-1} \bar{d}_i(t) \tag{14}$$

and e^k_j denotes the kth coordinate vector in $\Re^j$. Similar transformations can be found in Marino and Tomei (1995) and Krstic, Kanellakopoulos, and Kokotovic (1995). For state estimation, by following the standard procedures as in Wen (1994), we can obtain

$$\dot{v}_i^j = A_i^0 v_i^j + e^{n_i}_{n_i} w_i, \quad j = 0, \ldots, m_i, \tag{15}$$

$$\dot{\eta}_i = A_i^0 \eta_i + e^{n_i}_{n_i} y_i, \tag{16}$$

$$\Omega_i^{\mathrm{T}} = [v_i^{m_i}, \ldots, v_i^1, v_i^0, \Xi_i], \tag{17}$$

$$\Xi_i = -[(A_i^0)^{n_i-1}\eta_i, \ldots, A_i^0 \eta_i, \eta_i], \tag{18}$$

$$\xi_i^{n_i} = -(A_i^0)^{n_i} \eta_i, \tag{19}$$

where the vector $k_i = [k_i^1, \ldots, k_i^{n_i}]^{\mathrm{T}}$ is chosen so that the matrix $A_i^0 = A_i - k_i (e^1_{n_i})^{\mathrm{T}}$ is Hurwitz. Hence there exists a P_i such that $P_i A_i^0 + (A_i^0)^{\mathrm{T}} P_i = -2I$, $P_i = P_i^{\mathrm{T}} > 0$. With these designed filters our state estimate is

$$\hat{x}_i = \xi_i^{n_i} + \Omega_i^{\mathrm{T}} \theta_i, \tag{20}$$

$$\theta_i^{\mathrm{T}} = [b_i^{\mathrm{T}}, a_i^{\mathrm{T}}] \tag{21}$$

and the state estimation error $\varepsilon_i = x_i - \hat{x}_i$ satisfies

$$\dot{\varepsilon}_i = A_i^0 \varepsilon_i + f_i + d_i. \tag{22}$$

Let $V_{\varepsilon_i} = \varepsilon_i^{\mathrm{T}} P_i \varepsilon_i$. It can be shown that

$$\begin{aligned} \dot{V}_{\varepsilon_i} &= \varepsilon_i^{\mathrm{T}} [P_i A_i^0 + (A_i^0)^{\mathrm{T}} P_i] \varepsilon_i + 2\varepsilon_i^{\mathrm{T}} P_i (f_i + d_i) \\ &\leqslant -\varepsilon_i^{\mathrm{T}} \varepsilon_i + 2\|P_i d_i\|^2 + 2\|P_i f_i\|^2. \end{aligned} \tag{23}$$

Then system (11) can be expressed as

$$\dot{y}_i = b_i^{m_i} v_i^{m_i,2} + \xi_i^{n_i,2} + \bar{\delta}_i^{\mathrm{T}} \theta_i + \varepsilon_i^2 + f_i^1 + d_i^1, \tag{24}$$

$$\dot{v}_i^{m_i,q} = v_i^{m_i,q+1} - k_i^q v_i^{m_i,1}, \quad q = 2, \ldots, \rho_i - 1, \tag{25}$$

$$\dot{v}_i^{m_i,\rho_i} = v_i^{m_i,\rho_i+1} - k_i^{\rho_i} v_i^{m_i,1} + w_i, \tag{26}$$

where

$$\delta_i = [v_i^{m_i,2}, v_i^{m_i-1,2}, \ldots, v_i^{0,2}, \Xi_i^{(2)} - y_i (e^1_{n_i})^{\mathrm{T}}]^{\mathrm{T}}, \tag{27}$$

$$\bar{\delta}_i = [0, v_i^{m_i-1,2}, \ldots, v_i^{0,2}, \Xi_i^{(2)} - y_i (e^1_{n_i})^{\mathrm{T}}]^{\mathrm{T}} \tag{28}$$

and $v_i^{m_i,2}$, ε_i^2, $\xi_i^{n_i,2}$, Ξ_i^2 denote the second entries of $v_i^{m_i}$, ε_i, $\xi_i^{n_i}$, Ξ_i, respectively, f_i^1 and d_i^1 are the first elements of vectors f_i and d_i. All states of the local filters in (15) and (16) are available for feedback.

4. Design of adaptive controllers

In this section, we develop two adaptive backstepping design schemes. The system parameters b_{m_i}, θ_i are uncertain parameters. In Scheme I, there is no a priori information required from these parameters and thus they can be allowed totally uncertain. To ensure the boundedness of parameter estimates, two new terms are added in the adaptive law compared with conventional backstepping approaches. In Scheme II, we assume uncertain parameters are inside known compact sets, which is a priori information available. A projection operation, which is to replace the role of the newly added two terms in Scheme I, is used in the adaptive laws in this case. To illustrate the backstepping procedures, only the first scheme is elaborated in details.

4.1. Control Scheme I

As usual in backstepping approach, the following change of coordinates is made:

$$z_i^1 = y_i, \tag{29}$$

$$z_i^q = v_i^{m_i,q} - \alpha_i^{q-1}, \quad q = 2, 3, \ldots, \rho_i, \tag{30}$$

where α_i^{q-1} is the virtual control at the qth step of the ith loop and will be determined in later discussion. To illustrate the controller design procedures, we now give a brief description on the first step.

Step 1: We start with the equations for the stabilization error z_i^1 obtained from (24), (29) and (30) to get

$$\dot{z}_i^1 = b_i^{m_i} \alpha_i^1 + \xi_i^{n_i,2} + \bar{\delta}_i^{\mathrm{T}} \theta_i + \varepsilon_i^2 + f_i^1 + d_i^1 + b_i^{m_i} z_i^2. \tag{31}$$

The virtual control law α_i^1 is designed as

$$\alpha_i^1 = \hat{p}_i \bar{\alpha}_i^1, \tag{32}$$

$$\bar{\alpha}_i^1 = -\tfrac{3}{2} c_i^1 z_i^1 - l_i^1 z_i^1 - l_i^* z_i^1 (\psi_i(z_i^1))^2 - \xi_i^{n_i,2} - \bar{\delta}_i^{\mathrm{T}} \hat{\theta}_i, \tag{33}$$

where c_i^1, l_i^1 and l_i^* are positive design parameters, $\hat{\theta}_i$ is the estimate of θ_i, $\hat{p}_i$ is the estimate of $p_i = 1/b_i^{m_i}$.

Remark 3. The term $l_i^* z_i^1 (\psi_i(z_i^1))^2$ in (33) is designed to compensate the effects of interactions from other subsystems or the un-modelled part of its own subsystem. Note that the scheme in Wen (1994) does not have such a term and thus the result of Wen (1994) is not applicable to the systems considered here.

From (31) and (32) we have

$$\begin{aligned}\dot{z}_i^1 &= -\tfrac{3}{2}c_i^1 z_i^1 - l_i^1 z_i^1 - l_i^* z_i^1(\psi_i(z_i^1))^2 + \varepsilon_i^2 + \bar{\delta}_i^{\mathrm{T}}\tilde{\theta}_i \\ &\quad - b_i^{m_i}\bar{\alpha}_i^1\tilde{p}_i + b_i^{m_i} z_i^2 + f_i^1 + d_i^1 \\ &= -\tfrac{3}{2}c_i^1 z_i^1 - l_i^1 z_i^1 - l_i^* z_i^1(\psi_i(z_i^1))^2 + \varepsilon_i^2 + f_i^1 + d_i^1 \\ &\quad + (\delta_i - \hat{p}_i\bar{\alpha}_i^1 e^1_{n_i+m_i+1})^{\mathrm{T}}\tilde{\theta}_i - b_i^{m_i}\bar{\alpha}_i^1\tilde{p}_i + \hat{b}_i^{m_i} z_i^2,\end{aligned} \tag{34}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $e^1_{n_i+m_i+1} = [1, 0_{n_i}, 0_{m_i}]^{\mathrm{T}} \in R^{n_i+m_i+1}$.

Using $\tilde{p}_i = p_i - \hat{p}_i$, we obtain

$$b_i^{m_i}\alpha_i^1 = b_i^{m_i}\hat{p}_i\bar{\alpha}_i^1 = \bar{\alpha}_i^1 - b_i^{m_i}\tilde{p}_i\bar{\alpha}_i^1 \tag{35}$$

and

$$\begin{aligned}\bar{\delta}_i^{\mathrm{T}}\tilde{\theta}_i + b_i^{m_i} z_i^2 &= \bar{\delta}_i^{\mathrm{T}}\tilde{\theta}_i + \tilde{b}_i^{m_i} z_i^2 + \hat{b}_i^{m_i} z_i^2 \\ &= \bar{\delta}_i^{\mathrm{T}}\tilde{\theta}_i + (v_i^{m_i,2} - \alpha_i^1)(e^1_{n_i+m_i+1})^{\mathrm{T}}\tilde{\theta}_i + \hat{b}_i^{m_i} z_i^2 \\ &= (\delta_i - \hat{p}_i\bar{\alpha}_i^1 e^1_{n_i+m_i+1})^{\mathrm{T}}\tilde{\theta}_i + \hat{b}_i^{m_i} z_i^2.\end{aligned} \tag{36}$$

We consider the Lyapunov function

$$V_i^1 = \frac{1}{2}(z_i^1)^2 + \frac{1}{2}\tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\theta}_i + \frac{|b_i^{m_i}|}{2\gamma_i'}(\tilde{p}_i)^2 + \frac{1}{2\bar{l}_i^1}V_{\varepsilon_i}, \tag{37}$$

where Γ_i is a positive definite design matrix and γ_i' is a positive design parameter. We now examine the derivative of V_i^1

$$\begin{aligned}\dot{V}_i^1 &= z_i^1\dot{z}_i^1 - \tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\dot{\hat{\theta}}_i - \frac{|b_i^{m_i}|}{\gamma_i'}\tilde{p}_i\dot{\hat{p}}_i + \frac{1}{2\bar{l}_i^1}\dot{V}_{\varepsilon_i} \\ &\leqslant -\frac{3}{2}c_i^1(z_i^1)^2 + \hat{b}_i^{m_i} z_i^1 z_i^2 - l_i^*(z_i^1)^2(\psi_i(z_i^1))^2 \\ &\quad + \tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}[\Gamma_i(\delta_i - \hat{p}_i\bar{\alpha}_i^1 e^1_{n_i+m_i+1})z_i^1 - \dot{\hat{\theta}}_i] \\ &\quad - |b_i^{m_i}|\tilde{p}_i\frac{1}{\gamma_i'}[\gamma_i'\,\mathrm{sgn}(b_i^{m_i})\bar{\alpha}_i^1 z_i^1 + \dot{\hat{p}}_i] \\ &\quad - \frac{1}{2\bar{l}_i^1}\varepsilon_i^{\mathrm{T}}\varepsilon_i + \frac{1}{\bar{l}_i^1}(\|P_i d_i\|^2 + \|P_i f_i\|^2) \\ &\quad - l_i^1(z_i^1)^2 + (f_i^1 + d_i^1 + \varepsilon_i^2)z_i^1.\end{aligned} \tag{38}$$

Now we choose

$$\dot{\hat{p}}_i = -\gamma_i'\,\mathrm{sgn}(b_i^{m_i})\bar{\alpha}_i^1 z_i^1 - \gamma_i' l_i^p(\hat{p}_i - p_i^0), \tag{39}$$

$$\tau_i^1 = (\delta_i - \hat{p}_i\bar{\alpha}_i^1 e^1_{n_i+m_i+1})z_i^1, \tag{40}$$

where l_i^p and p_i^0 are two positive design constants.

