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# Exponential Stabilization of Extended Chained Forms

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**Abstract**

In this paper, an extended chained form is investigated and ultimate exponential stabilization is achieved. We found that this system is characterized by a time-varying parameter, which can be handled at will. A manifold based on this time-varying parameter is defined and a set of switch control laws is developed.

## 1 Introduction

In this paper, we address the feedback stabilization problem for extended chained forms as,

$$\begin{aligned} \dot{x}_1 &= v_1 \\ \dot{x}_2 &= v_2 \\ \dot{x}_i &= x_{i-1}v_1, \quad i \in \{3, \cdots, n\} \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{v}_1 &= u_1 \\ \dot{v}_2 &= u_2 \end{aligned} \tag{2}$$

by denoting $x = [x_1 \cdots x_n \, v_1 \, v_2]^T \in R^{n+2}$, the system may be written as $\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2$, where, $f(x) = v_1\frac{\partial}{\partial x_1} + v_2\frac{\partial}{\partial x_2} + \sum_{i=3}^{n} x_{i-1}v_1\frac{\partial}{\partial x_i}$ is the drift vector field, $g_1(x) = \frac{\partial}{\partial v_1}$ and $g_2(x) = \frac{\partial}{\partial v_2}$ are the input vector fields. Equation (1) is referred to as the nonholonomic chained system with $v_1$ and $v_2$ as the inputs while equations (1) and (2) together represent the extended chained system with $u_1$ and $u_2$ as the control inputs.

Chained system (1), which is first proposed in [1], is a canonical form that models the kinematic equations of a large class of mechanical systems. Necessary and sufficient conditions for converting a nonholonomic system into chained form are derived in [2]. As a result, all two-input regular nonholonomic systems in three or four dimensions (e.g., unicycle-type vehicles and car-like vehicles) are locally feedback equivalent to a chained system. It is shown in [3] that the kinematic model of vehicles with $n$ trailers can also be locally converted into a chained form. Being the dynamic extension, the extended chained system (1)(2) models the dynamic equations of the above systems.

It is known that nonholonomic systems are not stabilizable by a smooth static-state feedback [4]. So, more sophisticated stabilizers, such as time-varying and/or discontinuous feedbacks, have to be applied. Extensive results have been made for driftless nonholonomic systems [5–11]. In comparison with driftless nonholonomic systems, nonholonomic systems with drift terms are not open-loop stable and thus designing stabilizers for such systems is more challenging. Few researches have been attempted on this aspect. As a special nonholonomic system with drift terms, the dynamic extension of a driftless nonholonomic system may share some properties and results of the driftless nonholonomic system. It is shown in [12] that the controllability of a driftless system is inherited to its dynamic extension. Moreover, some stabilizers of a driftless nonholonomic system can be extended to that of its dynamic extension [13]. However, the understanding of such systems is still limited and many properties of such systems remain to be further explored. Feedback control of a general nonholonomic system with drift terms is still an open problem.

In this paper, the extended chained form is structured into two parts: a LTI subsystem and a LTV subsystem. By applying a $\sigma$ process to the LTV subsystem, and a conventional linear feedback stabilizer to the LTI subsystem, the property of the LTV subsystem is revealed with a new viewpoint. It is found that there exists a manifold, which is characterized by an ultimately exponentially converging time-varying parameter. Setting switching conditions by this manifold, a set of control laws is developed. When the extended chained form is outside this manifold, a control law is applied to drive the system into the manifold. When the extended chained form is inside this manifold, a control law smooth away from origin is applied to exponentially stabilize the LTV subsystem.