From the choice, the following useful property can be obtained:

$$\begin{aligned}& l_i^p\tilde{p}_i(\hat{p}_i - p_i^0) \\ &\quad = -l_i^p(\hat{p}_i - p_i)[\tfrac{1}{2}(\hat{p}_i - p_i) + \tfrac{1}{2}(\hat{p}_i + p_i) - p_i^0] \\ &\quad = -\tfrac{1}{2}l_i^p(\tilde{p}_i)^2 - \tfrac{1}{2}l_i^p(\hat{p}_i)^2 + \tfrac{1}{2}l_i^p(p_i)^2 + l_i^p\hat{p}_i p_i^0 - l_i^p p_i p_i^0 \\ &\quad = -\tfrac{1}{2}l_i^p(\tilde{p}_i)^2 - \tfrac{1}{2}l_i^p(\hat{p}_i)^2 + l_i^p\hat{p}_i p_i^0 - \tfrac{1}{2}l_i^p(p_i^0)^2 \\ &\qquad + \tfrac{1}{2}l_i^p(p_i^0)^2 - l_i^p p_i p_i^0 + \tfrac{1}{2}l_i^p(p_i)^2 \\ &\quad = -\tfrac{1}{2}l_i^p(\tilde{p}_i)^2 + \tfrac{1}{2}l_i^p(p_i - p_i^0)^2 - \tfrac{1}{2}l_i^p(\hat{p}_i - p_i^0)^2 \\ &\quad \leqslant -\tfrac{1}{2}l_i^p(\tilde{p}_i)^2 + \tfrac{1}{2}l_i^p(p_i - p_i^0)^2.\end{aligned} \tag{41}$$

Let $l_i^1 = 3\bar{l}_i^1$. Note that

$$-\bar{l}_i^1(z_i^1)^2 + f_i^1 z_i^1 \leqslant \frac{1}{4\bar{l}_i^1}\|f_i^1\|^2, \tag{42}$$

$$-\bar{l}_i^1(z_i^1)^2 + d_i^1 z_i^1 \leqslant \frac{1}{4\bar{l}_i^1}\|d_i^1\|^2, \tag{43}$$

$$\begin{aligned}-\bar{l}_i^1(z_i^1)^2 + \varepsilon_i^2 z_i^1 - \frac{1}{4\bar{l}_i^1}\varepsilon_i^{\mathrm{T}}\varepsilon_i &\leqslant -\bar{l}_i^1(z_i^1)^2 + \varepsilon_i^2 z_i^1 - \frac{1}{4\bar{l}_i^1}(\varepsilon_i^2)^2 \\ &= -\bar{l}_i^1\left(z_i^1 - \frac{1}{2\bar{l}_i^1}\varepsilon_i^2\right)^2 \leqslant 0.\end{aligned} \tag{44}$$

Then the following derivation for the derivative of V_i^1 can be carried out by using (39)–(44):

$$\begin{aligned}\dot{V}_i^1 &\leqslant -\frac{3}{2}c_i^1(z_i^1)^2 + \hat{b}_i^{m_i} z_i^1 z_i^2 - \frac{|b_i^{m_i}|}{2}l_i^p(\tilde{p}_i)^2 - \frac{1}{4\bar{l}_i^1}\varepsilon_i^{\mathrm{T}}\varepsilon_i \\ &\quad + \frac{|b_i^{m_i}|}{2}l_i^p(p_i - p_i^0)^2 + \tilde{\theta}_i^{\mathrm{T}}(\tau_i^1 - \Gamma_i^{-1}\dot{\hat{\theta}}_i) \\ &\quad - l_i^*(z_i^1\psi_i(z_i^1))^2 + \frac{1}{\bar{l}_i^1}\|P_i d_i\|^2 + \frac{1}{4\bar{l}_i^1}\|d_i^1\|^2 \\ &\quad + \frac{1}{\bar{l}_i^1}\|P_i f_i\|^2 + \frac{1}{4\bar{l}_i^1}\|f_i^1\|^2.\end{aligned} \tag{45}$$

Step q ($q = 2, \ldots, \rho_i$, $i = 1, \ldots, N$): Choose virtual control laws

$$\begin{aligned}\alpha_i^2 &= -\hat{b}_i^{m_i} z_i^1 - \left[c_i^2 + l_i^2\left(\frac{\partial\alpha_i^1}{\partial y_i}\right)^2\right] z_i^2 + \bar{B}_i^2 + \frac{\partial\alpha_i^1}{\partial\hat{\theta}_i}\Gamma_i\tau_i^2 \\ &\quad + \frac{\partial\alpha_i^1}{\partial\hat{\theta}_i}\Gamma_i l_i^\theta(\hat{\theta}_i - \theta_i^0),\end{aligned} \tag{46}$$

$$\begin{aligned}\alpha_i^q &= -z_i^{q-1} - \left[c_i^q + l_i^q\left(\frac{\partial\alpha_i^{q-1}}{\partial y_i}\right)^2\right] z_i^q + \bar{B}_i^q \\ &\quad + \frac{\partial\alpha_i^{q-1}}{\partial\hat{\theta}_i}\Gamma_i\tau_i^q + \frac{\partial\alpha_i^{q-1}}{\partial\hat{\theta}_i}\Gamma_i l_i^\theta(\hat{\theta}_i - \theta_i^0) \\ &\quad - \left(\sum_{k=2}^{q-1} z_i^k\frac{\partial\alpha_i^{k-1}}{\partial\hat{\theta}_i}\right)\Gamma_i\frac{\partial\alpha_i^{q-1}}{\partial y_i}\delta_i,\end{aligned} \tag{47}$$

$$\tau_i^q = \tau_i^{q-1} - \frac{\partial\alpha_i^{q-1}}{\partial y_i}\delta_i z_i^q, \tag{48}$$

where $c_i^q, l_i^q, q = 3, \ldots, \rho_i$ are positive design parameters, and $\bar{B}_i^q, q = 2, \ldots, \rho_i$ denotes some known terms and its detailed structure can be found in Krstic et al. (1995). Then the adaptive controller and parameter update laws are finally given by

$$w_i = \alpha_i^{\rho_i} - v_i^{m_i,\rho_i+1}, \tag{49}$$

$$\dot{\hat{\theta}}_i = \Gamma_i\tau_i^{\rho_i} + \Gamma_i l_i^\theta(\hat{\theta}_i - \theta_i^0), \tag{50}$$

where l_i^θ and θ_i^0 are positive design constants.

Note that if ψ_i is ρ_ith order differentiable, then $\alpha_i^{\rho_i}$ will be differentiable. So w_i is differentiable. Thus u_i is well defined and continuous from (4).

Table 1
Adaptive backstepping control Scheme I

Adaptive control laws:

$$\alpha_i^1 = \hat{p}_i \bar{\alpha}_i^1 \tag{T.1}$$

$$\bar{\alpha}_i^1 = -\tfrac{3}{2} c_i^1 z_i^1 - l_i^1 z_i^1 - l_i^* z_i^1 (\psi_i(z_i^1))^2 - \xi_i^{n_i,2} - \bar{\delta}_i^{\mathrm{T}} \hat{\theta}_i \tag{T.2}$$

$$\alpha_i^2 = -\hat{b}_i^{m_i} z_i^1 - \left[c_i^2 + l_i^2 \left(\frac{\partial \alpha_i^1}{\partial y_i} \right)^2 \right] z_i^2 + \bar{B}_i^2 + \frac{\partial \alpha_i^1}{\partial \hat{\theta}_i} \Gamma_i \tau_i^2 + \frac{\partial \alpha_i^1}{\partial \hat{\theta}_i} \Gamma_i l_i^\theta (\hat{\theta}_i - \theta_i^0) \tag{T.3}$$

$$\alpha_i^q = -z_i^{q-1} - \left[c_i^q + l_i^q \left(\frac{\partial \alpha_i^{q-1}}{\partial y_i} \right)^2 \right] z_i^q + \bar{B}_i^q + \frac{\partial \alpha_i^{q-1}}{\partial \hat{\theta}_i} \Gamma_i \tau_i^q + \frac{\partial \alpha_i^{q-1}}{\partial \hat{\theta}_i} \Gamma_i l_i^\theta (\hat{\theta}_i - \theta_i^0) - \left(\sum_{k=2}^{q-1} z_i^k \frac{\partial \alpha_i^{k-1}}{\partial \hat{\theta}_i} \right) \Gamma_i \frac{\partial \alpha_i^{q-1}}{\partial y_i} \delta_i, \quad q = 2, \ldots, \rho_i,\ i = 1, \ldots, N \tag{T.4}$$

$$w_i = \alpha_i^{\rho_i} - v_i^{m_i, \rho_i + 1} \tag{T.5}$$

Parameter update laws:

$$\dot{\hat{p}}_i = -\gamma_i' \operatorname{sgn}(b_i^{m_i}) \bar{\alpha}_i^1 z_i^1 - \gamma_i' l_i^p (\hat{p}_i - p_i^0) \tag{T.6}$$

$$\dot{\hat{\theta}}_i = \Gamma_i \tau_i^{\rho_i} + \Gamma_i l_i^\theta (\hat{\theta}_i - \theta_i^0) \tag{T.7}$$

$$\tau_i^q = \tau_i^{q-1} - \frac{\partial \alpha_i^{q-1}}{\partial y_i} \delta_i z_i^q \tag{T.8}$$

$$\tau_i^1 = (\delta_i - \hat{p}_i \bar{\alpha}_i^1 e_{n_i + m_i + 1}^1) z_i^1 \tag{T.9}$$

The designed adaptive controllers are summarized in Table 1.

Remark 4. From the analysis above, terms $\gamma_i' l_i^p (\hat{p}_i - p_i^0)$ and $\Gamma_i l_i^\theta (\hat{\theta}_i - \theta_i^0)$ in the adaptive controllers are used to handle the effects of hysteresis in order to ensure the boundedness of the parameter estimates. If projection operation is used as in Scheme II, such terms are not needed.

Remark 5. When going through the details of the design procedures, we note that in the equations concerning $\dot{z}_i^q, q = 1, 2, \ldots, \rho_i$, just functions f_i^1 from the interactions and d_i^1 due to the hysteresis effect appear, and they are always together with ε_i^2. This is because only $\dot{y}_i$ from the plant model (11) was used in the calculation of $\dot{\alpha}_i^q$ for steps $q = 2, \ldots, \rho_i$.

Remark 6. From our analysis, it can be noted that the design method can also be applied to system with perturbations satisfying similar boundedness properties to (10).

4.2. Control Scheme II

In this section, we assume uncertain parameters p_i and θ_i are inside known compact sets, which is the a priori information available as follows.

Assumption 5. Parameters p_i and θ_i are inside known compact sets Ω_{p_i} and Ω_{θ_i}.

Table 2
Adaptive backstepping control Scheme II

Adaptive control laws:

$$\alpha_i^1 = \hat{p}_i \bar{\alpha}_i^1 \tag{T.10}$$

$$\bar{\alpha}_i^1 = -\tfrac{3}{2} c_i^1 z_i^1 - l_i^1 z_i^1 - l_i^* z_i^1 (\psi_i(z_i^1))^2 - \xi_i^{n_i,2} - \bar{\delta}_i^{\mathrm{T}} \hat{\theta}_i \tag{T.11}$$

$$\alpha_i^2 = -\hat{b}_i^{m_i} z_i^1 - \left[c_i^2 + l_i^2 \left(\frac{\partial \alpha_i^1}{\partial y_i} \right)^2 \right] z_i^2 + \bar{B}_i^2 + \frac{\partial \alpha_i^1}{\partial \hat{\theta}_i} \Gamma_i \tau_i^2 \tag{T.12}$$

$$\alpha_i^q = -z_i^{q-1} - \left[c_i^q + l_i^q \left(\frac{\partial \alpha_i^{q-1}}{\partial y_i} \right)^2 \right] z_i^q + \bar{B}_i^q + \frac{\partial \alpha_i^{q-1}}{\partial \hat{\theta}_i} \Gamma_i \tau_i^q - \left(\sum_{k=2}^{q-1} z_i^k \frac{\partial \alpha_i^{k-1}}{\partial \hat{\theta}_i} \right) \Gamma_i \frac{\partial \alpha_i^{q-1}}{\partial y_i} \delta_i, \quad q = 2, \ldots, \rho_i,\ i = 1, \ldots, N \tag{T.13}$$

$$w_i = \alpha_i^{\rho_i} - v_i^{m_i, \rho_i + 1} \tag{T.14}$$

Parameter update laws:

$$\dot{\hat{p}}_i = Proj\{-\gamma_i' \operatorname{sgn}(b_i^{m_i}) \bar{\alpha}_i^1 z_i^1\} \tag{T.15}$$

$$\dot{\hat{\theta}}_i = Proj\{\Gamma_i \tau_i^{\rho_i}\} \tag{T.16}$$

$$\tau_i^q = \tau_i^{q-1} - \frac{\partial \alpha_i^{q-1}}{\partial y_i} \delta_i z_i^q \tag{T.17}$$

$$\tau_i^1 = (\delta_i - \hat{p}_i \bar{\alpha}_i^1 e_{n_i + m_i + 1}^1) z_i^1 \tag{T.18}$$

Thus we can use a smooth projection operation in the adaptive laws to ensure the estimates belonging to the compact sets for all the time. Such an operation can be found in Krstic et al. (1995). As shown in Krstic et al. (1995), the projection operation can ensure that the estimated parameter $\hat{p}_i(t) \in \Omega_{p_i}$ for all t, if $\hat{p}_i(0) \in \Omega_{p_i}$ and the estimated parameter vector $\hat{\theta}_i(t) \in \Omega_{\theta_i}$ for all t, if $\hat{\theta}_i(0) \in \Omega_{\theta_i}$.Thus, the boundedness of $\hat{\theta}_i$ and $\hat{p}_i$ are guaranteed for all t. Therefore, in this case, we do not need terms $\gamma_i' l_i^p (\hat{p}_i - p_i^0)$ and $\Gamma_i l_i^\theta (\hat{\theta}_i - \theta_i^0)$ in the controller design as in Scheme I.

As the controller design is similar to Scheme I, we only present the resulting control laws as summarized in Table 2.

5. Stability analysis

In this section, the stability of the overall closed-loop system consisting of the interconnected plants and decentralized controllers will be established.

5.1. Control Scheme I

Firstly, define $z_i(t) = [z_i^1, z_i^2, \ldots, z_i^{\rho_i}]^{\mathrm{T}}$. A mathematical model for each local closed-loop control system is derived from (34) and the rest of the design steps $2, \ldots, \rho_i$.

$$\dot{z}_i = A_{z_i} z_i + W_{\varepsilon i} (\varepsilon_i^2 + f_i^1 + d_i^1) + W_{\theta i}^{\mathrm{T}} \tilde{\theta}_i - b_i^{m_i} \bar{\alpha}_i^1 \tilde{p}_i e_{\rho_i}^1 - l_i^* z_i^1 (\psi_i(z_i^1))^2 e_{\rho_i}^1, \tag{51}$$

where A_{z_i} is a matrix having the similar structure as in the scalar systems given in Krstic et al. (1995):

$$A_{z_i}=\begin{bmatrix} -c_i^1-l_i^1 & \cdots & \cdots & 0 \\ -\hat{b}_i^{m_i} & \cdots & \cdots & \sigma_i^{2,\rho_i} \\ 0 & \cdots & \cdots & \sigma_i^{3,\rho_i} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & -c_i^{\rho_i}-l_i^{\rho_i}\left(\dfrac{\partial\alpha_i^{\rho_i-1}}{\partial y_i}\right)^2 \end{bmatrix}, \tag{52}$$

$$W_{\varepsilon i}=\begin{bmatrix} 1 \\ -\dfrac{\partial\alpha_i^1}{\partial y_i} \\ \vdots \\ -\dfrac{\partial\alpha_i^{\rho_i-1}}{\partial y_i} \end{bmatrix}, \quad W_{\theta i}^{\mathrm{T}}=W_{\varepsilon i}\delta_i^{\mathrm{T}}-\hat{p}_i\bar{\alpha}_i^1 e_{\rho_i}^1 {e_{\rho_i}^1}^{\mathrm{T}}, \tag{53}$$

where the terms $\sigma_i^{k,q}$ are due to the terms $(\partial\alpha_i^{k-1}/\partial\hat{\theta}_i)\Gamma_i(\tau_i^q-\tau_i^{q-1})$ in the z_i^q equation.

To show the system stability, the variables of the filters in (16) and the zero dynamics of subsystems should be included in the Lyapunov function. Under a similar transformation as in Wen (1994), the variables ζ_i associated with the zero dynamics of the ith subsystem can be shown to satisfy

$$\dot{\zeta}_i=A_i^{b_i}\zeta_i+\bar{b}_i z_i^1+\bar{f}_i, \tag{54}$$

where the eigenvalues of the $m_i\times m_i$ matrix $A_i^{b_i}$ are the zeros of the Hurwitz polynomial $N_i(s)$, $\bar{b}_i\in R^{m_i}$ and $\bar{f}_i\in R^{m_i}$ denoting the effects of the transformed interactions.

Now we define a Lyapunov function of the overall decentralized adaptive control system as

$$V=\sum_{i=1}^N V_i, \tag{55}$$

where

$$V_i=\sum_{q=1}^{\rho_i}\left(\frac{1}{2}(z_i^q)^2+\frac{1}{2\bar{l}_i^q}\varepsilon_i^{\mathrm{T}}P_i\varepsilon_i\right)+\frac{1}{2}\tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\theta}_i+\frac{|b_i^{m_i}|}{2\gamma_i'}\tilde{p}_i^2 + \frac{1}{2l_i^{\eta_i}}\eta_i^{\mathrm{T}}P_i\eta_i+\frac{1}{2l_i^{\zeta_i}}\zeta_i^{\mathrm{T}}P_i^{b_i}\zeta_i, \tag{56}$$

where $P_i^{b_i}$ satisfies $P_i^{b_i}(A_i^{b_i})+(A_i^{b_i})^{\mathrm{T}}P_i^{b_i}=-2I$, $l_i^{\eta_i}$ and $l_i^{\zeta_i}$ are constants satisfying

$$l_i^{\eta_i}\geqslant\frac{2\|P_i e_{n_i}^{n_i}\|^2}{c_i^1}, \tag{57}$$

$$l_i^{\zeta_i}\geqslant\frac{2\|P_i^{b_i}\bar{b}_i\|^2}{c_i^1}. \tag{58}$$

Note that

$$\begin{aligned}\Gamma_i\tau_i^{q-1}-\dot{\hat{\theta}}_i&=\Gamma_i\tau_i^{q-1}-\Gamma_i\tau_i^q+\Gamma_i\tau_i^q-\dot{\hat{\theta}}_i\\ &=\Gamma_i\frac{\partial\alpha_i^{q-1}}{\partial y_i}\delta z_i^q+(\Gamma_i\tau_i^q-\dot{\hat{\theta}}_i)\end{aligned} \tag{59}$$

and

$$\begin{aligned} l_i^\theta\tilde{\theta}_i^{\mathrm{T}}(\hat{\theta}_i-\theta_i^0)&=-l_i^\theta(\hat{\theta}_i-\theta_i)^{\mathrm{T}}(\tfrac{1}{2}(\hat{\theta}_i-\theta_i)+\tfrac{1}{2}(\hat{\theta}_i+\theta_i)-\theta_i^0)\\ &=-\tfrac{1}{2}l_i^\theta\|\tilde{\theta}_i\|^2-\tfrac{1}{2}l_i^\theta\|\hat{\theta}_i\|^2+\tfrac{1}{2}l_i^\theta\|\theta_i\|^2\\ &\quad+l_i^\theta\hat{\theta}_i^{\mathrm{T}}\theta_i^0-l_i^\theta\theta_i^{\mathrm{T}}\theta_i^0\\ &=-\tfrac{1}{2}l_i^\theta\|\tilde{\theta}_i\|^2-\tfrac{1}{2}l_i^\theta\|\hat{\theta}_i\|^2+l_i^\theta\hat{\theta}_i^{\mathrm{T}}\theta_i^0-\tfrac{1}{2}l_i^\theta\|\theta_i^0\|^2\\ &\quad+\tfrac{1}{2}l_i^\theta\|\theta_i^0\|^2-l_i^\theta\theta_i^{\mathrm{T}}\theta_i^0+\tfrac{1}{2}l_i^\theta\|\theta_i\|^2\\ &=-\tfrac{1}{2}l_i^\theta\|\tilde{\theta}_i\|^2+\tfrac{1}{2}l_i^\theta\|\theta_i-\theta_i^0\|^2-\tfrac{1}{2}l_i^\theta\|\hat{\theta}_i-\theta_i^0\|^2\\ &\leqslant-\tfrac{1}{2}l_i^\theta\|\tilde{\theta}_i\|^2+\tfrac{1}{2}l_i^\theta\|\theta_i-\theta_i^0\|^2. \end{aligned} \tag{60}$$