## 2 Subsystems and $\sigma$ Process

By reordering and partitioning states in (1)(2) as

$$x = \begin{bmatrix} x^1 \\ x^2 \end{bmatrix} = [v_1 \quad x_1 \quad | \quad v_2 \quad x_2 \quad \cdots \quad x_n]^T$$

extended chained system (1)(2) is rewritten as follows.

$$\dot{x}^1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} x^1 + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_1 \quad (3)$$

$$\dot{x}^2 = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & v_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & v_1 & 0 \end{bmatrix} x^2 + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_2 \quad (4)$$

System (3)(4) consists of two single input subsystems, i.e., LTI (linear time invariant) subsystem (3) and LTV (linear time-varying) subsystem (4) if $v_1$ is taken as a time function. We should note that subsystem (4) is not controllable at $v_1 = 0$ using only $u_2$. Therefore, the behavior of $v_1(t)$ around zero is crucial for the stabilization of subsystem (4). When an asymptotic stabilizer is applied to subsystem (3), time function $v_1(t)$ generally presents three kinds of behavior around zero:

(a) Resting at zero;

(b) Asymptotically converging to zero; or

(c) Crossing zero.

Clearly, in case (a), the stabilization of subsystem (4) is impossible (not controllable). In the contrast, it is possible in cases (b) and (c). However, the stabilization of subsystem (4) is more difficult when subsystem (3) asymptotically converges to zero (case (b)) because of the nonholonomic property and the existence of drift term in both subsystem (3) and subsystem (4).

To overcome this obstacle, we consider discontinuous feedback control laws with the aid of $\sigma$ process, which is introduced to treat the driftless nonholonomic systems in [11]. Using the $\sigma$ process, the smooth system is transformed into discontinuous one. The transformed system can be dealt with using a smooth feedback law. Then, the control law is transformed back to the original coordinates. In the original coordinates, the developed control law is discontinuous.

We use the $\sigma$ process of the following rational discontinuous coordinate transformation for subsystem (4).

$$z = [z_1 \cdots z_n]^T = T(x) = \begin{bmatrix} v_2 & x_2 & \frac{x_3}{v_1} & \cdots & \frac{x_n}{v_1^{n-2}} \end{bmatrix}^T \quad (5)$$

Subsystem (4) is transformed to

$$\dot{z} = \begin{bmatrix} 0 & & & & \\ 1 & 0 & & & 0 \\ & 1 & \omega(t) & & \\ & & \ddots & \ddots & \\ 0 & & & 1 & (n-2)\omega(t) \end{bmatrix} z + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_2 \quad (6)$$

with

$$\omega(t) = -u_1(t)/v_1(t) \quad (7)$$

The transformed subsystem (6) is then a LTV controllable system if $\omega(t)$ is taken as a time function and well defined. Compared with (4), subsystem (6) exhibits some different properties. The most important one is that when $v_1 \to 0$, subsystem (4) will erode into being uncontrollable while subsystem (6) remains completely controllable because the parameter $\omega(t)$ will tend to a constant when $u_1(t)$ is appropriately designed as shown later. As a result, the control $u_2$ can be designed to stabilize subsystem (6) and hence subsystem (4) when $v_1$ is asymptotically converging to zero (case (b)).

**Remark 1** *One may check that, the asymptotic (exponential) stability of the transformed system (3)(6) implies the boundedness and asymptotic (exponential) convergence to zero of the original system (3)(4). However, we cannot conclude the Lyapunov stability of the original system (3)(4). This problem arises out of the discontinuous transformation (5), which maps a neighborhood of $z$ into a sub-manifold in a neighborhood of $x^2$ but not a neighborhood of $x^2$. Nevertheless, if states $x$ start from a manifold in which both subsystems (3) and (6) are stable under a feedback law, this manifold will remain invariant to states $x$, and moreover, states $x$ are bounded in this invariant manifold and converge to zero under the same feedback law. For this reason, we say that such feedback law stabilize the original system (3)(4). Therefore, the lack of the stability in Lyapunov sense of the original system (3)(4) does not limit the practical application of the feedback laws based on the transformed system (3)(6).* □

## 3 Stabilization Algorithm

Since the subsystem (6) is characterized by the parameter $\omega(t)$, the behavior of $\omega(t)$ is crucial for the design of stabilization control laws. Clearly, $\omega(t)$ is not bounded when $v_1(t)$ crosses the hyperplane $v_1 = 0$. Nevertheless, $\omega(t)$ can be handled by control input $u_1$ and different control law $u_1$ produces different behavior of $\omega(t)$. Here, we focus on the behavior of $\omega(t)$ when the following control law is applied to subsystem (3).