From (23), (45), (T.10)–(T.16), (54), (59) and (60), the derivative of V_i in (56) is given by

$$\begin{aligned}\dot{V}_i\leqslant&-\sum_{q=1}^{\rho_i}c_i^q(z_i^q)^2-\frac{1}{2}l_i^\theta\|\tilde{\theta}_i\|^2+\frac{1}{2}l_i^\theta\|\theta_i-\theta_i^0\|^2\\ &+\sum_{q=1}^{\rho_i}\frac{1}{\bar{l}_i^q}(\|P_i d_i\|^2+\|P_i f_i\|^2)-\frac{|b_i^{m_i}|}{2}l_i^p(\tilde{p}_i)^2\\ &+\frac{|b_i^{m_i}|}{2}l_i^p(p_i-p_i^0)^2-l_i^*(z_i^1)^2\psi_i^2(z_i^1)-\frac{1}{4\bar{l}_i^1}\varepsilon_i^{\mathrm{T}}\varepsilon_i\\ &+\frac{1}{4\bar{l}_i^1}(\|f_i^1\|^2+\|d_i^1\|^2)+\sum_{q=2}^{\rho_i}\left(-\frac{1}{2\bar{l}_i^q}\varepsilon_i^{\mathrm{T}}\varepsilon_i\right.\\ &\left.-l_i^q\left(\frac{\partial\alpha_i^{q-1}}{\partial y_i}\right)^2(z_i^q)^2+\frac{\partial\alpha_i^{q-1}}{\partial y_i}(f_i^1+d_i^1+\varepsilon_i^2)z_i^q\right)\\ &-\frac{1}{2}c_i^1(z_i^1)^2-\frac{1}{l_i^{\eta_i}}\|\eta_i\|^2+\frac{1}{l_i^{\eta_i}}\eta_i^{\mathrm{T}}P_i e_{n_i}^{n_i}y_i\\ &-\frac{1}{l_i^{\zeta_i}}\|\zeta_i\|^2+\frac{1}{l_i^{\zeta_i}}\zeta_i^{\mathrm{T}}P_i^{b_i}\bar{b}_i z_i^1+\frac{1}{l_i^{\zeta_i}}\zeta_i^{\mathrm{T}}P_i^{b_i}\bar{f}_i.\end{aligned} \tag{61}$$

Using the inequality $ab\leqslant(a^2+b^2)/2$, we have

$$-\bar{l}_i^q\left(\frac{\partial\alpha_i^{q-1}}{\partial y_i}\right)^2(z_i^q)^2+\frac{\partial\alpha_i^{q-1}}{\partial y_i}f_i^1 z_i^q\leqslant\frac{1}{4\bar{l}_i^q}\|f_i^1\|^2, \tag{62}$$

$$-\bar{l}_i^q\left(\frac{\partial\alpha_i^{q-1}}{\partial y_i}\right)^2(z_i^q)^2+\frac{\partial\alpha_i^{q-1}}{\partial y_i}d_i^1 z_i^q\leqslant\frac{1}{4\bar{l}_i^q}\|d_i^1\|^2, \tag{63}$$

$$-\bar{l}_i^q\left(\frac{\partial\alpha_i^{q-1}}{\partial y_i}\right)^2(z_i^q)^2+\frac{\partial\alpha_i^{q-1}}{\partial y_i}\varepsilon_i^2 z_i^q-\frac{1}{4\bar{l}_i^q}\varepsilon_i^{\mathrm{T}}\varepsilon_i\leqslant 0 \tag{64}$$

and

$$\begin{aligned}&-\frac{1}{2l_i^{\eta_i}}\|\eta_i\|^2+\frac{1}{l_i^{\eta_i}}\eta_i^{\mathrm{T}}P_i e_{n_i}^{n_i}z_i^1-\frac{1}{4}c_i^1(z_i^1)^2\\ &\leqslant-\frac{\|\eta_i\|^2}{2(l_i^{\eta_i})^2}\left(l_i^{\eta_i}-\frac{2\|P_i e_{n_i}^{n_i}\|^2}{c_i^1}\right)\leqslant 0,\end{aligned} \tag{65}$$

$$-\frac{1}{2l_i^{\zeta_i}}\|\zeta_i\|^2+\frac{1}{l_i^{\zeta_i}}\zeta_i^{\mathrm{T}}P_i^{b_i}\bar{b}_iz_i^1-\frac{1}{4}c_i^1(z_i^1)^2$$

$$\leqslant -\frac{\|\zeta_i\|^2}{2(l_i^{\zeta_i})^2}\left(l_i^{\zeta_i}-\frac{2\|P_i^{b_i}\bar{b}_i\|^2}{c_i^1}\right)\leqslant 0, \tag{66}$$

$$-\frac{1}{4l_i^{\zeta_i}}\|\zeta_i\|^2+\frac{1}{l_i^{\zeta_i}}\|\zeta_i\|\|P_i^{b_i}\bar{f}_i\|\leqslant\frac{1}{l_i^{\zeta_i}}\|P_i^{b_i}\bar{f}_i\|^2. \tag{67}$$

Then, the derivative of the V_i satisfies

$$\begin{aligned}\dot{V}_i\leqslant & -\sum_{q=1}^{\rho_i}c_i^q(z_i^q)^2-\frac{1}{2}l_i^\theta\|\tilde{\theta}_i\|^2-\frac{|b_i^{m_i}|}{2}l_i^p(\tilde{p}_i)^2\\ & -\sum_{q=1}^{\rho_i}\frac{1}{4\bar{l}_i^q}\varepsilon_i^{\mathrm{T}}\varepsilon_i-\frac{1}{2l_i^{\eta_i}}\|\eta_i\|^2-\frac{1}{4l_i^{\zeta_i}}\|\zeta_i\|^2\\ & -l_i^*(z_i^1)^2(\psi_i(z_i^1))^2+\sum_{q=1}^{\rho_i}\frac{1}{\bar{l}_i^q}\left(\|P_if_i\|^2+\frac{1}{4}\|f_i\|^2\right)\\ & +\frac{1}{l_i^{\zeta_i}}\|P_i^{b_i}\bar{f}_i\|^2+M_i^*,\end{aligned} \tag{68}$$

where $D_{i,\max}$ denotes the bound of $d_i(t)$, and

$$M_i^*=M_i+\sum_{q=1}^{\rho_i}\frac{1}{4\bar{l}_i^q}(4\|P_i\|^2+1)D_{i,\max}^2, \tag{69}$$

$$M_i=\frac{|b_i^{m_i}|}{2}l_i^p(p_i-p_i^0)^2+\frac{1}{2}l_i^\theta\|\theta_i-\theta_i^0\|^2. \tag{70}$$

Remark 7. Due to the presence of hysteresis, an extra term M_i^* appears in (68) compared to the analysis in Wen (1994). The handling of M_i^* is elaborated after (75).

From Assumption 4, we can show that

$$\begin{aligned}&\sum_{q=1}^{\rho_i}\frac{1}{\bar{l}_i^q}\left(\|P_if_i\|^2+\frac{1}{4}\|f_i\|^2\right)+\frac{1}{l_i^{\zeta_i}}\|P_i^{b_i}\bar{f}_i\|^2\\ &\quad\leqslant\sum_{j=1}^{N}\gamma_{ij}|z_j^1\psi_j(z_j^1)|^2,\end{aligned} \tag{71}$$

where $\gamma_{ij}=O(\bar{\gamma}_{ij}^2)$ indicating the coupling strength from the jth subsystem to the ith subsystem depending on $\bar{l}_i^q, l_i^{\zeta_i}, \|P_i\|, \|P_i^{b_i}\|$ and $\|T_j^{-1}\|$, $j=1,2,\ldots,N$. $O(\bar{\gamma}_{ij}^2)$ denotes that γ_{ij} and $O(\bar{\gamma}_{ij}^2)$ are in the same order mathematically. Clearly there exist a constant γ_{ij}^* such that for each constant γ_{ij} satisfying $\gamma_{ij}\leqslant\gamma_{ij}^*$,

$$l_i^*\geqslant\sum_{j=1}^{N}\gamma_{ji} \tag{72}$$

if

$$l_i^*\geqslant\sum_{j=1}^{N}\gamma_{ji}^*. \tag{73}$$

Constant γ_{ij}^* stands for a upper bound of γ_{ij}. Now taking the summation of the first term in (68) into account and using (71) and (72), we get

$$\begin{aligned}&\sum_{i=1}^{N}-\left[l_i^*(z_i^1)^2(\psi_i(z_i^1))^2-\frac{1}{l_i^{\zeta_i}}\|P_i^{b_i}\bar{f}_i\|^2\right.\\ &\quad\left.-\sum_{k=1}^{\rho_i}\frac{1}{\bar{l}_i^k}\left(\|P_if_i\|^2+\frac{1}{4}\|f_i\|^2\right)\right]\\ &\quad\leqslant\sum_{i=1}^{N}-\left[l_i^*-\sum_{j=1}^{N}\gamma_{ji}\right]|z_i^1\psi_i(z_i^1)|^2\leqslant 0.\end{aligned} \tag{74}$$

Then

$$\begin{aligned}\dot{V}\leqslant\sum_{i=1}^{N}&\left[-\sum_{q=1}^{\rho_i}c_i^q(z_i^q)^2-\frac{1}{2}l_i^\theta\|\tilde{\theta}_i\|^2-\frac{|b_i^{m_i}|}{2}l_i^p(\tilde{p}_i)^2\right.\\ &\left.-\sum_{q=1}^{\rho_i}\frac{1}{4\bar{l}_i^q}\varepsilon_i^{\mathrm{T}}\varepsilon_i-\frac{1}{2l_i^{\eta_i}}\|\eta_i\|^2-\frac{1}{4l_i^{\zeta_i}}\|\zeta_i\|^2+M_i^*\right].\end{aligned} \tag{75}$$

Remark 8. The summation in (74) is one of the key steps in the stability analysis. Note that this results in the cancellation of the interaction effects from other subsystems. The approach in Wen (1994) cannot be applied here due to non-Lipschitz type nonlinear interactions.

Notice that

$$\begin{aligned}&-\sum_{q=1}^{\rho_i}c_i^q(z_i^q)^2-\frac{1}{2}l_i^\theta\|\tilde{\theta}_i\|^2-\frac{|b_i^{m_i}|}{2}l_i^p(\tilde{p}_i)^2\\ &\quad-\sum_{q=1}^{\rho_i}\frac{1}{4\bar{l}_i^q}\varepsilon_i^{\mathrm{T}}\varepsilon_i-\frac{1}{2l_i^{\eta_i}}\|\eta_i\|^2-\frac{1}{4l_i^{\zeta_i}}\|\zeta_i\|^2\\ &\quad\leqslant -f_i^-\bar{V}_i\end{aligned} \tag{76}$$

and

$$\begin{aligned}V_i=&\sum_{q=1}^{\rho_i}\frac{1}{2}(z_i^q)^2+\frac{1}{2}\tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\theta}_i+\frac{|b_i^{m_i}|}{2\gamma_i'}(\tilde{p}_i)^2\\ &+\sum_{q=1}^{\rho_i}\frac{1}{2\bar{l}_i^q}\varepsilon_i^{\mathrm{T}}P_i\varepsilon_i+\frac{1}{2l_i^{\eta_i}}\eta_i^{\mathrm{T}}P_i\eta_i+\frac{1}{2l_i^{\zeta_i}}\zeta_i^{\mathrm{T}}P_i^{b_i}\zeta_i\\ \leqslant & f_i^+\bar{V}_i,\end{aligned} \tag{77}$$

where

$$\bar{V}_i=\sum_{q=1}^{\rho_i}(z_i^q)^2+\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i+(\tilde{p}_i)^2+\sum_{q=1}^{\rho_i}\varepsilon_i^{\mathrm{T}}\varepsilon_i+\eta_i^{\mathrm{T}}\eta_i+\zeta_i^{\mathrm{T}}\zeta_i, \tag{78}$$

$$f_i^- = \min\left\{c_i^q,\ \frac{1}{2}l_i^\theta,\ \frac{|b_i^{m_i}|}{2}l_i^p,\ \frac{1}{4\bar{l}_i^q},\ \frac{1}{2l_i^{\eta_i}},\ \frac{1}{4l_i^{\zeta_i}}\right\}, \tag{79}$$

$$f_i^+ = \max\left\{\frac{1}{2},\ \frac{1}{2}\lambda_i^{q,\max}(\Gamma_i),\ \frac{|b_i^{m_i}|}{2\gamma_i'},\ \frac{1}{2l_i^{\zeta_i}}\lambda_i^{q,\max}(P_i^{b_i}),\ \frac{1}{2\min(\bar{l}_i^q, l_i^{\eta_i})}\lambda_i^{q,\max}(P_i)\right\},\quad q=1,\ldots,\rho_i, \tag{80}$$

where $\lambda_i^{q,\max}(P_i)$, $\lambda_i^{q,\max}(P_i^{b_i})$ and $\lambda_i^{q,\max}(\Gamma_i)$ are the maximum eigenvalues of P_i, $P_i^{b_i}$ and Γ_i, respectively. Therefore, from (75) we obtain

$$\dot{V} \leqslant -f^*V + M^*, \tag{81}$$

where $f^* = \sum_{i=1}^N f_i^- / \sum_{i=1}^N f_i^+$, $M^* = \sum_{i=1}^N M_i^*$ is a bounded term. By direct integrations of the differential inequality (81), we have

$$V \leqslant V(0)\mathrm{e}^{-f^*t} + \frac{M^*}{f^*}(1-\mathrm{e}^{-f^*t}) \leqslant V(0) + \frac{M^*}{f^*}. \tag{82}$$

This shows that V is uniformly bounded. Thus $z_i^1, z_i^2, \ldots, \hat{p}_i, \hat{\theta}_i$, $\varepsilon_i, \zeta_i, v_i^j, \eta_i$ and x_i are bounded as in Wen (1994) and Krstic et al. (1995). Therefore boundedness of all signals in the system is ensured as formally stated in the following theorem.

Theorem 1. *Consider the closed-loop adaptive system consisting of the plant* (1) *under Assumptions* 1–4, *the controller* (49), *the estimator* (39), (50), *and the filters* (15) *and* (16). *There exist a constant γ_{ij}^* such that for each constant γ_{ij} satisfying $\gamma_{ij} \leqslant \gamma_{ij}^*$, $i, j = 1, \ldots, N$, all the signals in the system are globally uniformly bounded.*

Remark 9. Parameter l_i^* can be chosen as any positive value and the condition that $\gamma_{ij} \leqslant \gamma_{ij}^*$ has the implication that the designed local controllers are able to stabilize any interconnected system with coupling strength satisfying (73). This implication is similar to the interpretations of the results in Datta and Ioannou (1992), Ioannou (1986), Wen (1994), Wen and Hill (1992), and Wen and Soh (1997), where sufficiently weak interactions are allowed. Thus the result is qualitative in nature, which shows the robustness of designed local controllers against interactions.

We now derive a bound for the vector $z_i(t)$ where $z_i(t) = [z_i^1, z_i^2, \ldots, z_i^{\rho_i}]^{\mathrm{T}}$. Firstly, the following definitions are made:

$$c_i^0 = \min_{1\leqslant q\leqslant \rho_i} c_i^q, \quad d_0 = \sum_{i=1}^N\sum_{q=1}^{\rho_i}\frac{1}{4\bar{l}_i^q}, \tag{83}$$

$$\|z_i\|_{[0,T]} = \sqrt{\frac{1}{T}\int_0^{\mathrm{T}}\|z_i(t)\|^2\,\mathrm{d}t}. \tag{84}$$

Note that definition (84) is similar to the root mean square value used in electric circuit.

Define

$$V_\rho = \sum_{i=1}^N\sum_{q=1}^{\rho_i}\left(\frac{1}{2}(z_i^q)^2 + \frac{1}{2\bar{l}_i^q}\varepsilon_i^{\mathrm{T}}P_i\varepsilon_i\right) + \frac{1}{2}\tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\theta}_i + \frac{|b_i^{m_i}|}{2\gamma_i'}(\tilde{p}_i)^2. \tag{85}$$

Following similar analysis to (68), the derivative of V_ρ can be given as

$$\dot{V}_\rho \leqslant -f^*V_\rho + M^* \leqslant c_i^0\|z_i\|^2 + M^*. \tag{86}$$

Integrating both sides, we obtain

$$\|z_i\|_{[0,T]} \leqslant \frac{1}{c_i^0}\left[\frac{|V_\rho(0)-V_\rho(T)|}{T} + \sum_{i=1}^N M_i + d_0\frac{1}{T}\sum_{i=1}^N\rho_i(4\|P_i\|^2+1)\int_0^{\mathrm{T}}(d_i(t))^2\,\mathrm{d}t\right]. \tag{87}$$

On the other hand, from (56), we have

$$\begin{aligned}\frac{|V_\rho(0)-V_\rho(T)|}{T} &\leqslant \frac{1-\mathrm{e}^{-f^*T}}{T}\left(\frac{M}{f^*}+V_\rho(0)\right) + \frac{d_0}{T}\sum_{i=1}^N\rho_i(4\|P_i\|^2+1)\int_0^{\mathrm{T}}\mathrm{e}^{-f^*(T-t)}(d_i(t))^2\,\mathrm{d}t\\ &\leqslant M + f^*V_\rho(0) + \frac{1}{T}d_0\sum_{i=1}^N\rho_i(4\|P_i\|^2+1)\\ &\quad\times\int_0^{\mathrm{T}}\mathrm{e}^{-f^*(T-t)}(d_i(t))^2\,\mathrm{d}t \quad \text{for all } T\geqslant 0,\end{aligned} \tag{88}$$

where we have used the fact that $\mathrm{e}^{-f^*(T-t)} \leqslant 1$ and $(1-\mathrm{e}^{-f^*T})/T \leqslant f^*$, and $M = \sum_{i=1}^N M_i$. Then a bound for $\|z_i\|_{[0,T]}$ is established

$$\|z_i\|_{[0,T]} \leqslant 2V_\rho(0) + \frac{1}{c_i^0}\sum_{i=1}^N(|b_i^{m_i}|l_i^p(p_i-p_i^0)^2 + l_i^\theta\|\theta_i-\theta_i^0\|^2) + \frac{1}{c_i^0}d_0\sum_{i=1}^N 2\rho_i(4\|P_i\|^2+1)D_{i,\max}^2 \tag{89}$$

using the fact that $f^*/c_i^0 \leqslant 2$. The initial value of the Lyapunov function is

$$V_\rho(0) = \sum_{i=1}^N\frac{1}{2}\left[\|z_i(0)\|^2 + \|\tilde{\theta}_i(0)\|^2_{\Gamma_i^{-1}} + \frac{|b_i^{m_i}|}{\gamma_i'}|\tilde{p}_i(0)|^2 + d_i^0\|\varepsilon_i(0)\|^2_{P_i}\right], \tag{90}$$

where $d_i^0 = \sum_{q=1}^{\rho_i}1/\bar{l}_i^q$, $\|\tilde{\theta}_i(0)\|^2_{\Gamma_i^{-1}} = \tilde{\theta}_i(0)^{\mathrm{T}}\Gamma_i^{-1}\tilde{\theta}_i(0)$ and $\|\varepsilon_i(0)\|^2_{P_i} = \varepsilon_i(0)^{\mathrm{T}}P_i\varepsilon_i(0)$.

Following similar ideas to Krstic et al. (1995, p. 455), where $z(0)$ is set to zero by appropriately initializing the reference trajectory for single-loop case, we can set z_i^q, $q=2,\ldots,\rho_i$ to zero by suitably initializing our designed filters (15) and (16) as follows:

$$v_i^{m_i,q}(0)=\alpha_i^{q-1}(y_i(0),\hat{\theta}_i(0),\hat{p}_i(0),\eta_i(0),v_i^{m_i,q-1}(0),\bar{v}_i^{m_i-1,2}(0)),\quad q=1,2,\ldots,\rho_i. \tag{91}$$

Thus, by setting $z_i^q(0)=0$, $q=2,\ldots,\rho_i$, a bound for $\|z_i\|_{[0,T]}$ is established and stated in the following theorem.

Theorem 2. *Consider the initial values* $z_i^q(0)=0$ $(q=2,\ldots,\rho_i,\ i=1,\ldots,N)$, *the bound* $\|z_i\|_{[0,T]}$ *satisfies*

$$\begin{aligned}\|z_i\|_{[0,T]}\leqslant&\sum_{i=1}^{N}y_i(0)+\|\tilde{\theta}_i(0)\|^2_{\Gamma_i^{-1}}+\frac{|b_i^{m_i}|}{\gamma_i'}|\tilde{p}_i(0)|^2\\&+\frac{1}{c_i^0}\sum_{i=1}^{N}(|b_i^{m_i}|l_i^p(p_i-p_i^0)^2+l_i^\theta\|\theta_i-\theta_i^0\|^2)\\&+\frac{1}{c_i^0}d_0\sum_{i=1}^{N}2\rho_i(4\|P_i\|^2+1)D_{i,\max}^2\\&+d_i^0\|\varepsilon_i(0)\|^2_{P_i}.\end{aligned}\tag{92}$$

Proof. Using (79), (80) and (87)–(90), the fact that $f^*/c_i^0\leqslant 2$, (92) can be obtained. □

Remark 10. Regarding the above bound, the following conclusions can be drawn by noting that $\tilde{\theta}_i(0)$, $\tilde{p}_i(0)$, $\varepsilon_i(0)$ and $y_i(0)$ are independent of $c_i^0,\Gamma_i,\gamma_i',l_i^\theta,l_i^p$.

- The transient performance in the sense of truncated norm given in (92) depends on the initial estimation errors $\tilde{\theta}_i(0)$, $\tilde{p}_i(0)$ and $\varepsilon_i(0)$. The closer the initial estimates to the true values, the better the transient performance.
- This bound can also be systematically reduced down to a lower bound depending $y_i(0)$ by increasing Γ_i,γ_i',c_i^0 and decreasing l_i^p,l_i^θ.
- This bound is depending on the effect of hysteresis.