$$u_1 = -(\lambda + \bar{\omega})v_1 - \lambda\bar{\omega}x_1 \qquad \forall\ \lambda > \bar{\omega} > 0 \quad (8)$$

Clearly, feedback (8) exponentially stabilizes subsystem (3). The solution of $x^1 = [v_1\, x_1]^T$ is

$$\begin{bmatrix} v_1(t) \\ x_1(t) \end{bmatrix} = \begin{bmatrix} \frac{\lambda e^{-\lambda t} - \bar{\omega} e^{-\bar{\omega} t}}{\lambda - \bar{\omega}} & \frac{\bar{\omega}\lambda\left(e^{-\lambda t} - e^{-\bar{\omega} t}\right)}{\lambda - \bar{\omega}} \\ \frac{e^{-\bar{\omega} t} - e^{-\lambda t}}{\lambda - \bar{\omega}} & \frac{\lambda e^{-\bar{\omega} t} - \bar{\omega} e^{-\lambda t} +}{\lambda - \bar{\omega}} \end{bmatrix} \begin{bmatrix} v_1(t_0) \\ x_1(t_0) \end{bmatrix} \quad (9)$$

Then, combining (7)-(9) leads to

$$\omega(t) = \bar{\omega} + \lambda - \frac{\lambda(\omega(t_0) - \lambda) - \bar{\omega}(\omega(t_0) - \bar{\omega})e^{-(\lambda - \bar{\omega})t}}{(\omega(t_0) - \lambda) - (\omega(t_0) - \bar{\omega})e^{-(\lambda - \bar{\omega})t}} \quad (10)$$

With different initial value $\omega(t_0)$, the behavior of $\omega(t)$ is shown in Figure 1: (a) if $\omega(t_0) > \lambda$, $\omega(t)$ goes to infinity first and then exponentially converges to $\bar{\omega}$ (as shown by $A_1$ and $A_2$); (b) if $\omega(t_0) = \lambda$, $\omega(t)$ remains on constant $\lambda$ (as shown by $B$); (c) if $\omega(t_0) < \lambda$, $\omega(t)$ exponentially converges to $\bar{\omega}$ directly (as shown by $C$ or $D$).

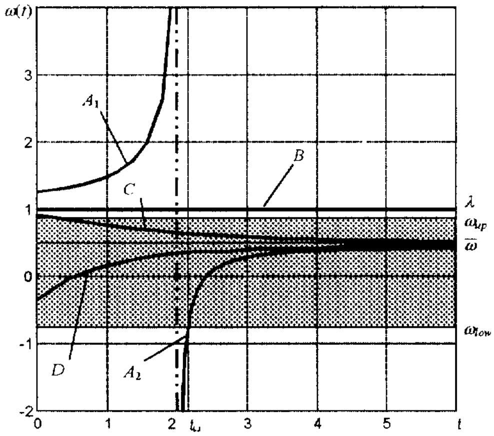


**Figure 1:** Behavior of $\omega(t)$

Formally, omitting the proof, these observations can be expressed by the following lemma.

**Lemma 1** *Using control law (8) in system (3), the open connected set (the shaded area):*

$$\Omega = \{\omega \in R: \omega_{low} < \omega < \omega_{up} \forall -\infty < \omega_{low} < \bar{\omega} < \omega_{up} \leq \lambda\} \quad (11)$$