5.2. Control Scheme II

Now we define a Lyapunov function of the overall decentralized adaptive control system as

$$V=\sum_{i=1}^{N}V_i,\tag{93}$$

where

$$\begin{aligned}V_i=&\sum_{q=1}^{\rho_i}\left(\frac{1}{2}(z_i^q)^2+\frac{1}{2\bar{l}_i^q}V_{\varepsilon_i}\right)+\frac{1}{2}\tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\theta}_i+\frac{|b_i^{m_i}|}{2\gamma_i'}(\tilde{p}_i)^2\\&+\frac{1}{2l_i^{\eta_i}}\eta_i^{\mathrm{T}}P_i\eta_i+\frac{1}{2l_i^{\zeta_i}}\zeta_i^{\mathrm{T}}P_i^{b_i}\zeta_i.\end{aligned}\tag{94}$$

Similar to the procedure of Scheme I, by using the properties that $-\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}Proj(\tau)\leqslant-\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tau$, the derivative of the V_i satisfies

$$\begin{aligned}\dot{V}_i\leqslant&-\sum_{q=1}^{\rho_i}\left[c_i^q(z_i^q)^2-\frac{1}{4\bar{l}_i^q}\varepsilon_i^{\mathrm{T}}\varepsilon_i\right]-\frac{1}{2l_i^{\eta_i}}\eta_i^{\mathrm{T}}\eta_i-\frac{1}{4l_i^{\zeta_i}}\zeta_i^{\mathrm{T}}\zeta_i\\&-|b_i^{m_i}|\tilde{p}_i\frac{1}{\gamma_i'}[\gamma_i'\operatorname{sgn}(b_i^{m_i})\bar{\alpha}_i^1z_i^1+\dot{\hat{p}}_i]+\frac{1}{l_i^{\zeta_i}}\|P_i^{b_i}\bar{f}_i\|^2\\&+\tilde{\theta}_i^{\mathrm{T}}(\tau_i^{\rho_i}-\Gamma_i^{-1}\dot{\hat{\theta}}_i)-l_i^*(z_i^1)^2(\psi_i(z_i^1))^2+M_i^*\\&+\sum_{q=1}^{\rho_i}\frac{1}{\bar{l}_i^q}\left(\|P_if_i\|^2+\frac{1}{4}\|f_i\|^2\right)\\\leqslant&-\sum_{q=1}^{\rho_i}\left[c_i^q(z_i^q)^2-\frac{1}{4\bar{l}_i^q}\varepsilon_i^{\mathrm{T}}\varepsilon_i\right]-\frac{1}{2l_i^{\eta_i}}\eta_i^{\mathrm{T}}\eta_i-\frac{1}{4l_i^{\zeta_i}}\zeta_i^{\mathrm{T}}\zeta_i\\&-l_i^*(z_i^1)^2(\psi_i(z_i^1))^2+\sum_{q=1}^{\rho_i}\frac{1}{\bar{l}_i^q}\left(\|P_if_i\|^2+\frac{1}{4}\|f_i\|^2\right)\\&+\frac{1}{l_i^{\zeta_i}}\|P_i^{b_i}\bar{f}_i\|^2+M_i^*,\end{aligned}\tag{95}$$

where

$$M_i^*=\sum_{q=1}^{\rho_i}\frac{1}{4\bar{l}_i^q}(4\|P_i\|^2+1)D_{i,\max}^2.\tag{96}$$

From (71)–(74), the derivative of the V satisfies

$$\begin{aligned}\dot{V}\leqslant\sum_{i=1}^{N}\Bigg[&-\sum_{q=1}^{\rho_i}\left(c_i^q(z_i^q)^2-\frac{1}{4\bar{l}_i^q}\varepsilon_i^{\mathrm{T}}\varepsilon_i\right)-\frac{1}{2l_i^{\eta_i}}\eta_i^{\mathrm{T}}\eta_i\\&-\frac{1}{4l_i^{\zeta_i}}\zeta_i^{\mathrm{T}}\zeta_i+M_i^*\Bigg].\end{aligned}\tag{97}$$

This shows that $z_i^1,z_i^2,\ldots,z_i^{\rho_i},\varepsilon_i,\zeta_i,\lambda_i,\eta_i$ and x_i are bounded. With the projection operation, $\tilde{\theta}_i$ and $\tilde{p}_i$ are bounded. Therefore boundedness of all signals in the system is ensured as formally stated in the following theorem.

Theorem 3. *Consider the closed-loop adaptive system consisting of the plant* (1) *under Assumptions* 1–5, *the controller* (T.14), *the estimator* (T.15), (T.16), *and the filters* (15) *and* (16). *There exist* γ_{ij}^* *such that for all* $\gamma_{ij}\leqslant\gamma_{ij}^*$, $i,j=1,\ldots,N$, *all the signals in the system are uniformly bounded. A bound for* $\|z_i\|_{[0,T]}$ *is established as*

$$\begin{aligned}\|z_i\|_{[0,T]}\leqslant\sum_{i=1}^{N}\Bigg[&y_i(0)+\|\tilde{\theta}_i(0)\|^2_{\Gamma_i^{-1}}+\frac{|b_i^{m_i}|}{\gamma_i'}|\tilde{p}_i(0)|^2\\&+d_i^0\|\varepsilon_i(0)\|^2_{P_i}+\frac{1}{c_i^0}d_0\sum_{i=1}^{N}2\rho_i(4\|P_i\|^2+1)D_{i,\max}^2\Bigg]\end{aligned}\tag{98}$$

by setting $z_i^q(0)=0$, $q=2,\ldots,\rho_i$, $i=1,\ldots,N$.

Remark 11. The condition that $\gamma_{ij} \leqslant \gamma_{ij}^*$ now has the following two implications:

(1) If we know $\bar\gamma_{ij}$, then we can get an estimate of its bound γ_{ij}^* which depends on $\bar l_i^q$, $l_i^{\zeta_i}$, $\|P_i\|$, $\|P_i^{b_i}\|$ and the bound of $\|T_j^{-1}\|$, $j = 1, 2, \ldots, N$ and design l_i^* according to (72). This means that the coupling strength of the interconnection between subsystems can be allowed arbitrarily strong.
(2) If $\bar\gamma_{ij}$ is unknown, we have similar implication to Remark 9.

If the system has no hysteresis, then $d_i(t) = 0$ and we have the following corollary.

Corollary 1. *Consider the closed-loop decentralized adaptive control system consisting of the plant* (1) *without input hysteresis under Assumptions* 1–5 *and the controller* (T.14), *the estimator* (T.15) *and* (T.16), *and the filters* (15) *and* (16). *All the states of the system asymptotically approach to zero and the bound* $\|z_i\|_2$ *is given by*

$$\|z_i\|_2 \leqslant \frac{1}{2\sqrt{c_i^0}}\left(\sum_{i=1}^{N} y_i(0) + \|\tilde\theta_i(0)\|^2_{\Gamma_i^{-1}} + \frac{|b_i^{m_i}|}{\gamma_i'}|\tilde p_i(0)|^2 + d_i^0|\varepsilon_i(0)|^2_{P_i}\right)^{1/2} \tag{99}$$

by setting $z_i^q(0) = 0,\ q = 1, 2, \ldots, \rho_i,\ i = 1, \ldots, N.$

Proof. In the absence of hysteresis the term $d_i(t) = 0$, so $M_i^* = 0$ in (97). We have

$$\dot V \leqslant \sum_{i=1}^{N}\left[-\sum_{q=1}^{\rho_i} c_i^q (z_i^q)^2 - \sum_{q=1}^{\rho_i}\frac{1}{4\bar l_i^q}\varepsilon_i^{\mathrm T}\varepsilon_i - \frac{1}{2l_i^{\eta_i}}\eta_i^{\mathrm T}\eta_i - \frac{1}{4l_i^{\zeta_i}}\zeta_i^{\mathrm T}\zeta_i\right] \leqslant -c_i^0\|z_i\|_2^2 \leqslant 0, \tag{100}$$

where $\|z_i\|_2^2 = \int_0^\infty \|z_i\|^2\,\mathrm d\tau$. This proves that the uniform stability and the uniform boundedness of z_i^q, $\hat p_i$, $\hat\theta_i$, ε_i, ζ_i, η_i, v_i^j, x_i and u_i. Following the similar argument as in Wen (1994) and Krstic et al. (1995), it can be shown that both $\dot V$ and $\ddot V$ are bounded as well as $\dot V$ is integrable over $[0, \infty]$. Therefore, $\dot V$ tends to zero and thus the system states x_i converge to zero from (100). Also (99) can be obtained clearly.

Remark 12. In the absence of hysteresis, the L_2 norm of the system states is shown to be bounded by a function of design parameters. This implies that the transient system performance in terms of L_2 bounds can be adjusted by choosing suitable design parameters. This result further extends that presented in Wen (1994), where only first order interactions considered and no transient performance like (99) is available.

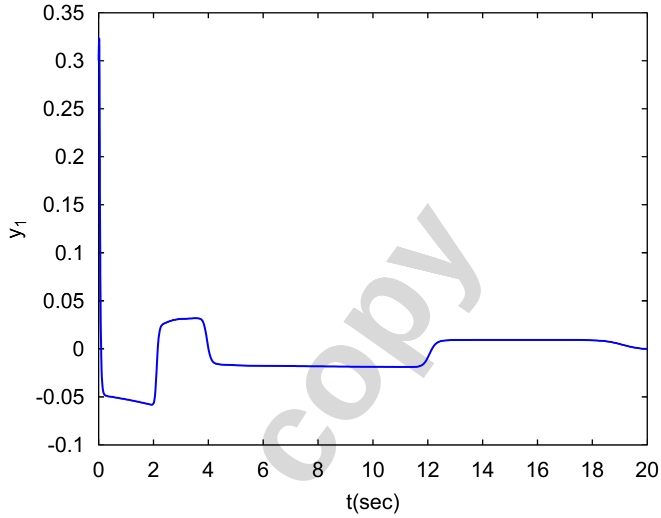

Fig. 2. Output y_1 with considering hysteresis using Scheme I.

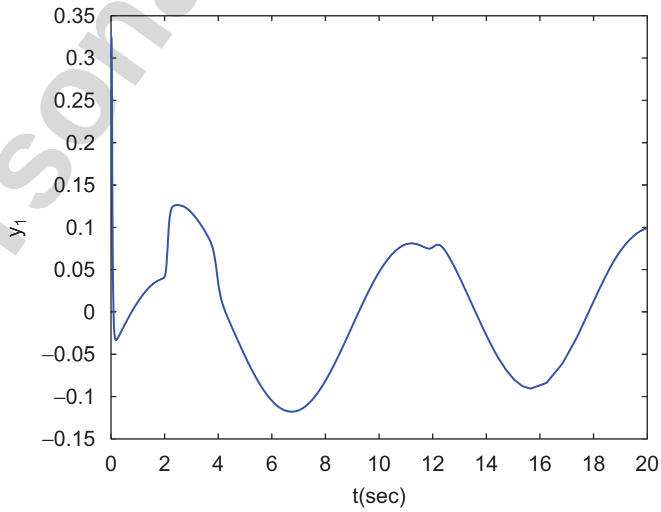

Fig. 3. Output y_1 without considering hysteresis.

Remark 13. Following similar analysis for the L_2 bound and the approaches in Krstic et al. (1995), a bound on $\|z_i\|_\infty$ can also be established and this bound can be adjusted by choosing suitable design parameters.

6. An illustrative example

We consider the following interconnected system with three subsystems:

$$\dot x_1 = a_1x_1 + b_1u_1 + f_1, \quad y_1 = x_1, \tag{101}$$

$$\dot x_2 = a_2x_2 + b_2u_2 + f_2, \quad y_2 = x_2, \tag{102}$$

$$\dot x_3 = a_3x_3 + b_3u_3 + f_3, \quad y_3 = x_3, \tag{103}$$

$$u_1 = BH_1(w_1), \quad u_2 = BH_2(w_2), \quad u_3 = BH_3(w_3), \tag{104}$$

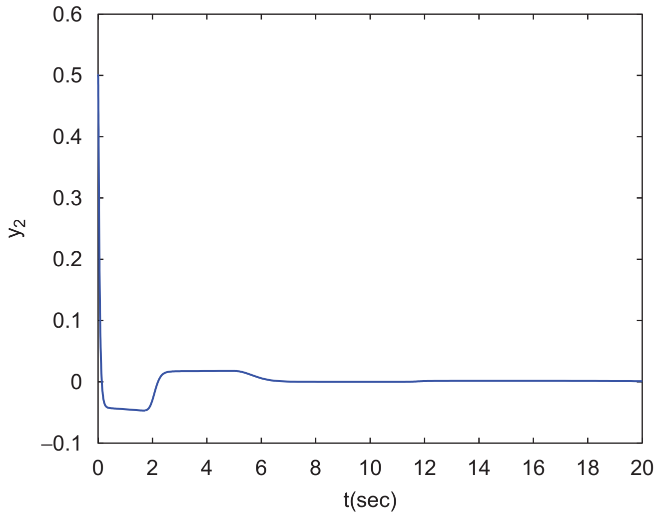

Fig. 4. Output y_2 with considering hysteresis using Scheme I.

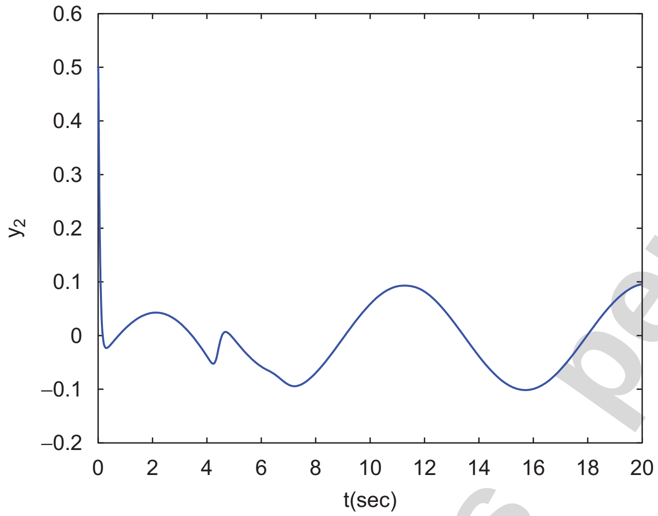

Fig. 5. Output y_2 without considering hysteresis.

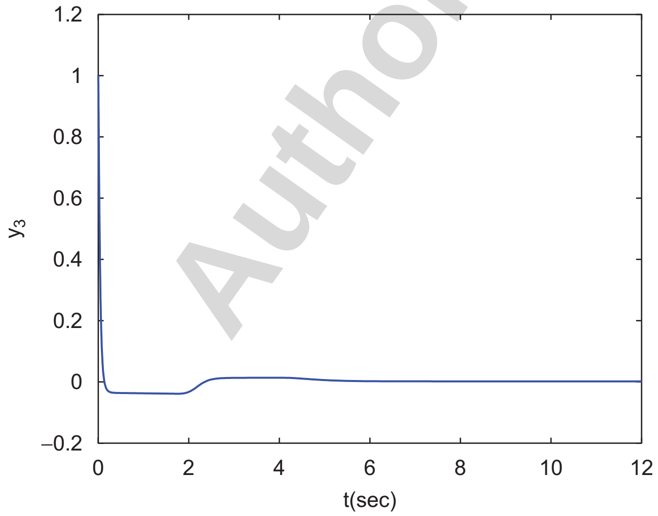

Fig. 6. Output y_3 with considering hysteresis using Scheme I.

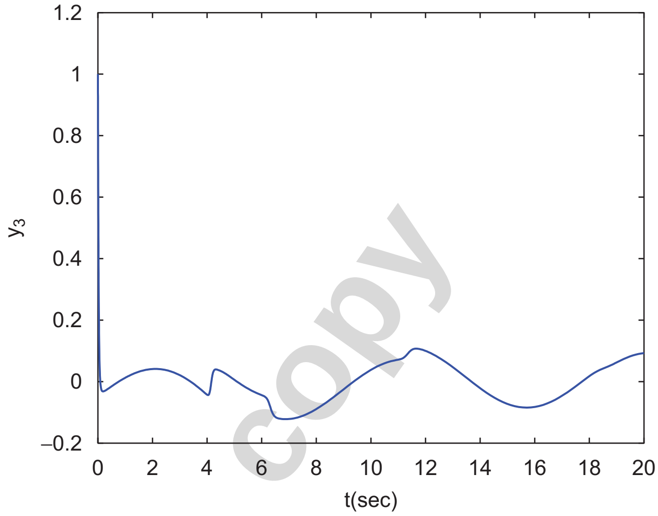

Fig. 7. Output y_3 without considering hysteresis.

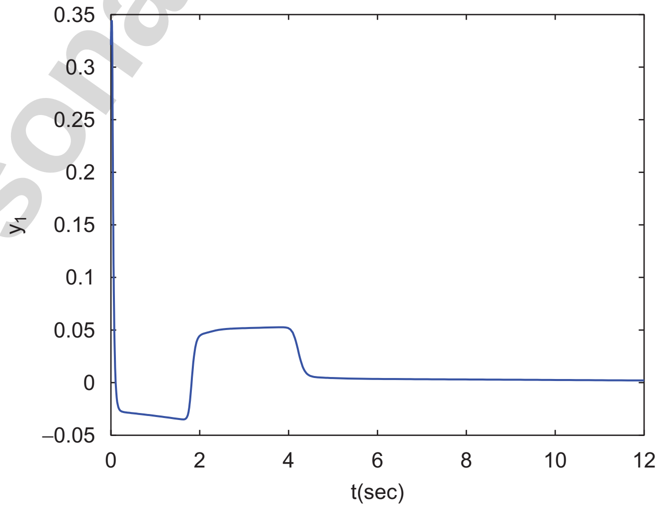

Fig. 8. Output y_1 in the presence of hysteresis using Scheme II.

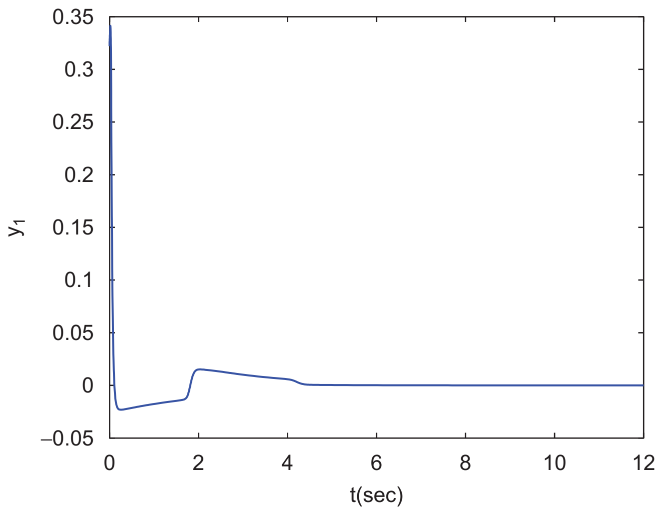

Fig. 9. Output y_1 in the absence of hysteresis using Scheme II.

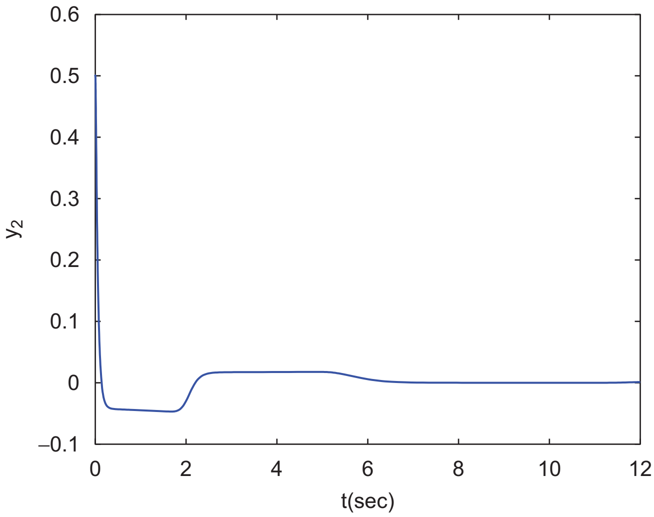

Fig. 10. Output y_2 in the presence of hysteresis using Scheme II.

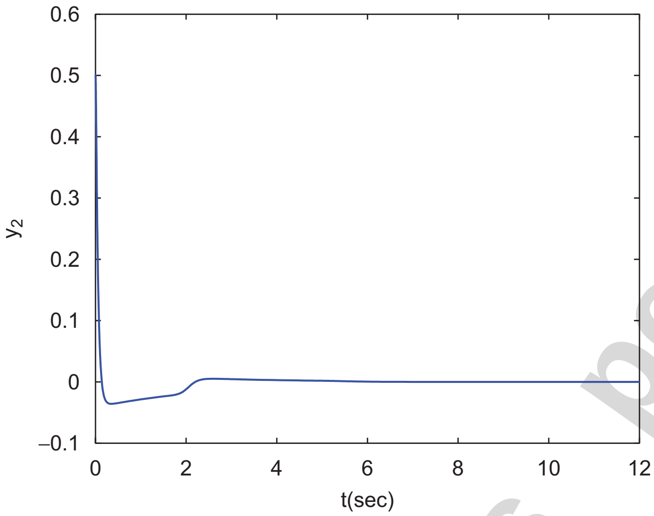

Fig. 11. Output y_2 in the absence of hysteresis using Scheme II.

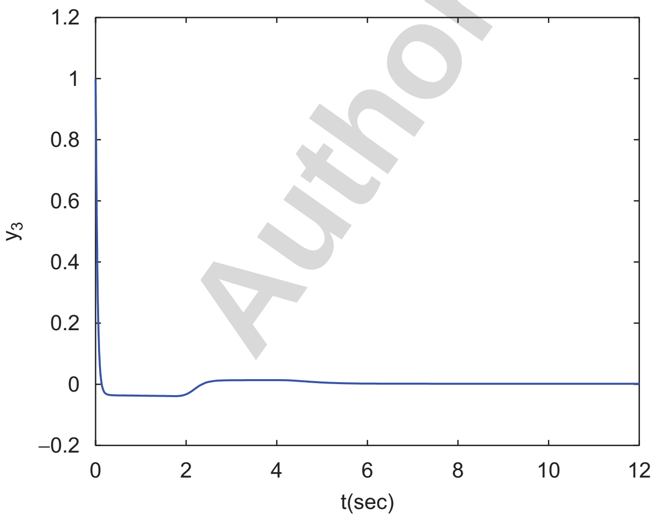

Fig. 12. Output y_3 in the presence of hysteresis using Scheme II.