*is invariant. Moreover, $\omega(t)$ ultimately exponentially converges to $\bar{\omega}$, if initial values of $v_1(t_0)$ and $x_1(t_0)$ satisfy*

$$v_1(t_0) + \lambda x_1(t_0) \neq 0 \quad (12)$$

□

This Lemma and above observations show that $\omega(t)$ always runs into region $\Omega$ in a finite time (e.g. $t_\Omega$ for $A_1$ and $A_2$ in Figure 1) and remains in region $\Omega$ for all $t > t_\Omega$, if initial condition (12), which is equivalent to $\omega(t_0) \neq \lambda$ for $v_1(t_0) \neq 0$ (except $B$ in Figure 1), is satisfied. This region $\Omega$ actually defines an invariant manifold $\mathcal{M}$ of configuration $x$ as

$$\mathcal{M} = \{x \in R^{n+2}: \ v_1 \neq 0 \text{ and } \omega(t) \in \Omega\} \quad (13)$$

When $x(t_0)$ starts from manifold $\mathcal{M}$, parameter $\omega(t)$ is bounded in $\Omega$ such that subsystem (6) can be taken as a conventional LTV system. In this case, the stabilization problem of the extended chained system may be solved by using control law (8) to subsystem (3) and a stabilizing control law to subsystem (6). To solve the stabilization problem globally, we can use another set of control laws to drive the system from arbitrary initial configuration $x(t_0) \notin \mathcal{M}$ get into the manifold $\mathcal{M}$ in finite time (suppose $t < t_\Omega$).

Based on this analysis, a stabilization algorithm for the extended chained system is proposed as a set of switch control laws as follows.

**Algorithm 1** *Consider system (3)(4). Apply following switch control law C1 to stabilize subsystem (3):*

*C11:* $u_1 = U(x)$, *if* $\|x\| > \varepsilon$ *and* $v_1(0) + \lambda x_1(0) = 0$ *and* $0 \leq t < t_U$.

*C12:* $u_1 = -(\lambda + \bar{\omega})v_1 - \lambda\bar{\omega}x_1$, *elsewhere.*

*where,* $\lambda > \bar{\omega} > 0$. *The constant* $\varepsilon > 0$ *is a given stabilization tolerance (i.e., if* $\|x\| < \varepsilon$*, no further control effort is required).* $t_U$ *is a chosen finite instant and* $U(x)$ *is designed such that* $v_1(t_U) + \lambda x_1(t_U) \neq 0$.

*Then, subsystem (4) is stabilized by the switch control law C2 as follows.*

*C21: If* $\|x\| > \varepsilon$ *and* $x \notin \mathcal{M}$*, a control law* $u_2$*, which maintains the boundedness of subsystem (4), is applied.*

*C22: If* $\|x\| > \varepsilon$ *and* $x \in \mathcal{M}$*, a control law* $u_2$*, which exponentially stabilize subsystem (6) and hence subsystem (4), is applied.*

*C23: If* $\|x\| \leq \varepsilon$*, a control law* $u_2 = 0$ *is applied.* ■

The effectiveness of Algorithm 1 is supported by the following Theorem.

**Theorem 1** *Control law C1 ultimately exponentially stabilizes subsystem (3) and ensures configuration $x$ getting into invariant manifold $\mathcal{M}$ in finite time from arbitrary initial condition $x(0)$ if $\|x(0)\| > \varepsilon$.*

**Proof:** The claim for the ultimate exponential stabilization of subsystem (3) is clear, because control law C11 works at most in finite time $0 \leq t < t_U$ and then control law C12 exponentially stabilizes subsystem (3) for $t \geq t_U$.

The second claim is valid because: if $\|x(0)\| > \varepsilon$ and $v_1(0) + \lambda z_1(0) = 0$, control law C11 works in finite time $0 \leq t < t_U$ such that initial condition (12) is always satisfied at instant $t_U$. Then, Lemma 1 shows that time-varying parameter $\omega(t)$ gets into invariant subset $\Omega$ in finite time under control law C12. By the definition of manifold $\mathcal{M}$ in (13), it is equivalent to configuration $x$ getting into invariant manifold $\mathcal{M}$ in finite time. ■

**Remark 2** *Design of control law C11 is rather liberty. One may verify that the simple linear feedback*

$$U(x) = -\lambda_1(v_1 - \bar{v}) \quad (14)$$

*where $\lambda_1 > 0$ and $\bar{v}$ are constants, satisfies the requirement.* □

**Remark 3** *Finite instant $t_{II}$ is usually designed such that the stabilization of the system with initial condition $x(t_{II})$ requires a smaller control effort.* □

**Remark 4** *Note that control law C1 will set up an initial value of $v_1 \neq 0$ if $\|x(0)\| > \varepsilon$. It is achieved by the combination of control law C11 and C12. When $x^1(0) = 0$ and $\|x(0)\| > \varepsilon$, control law C11 will set up an initial value of $v_1 \neq 0$. On the contrary, when $x^1(0) \neq 0$ and $\|x(0)\| > \varepsilon$, control law C12 will set up an initial value of $v_1 \neq 0$.* □

**Remark 5** *It is straightforward that switch control law C2 in Algorithm 1 ultimately exponentially stabilizes subsystem (4). Clearly, control law C21 works only in finite time $0 \leq t < t_\Omega$. When configuration $x$ gets into invariant manifold $\mathcal{M}$, control law C22 exponentially stabilizes subsystem (4) for $t \geq t_\Omega$. Since control law C22 is developed based on subsystem (6), it is discontinuous at origin. So, control law C23 is applied when $\|x\| \leq \varepsilon$.* □

**Remark 6** *We should note that, to get better stabilizing performance of the extended chained system, the convergence rate of subsystem (4) should be comparable with that of subsystem (3). The analysis in Section 2 shows that subsystem (4) is not controllable if $v_1 = 0$. However, we know that control law C1 drives $v_1 \in x^1$ exponentially converge to zero. It indicates that subsystem (4) will lose its controllability at last. Therefore, the stabilization of subsystem (4) should be faster than or at least as fast as that of subsystem (3). For this reason, stabilizer C22 is designed to stabilize subsystem (4) exponentially.* □

## 4 Application to a Car-Like Robot

The dynamic model of a car-like robot as shown in Figure 2 is

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \frac{v}{l}\tan\gamma \\ \dot{\gamma} &= \omega \\ \dot{v} &= u_d \\ \dot{\omega} &= u_s \end{aligned} \tag{15}$$

where, the triplet $(x, y, \theta)$ represents the posture of the robot; $v$ is the linear velocity of the robot; $\gamma$ and $\omega$ represent the steering angle and the steering rate of the robot wheel respectively; $l$ is the wheel-base; $u_d$ and $u_s$ are control inputs.

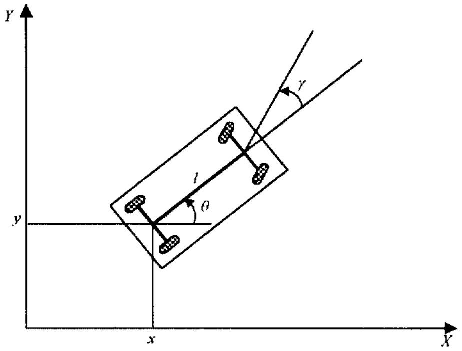


**Figure 2:** Pictorial of a car-like robot

Using the local coordinates transformation

$$x_1 = x \qquad x_2 = \frac{\tan\gamma}{l\cos^3\theta} \qquad x_3 = \tan\theta \qquad x_4 = y$$
$$v_1 = v\cos\theta \qquad v_2 = \frac{l\omega\cos\theta + 3v\sin\theta\sin^2\gamma}{l^2\cos^4\theta\cos^2\gamma}$$

and an corresponding input change, system (15) can be put into extended chained form (3)(4) for $n = 4$.

Simulations are carried for the parking control of system (15) to verify Algorithm 1. In the simulation, (14) is applied as control law C11 with $\lambda_1 = 2$, $\bar{v} = 25$ and $t_{II} = \operatorname{sech}(x_1(0))$. Control law C12 is taken with $\lambda = 2$ and $\bar{\omega} = 0.5$. Based on control law C12, region $\Omega$ is chosen with $\omega_{up} = \lambda = 2$ and $\omega_{low} = \bar{\omega} - 1.5 = -1$.