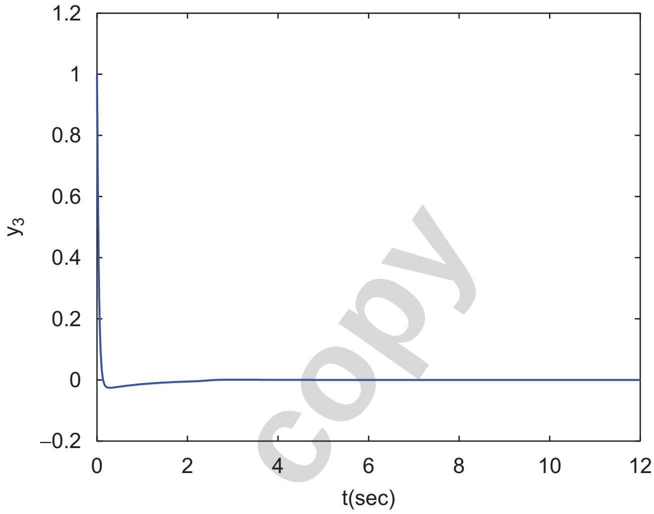

Fig. 13. Output y_3 in the absence of hysteresis using Scheme II.

where $a_1 = 1, b_1 = 1, a_2 = 0.5, b_2 = 1, a_3 = 2, b_3 = 1$, the nonlinear interaction terms $f_1 = y_2 + \sin(y_2) + 0.2y_3, f_2 = 0.2y_1^2 + y_3$, $f_3 = y_1 + 0.5y_2^2$, $BH_1(w_1)$, $BH_2(w_2)$ and $BH_3(w_3)$ are the backlash hysteresis described by (4) with parameters $\alpha_1' = 1, c_1' = 2, h_1 = 0.2, \alpha_2' = 1, c_2' = 1, h_2 = 0.2, \alpha_3' = 1.2, c_3' = 1, h_3 = 0.3$. These parameters are not needed to be known in the controller design. The objective is to stabilize system (101)–(103). The controller (49) and the estimator (39), (50) are implemented, where $\hat{p}_i$ and $\hat{\theta}_i$ are estimates of $p_i = 1/b_i c_i'$ and $\theta_i = a_i, i = 1, 2$, respectively. The design parameters are chosen as $c_1^1 = c_2^1 = c_3^1 = 10, l_1^1 = l_1^2 = l_1^3 = 5, l_1^* = l_2^* = l_3^* = 5$, $\gamma_1 = 2, \gamma_2 = 2, \gamma_3 = 2, \Gamma_1 = \Gamma_2 = \Gamma_3 = 1$. The initials are set as $y_1(0) = 0.3$, $y_2(0) = 0.5$, $y_3(0) = 1.0$. Clearly, the result in Adly (1995) is not applicable here due to the presence of hysteresis and the fact that f_2 and f_3 do not satisfy the first-order bounding condition.

In order to illustrate the effects of hysteresis, we observe system performances by applying controllers designed without considering hysteresis and with our proposed Scheme I, respectively. The simulation results presented in Figs. 2,4,6 and 3,5,7 show the system outputs y_1, y_2 and y_3 with Scheme I and without considering hysteresis, respectively. Clearly, poor performance is observed if hysteresis is not taken into account in controller design. In fact, system stability is not even ensured theoretically in this case (Figs. 3,5,7). When Scheme II is applied, we study the cases in the presence or absence of hysteresis. Figs. 8–13 show the system outputs, which show that $|y_i| \to 0$ in the absence of hysteresis. All the simulation results verify that our proposed two schemes are effective to cope with hysteresis nonlinearity and high-order nonlinear interactions.

7. Conclusion

In this paper, decentralized adaptive output feedback stabilization of a class of interconnected subsystems with the input of each loop preceded by unknown backlash-like hysteresis nonlinearity is considered. Each local adaptive controller is designed based on a general transfer function of the local subsystem with arbitrary relative degree by developing two adaptive control schemes. The effects of hysteresis and interactions are considered in the design. The nonlinear interactions between subsystems are allowed to satisfy higher-order nonlinear bounds. In Scheme I, the term multiplying the control and the system parameters are not assumed to be within known intervals. Two new terms are added in the parameter updating law, compared to the standard backstepping approach. In Scheme II, uncertain parameters are assumed inside known compact sets. Thus we use projection operation in the adaptive laws. It is shown that the designed local adaptive controllers with both schemes stabilize the overall interconnected systems. We also derive an explicit bound on the root mean square performance of the system states in terms of design parameters. This implies that the transient system performance can be adjusted by choosing suitable design parameters. With Scheme II in the absence of hysteresis, perfect stabilization is ensured and the L_2 norm of the system states is also shown to be bounded by a function of design parameters. The strengths can be allowed arbitrary strong if their upper bounds are available in this case. Simulation results illustrate the effectiveness of our schemes by comparing the cases with and without considering hysteresis in controller design, as well as examining the outputs in the presence and absence of hysteresis when Scheme II is employed. To further improve system performance, it is worthy to take the detailed structural information of the hysteresis into account in the controller design, instead of only considering its effects. It will be also of interest to extend our results to the cases where the interconnections satisfy more general bounding conditions.

References

Adly, A. A. (1995). Performance simulation of hysteresis motors using accurate rotor media models. *IEEE Transactions on Magnetics*, *31*, 3542–3544.

Ahmad, N. J., & Khorrami, F. (1999). Adaptive control of systems with backlash hysteresis at the input. In *Proceedings of the American control conference* (pp. 3018–3022).

Brokate, M., & Sprekels, J. (1996). *Hysteresis and phase transition*. Berlin: Springer.

Coleman, B. D., & Hodgdon, M. L. (1986). A constitutive relation for rate-independent hysteresis in ferromagnetically soft materials. *International Journal of Engineering Science*, *24*, 897–919.

Datta, A., & Ioannou, P. (1991). Decentralized indirect adaptive control of interconnected systems. *International Journal of Adaptive Control and Signal Processing*, *5*, 259–281.

Datta, A., & Ioannou, P. (1992). Decentralized adaptive control. In C. T. Leondes (Ed.), *Advances in control and dynamic systems* New York: Academic Press.

Gavel, D. T., & Siljak, D. D. (1989). Decentralized adaptive control: Structural conditions for stability. *IEEE Transactions on Automatic Control*, *34*, 413–426.

Hill, D. J., Wen, C., & Goodwin, G. C. (1988). Stability analysis of decentralized robust adaptive control. *System and Control Letters*, *11*, 277–284.

Hodgdon, M. L. (1988a). Applications of a theory ferromagnetic hysteresis. *IEEE Transactions on Magnetics*, *24*, 218–221.

Hodgdon, M. L. (1988b). Mathematical theory and calculations of magnetic hysteresis curves. *IEEE Transactions on Magnetics*, *24*, 3120–3122.

Hong, S. K., Kim, H. K., Kim, H. S., & Jung, H. K. (2000). Torque calculation of hysteresis motor using vector hysteresis model. *IEEE Transactions on Magnetics*, *36*, 1932–1935.

Huseyin, O., Sezer, M. E., & Siljak, D. D. (1982). Robust decentralized control using output feedback. *IEE Proceedings on Control Theory & Applications*, 310–314.

Ioannou, P. (1986). Decentralized adaptive control of interconnected systems. *IEEE Transactions on Automatic Control*, *31*, 291–298.

Krstic, M., Kanellakopoulos, I., & Kokotovic, P. V. (1995). *Nonlinear and adaptive control design*. New York: Wiley.

Lamba, H., Grinfeld, M., McKee, S., & Simpson, R. (1997). Subharmonic ferroresonance in an lcr circuit with hysteresis. *IEEE Transactions on Magnetics*, *33*, 2495–2500.

Macki, J. W., Nistri, P., & Zecca, P. (1993). Mathematical models for hysteresis. *SIAM Review*, *35*, 94–123.

Marino, R., & Tomei, P. (1995). *Nonlinear control design: Geometric, adaptive and robust*. New York: Prentice-Hall.

Mittal, S., & Menq, C.-H. (2000). Hysteresis compensation in electromagnetic actuators through preisach model inversion. *IEEE/ASME Transactions on Mechatronics*, *5*, 394–409.

Moheimani, S. O. R., & Goodwin, G. C. (2001). Guest editorial introduction to the special issue on dynamics and control of smart structures. *IEEE Transactions on Control Systems Technology*, *9*, 3–4.

Pare, T. E., & How, J. P. (1998). Robust stability and performance analysis of systems with hysteresis nonlinearities. In *Proceedings of American control conference* (pp. 1904–1908).

Sezer, M. E., & Siljak, D. D. (1981a). On decentralized stabilization and structure of linear large scale systems. *Automatica*, *17*, 641–644.

Sezer, M. E., & Siljak, D. D. (1981b). Robustness of suboptimal control: Gain and phase margin. *IEEE Transactions on Automatic Control*, *26*, 907–911.

Shi, L., & Singh, S. K. (1992). Decentralized adaptive controller design for large-scale systems with higher order interconnections. *IEEE Transactions on Automatic Control*, *37*, 1106–1118.

Stepanenko, Y., & Su, C. Y. (1998). Intelligent control of piezoelectric actuators. In *Proceeding of the 37th IEEE conference on decision and control* (pp. 4234–4239) USA.

Su, C. Y., Stepanenko, Y., Svoboda, J., & Leung, T. P. (2000). Robust adaptive control of a class of nonlinear systems with unknown backlash-like hysteresis. *IEEE Transactions on Automatic Control*, *45*(12), 2427–2432.

Sun, X., Zhang, W., & Jin, Y. (1992). Stable adaptive control of backlash nonlinear systems with bounded disturbance. In *Proceedings of the IEEE conference on decision and control* (pp. 274–275).

Tan, X., & Baras, J. S. (2005). Adaptive identification and control of hysteresis in smart materials. *IEEE Transactions on Automatic Control*, *50*, 1469–1480.

Tao, G., & Kokotovic, P. V. (1995). Adaptive control of plants with unknown hysteresis. *IEEE Transactions on Automatic Control*, *40*, 200–212.

Tao, G., & Kokotovic, P. V. (1996). *Adaptive control of systems with actuator and sensor nonlinearities*. New York: Wiley.

Wen, C. (1994). Decentralized adaptive regulation. *IEEE Transactions on Automatic Control*, *39*, 2163–2166.

Wen, C., & Hill, D. J. (1992). Global boundedness of discrete-time adaptive control just using estimator projection. *Automatica*, *28*, 1143–1157.

Wen, C., & Soh, Y. C. (1997). Decentralized adaptive control using integrator backstepping. *Automatica*, *33*, 1719–1724.

Zhang, Y., Wen, C., & Soh, Y. C. (2000). Robust decentralized adaptive stabilization of interconnected systems with guaranteed transient performance. *Automatica*, *36*, 907–915.

Zhou, J., Wen, C., & Zhang, Y. (2004). Adaptive backstepping control of a class of uncertain nonlinear systems with unknown backlash-like hysteresis. *IEEE Transactions on Automatic Control*, *49*, 1751–1757.

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