Control law C21, which stabilizes subsystem (4) if $|v_1(t)|$ does not asymptotically tend to zero, is developed as

$$\begin{aligned} u_2 = & -k_\alpha\left(v_2 + k_w|v_1|\left(k_1x_4 + x_2\right) + \left(k_1 + k_0\right)v_1x_3\right) \\ & -k_w|v_1|v_2 - \left(k_1 + k_0\right)v_1^2x_2 \\ & -k_1k_w|v_1|v_1x_3 - \frac{k_1x_4 + x_2}{k_0k_1} \end{aligned} \tag{16}$$

where we choose $k_\alpha = 15$, $k_w = 2$, $k_0 = 1.5$ and $k_1 = 1$.

Control law C22, which stabilizes subsystem (6), is developed as

$$u_2(z,t) = -c_1(z_1 - \alpha_1) - z_2 + \alpha_2(z,t) + \dot{\alpha}_1(z,t) \tag{17}$$

with

$$\begin{aligned} \alpha_2 &= -\left(1 + c_3c_4 + 4\omega^2 + 2c_3\omega + 4c_4\omega + 2\dot{\omega}\right)z_4 \\ &\quad -\left(c_3 + \omega + 2\omega + 2c_4\right)z_3 \\ \alpha_1 &= c_2\alpha_2 + \dot{\alpha}_2 - c_2z_2 - z_3 - \left(2\omega + c_4\right)z_4 \end{aligned}$$

where we choose $c_1 = 50, c_2 = 1, c_3 = 100$ and $c_4 = 10$.

The initial state is set to

$$(x(0),\ y(0),\ \theta(0),\ \gamma(0), v(0), \omega(0)) = (0,\ 10,\ 0,\ 0,\ 0,\ 0)$$

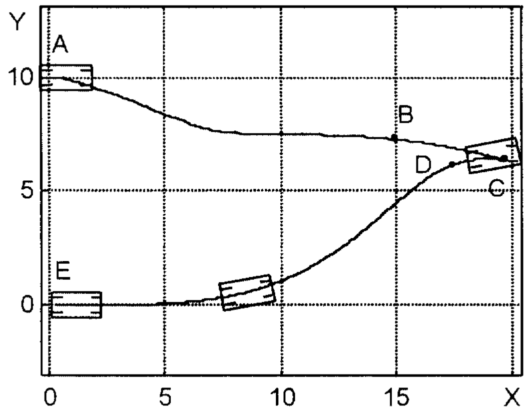


**Figure 3:** Parallel parking control of a car-like robot: Cartesian motion

Clearly, it is a typical parallel parking control task. Figure 3 displays the parking locus of the car-like robot, which stops at the origin at the end. In this maneuver, control laws switch between different phases. In the beginning phase, i.e., $AB$ phase, control law C11 works as $u_1$ to drive the robot to set up an initial motion and control law C21 works as $u_2$ to maintain the boundedness of the states in subsystem (4). In the middle phase, i.e., BC plus CD phase, control law $u_1$ switches to C12 to stabilize states in subsystem (3) and control law C21 still works to maintain the boundedness of the states in subsystem (4). In the last phase, i.e., DE phase, configuration $x$ has been converged into the invariant manifold $\mathcal{M}$, $u_2$ switches to control law C22, which works with C12 together to stabilize the whole configurations to the origin. Figure 3 shows that the switch control laws given by Algorithm 1 produce a rather natural parking maneuver of the car-like robot.

## 5 Conclusions

As a nonholonomic system with drift terms, the extended chained form is ultimately exponentially stabilized by a set of switching feedback control laws. In the present paper, with a $\sigma$ process to the studied nonholonomic system, the system's structure is understood in a new point of view. A crucial time-varying parameter, which defines a manifold of the nonholonomic system, is found. This time-varying parameter can be handled by one of the system's control inputs such that its changing rule is handled at will by suitable design that control input. According to different phases of the changes of this time-varying parameter, the nonholonomic system may be transformed into certain different standard forms. The transformed standard forms may be dealt with some known control techniques such that a set of switch control laws is developed.

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