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Discrete-Time Model Reference Learning Control: Theory and Experiment

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Abstract. Many modern industrial installations include digital computers as an integral part of the operations. Digital computers are extensively used to implement complex control algorithms to control the functioning of the system. The discretization of the nonlinear dynamic equations like robot dynamics results in an extremely complicated discrete dynamic equations. Therefore, it will be difficult to design a discrete-time controller to give good tracking performances in the presence of certain uncertainties. In this paper, a discrete-time Model Reference Learning Control (MRLC) algorithm is presented for a class of nonlinear and time varying discrete-time system. Sufficient conditions for guaranteeing the convergence of the discrete-time MRLC system are derived. The robustness of the learning system to measurement noise, dynamics fluctuation and re-initialization error is studied. Experimental results of an industrial robot SEIKO TT3000 are presented to verify the theoretical analysis.

Keywords: Iterative learning (repetitive) control, model reference, convergence, discrete time.

1. Introduction

Learning control [3], [13] is a concept for controlling uncertain dynamic system in an iterative manner. It arises from the recognition that robotic manipulators are usually used to perform repetitive tasks. Many research efforts have been devoted towards defining and analyzing learning control schemes [1]–[8], [13]–[15]. A recent survey of the works by Arimoto can be found in [12].

So far, most researches on learning control have been focused on the problem of trajectory learning control [12] where the system is designed to track a given trajectory as the operation is repeated. Examples of such applications are motion tracking control [1] [2] [3] [8] [13] and simultaneous motion and force trajectories tracking control [4] [5] [6] [7] [14] of robots employed to perform repetitive tasks. However, in certain applications like impedance control [9] of robotic manipulators, the control objective is specified explicitly by a desired model rather than a desired trajectory. Impedance control is one of the major approaches used in the controller design for force control [12] problem of robotic manipulator. It provides a unified approach to all aspects of manipulation [9]. Both free motion and contact tasks can be controlled using a single control algorithm. Impedance control does not attempt to track motion and force trajectories but rather to regulate the mechanical impedance [9] specified by a desired model.

Recently, a learning concept call Model Reference Learning Control (MRLC) is proposed by Cheah and Wang [6]. In contrast to the previous learning control approaches, a reference system model for specifying the desired response is given and a learning algorithm is designed to make the system response converges to that specified by the model as the operation is repeated. This allows the control designer to specify output performance at system level rather at trajectory level, adding extra freedom to the specification of control objective. Furthermore, MRLC scheme widens the potential researches and applications of the learning controller. In particular, it allows the research into an important class of learning control and force control problem, the learning impedance control of robotic manipulator [5], [14].

The model reference learning controllers presented in [5], [6], [14] are continuous time learning controllers. However, most modern industrial controllers are implemented in discrete time and invariably including sampling operations. Therefore, it is of theoretical and practical importance to formulate and develop the Model Reference Learning Controller in discrete time. In this paper, a discrete-time MRLC scheme that can be readily implemented in digital computer is proposed for a class of discrete-time reference model and discrete-time nonlinear dynamic systems. We shall prove that the robot is able to improve the performance in terms of reference model error by appropriately adjusting its control inputs based on the previous operation result. We study in detail the robustness of the proposed algorithm to dynamics fluctuation, measurement noise and error in initial condition. To verify the theoretical analysis, the proposed MRLC scheme is applied to an industrial robot SEIKO TT3000 and experimental results are presented.

The remaining of this paper is organised as follows: Section 2 presents the discrete-time dynamic equations and problem formulation, Section 3 presents the discrete-time model reference learning controller and analyses its robustness, section 4 presents the experimental results of an industrial robot SEIKO TT3000 and section 5 concludes the paper.

2. Discrete-Time Learning Control Formulation

Let us consider a class of nonlinear time varying discrete-time dynamic systems described by the following difference equations,

$$\begin{aligned} x_k(j+1) &= \mu(x_k(j), j) + \nu(x_k(j), j)u_k(j) + \nu(x_k(j), j)d_k(j) \\ &\quad + \rho(x_k(j), j)r_k(j), \end{aligned} \tag{1}$$

where $j = 0, 1, \dots, N$ is the discrete-time index at each discrete instant of time, $x_k(j) \in R^n$ denotes the state variable at the k^{th} operation, $\mu(x_k(j), j) \in R^n$, $\nu(x_k(j), j) \in R^{n\times p}$, $\rho(x_k(j), j) \in R^{n\times m}$ are nonlinear system dynamic terms, $u_k(j) \in R^p$ denotes the control input, $d_k(j)$ represents the fluctuation or disturbance in the system dynamics when the operation is not repeated under the same condition, and $r_k(j) \in R^m$ denotes a force variable described by a nonlinear stiffness relationship,

$$r_k(j) = k_s(x_s(j) - x_k(j)), \tag{2}$$

where we assume that $k_s(\cdot) \in R^m$ is an unknown nonlinear stiffness function, $m \leq n$, $p \leq n$ and $x_s(j) \in R^n$ is an unknown static position. If the structure and parameter of the system is known, a desired feedforward control input can be calculated such that the system follows the desired output exactly. However, since the structure and parameter of the system are rarely known with accuracy, the system is subjected to unmodeled dynamics and disturbances. Learning control is an iterative approach to finding the desired feedforward control input when the operation is repeated. Since a pure feedforward control is not stable, it is necessary to incorporate a feedback action to stabilize the closed-loop dynamics. Hence, the control input is in general described by [2]:

$$u_k(j) = K(x_d(j) - x_k(j)) + m_k(j), \tag{3}$$

where we assume that $K(\cdot)$ is a feedback control law which has been designed for stability for the control system, $x_d(t) \in R^n$ is the reference state and $m_k(t)$ describes the feedforward learning control input. Substitute equation (3) into equation (1), we have,

$$\begin{aligned} x_k(j+1) &= \mu'(x_k(j), j) + \nu(x_k(j), j)m_k(j) + \nu(x_k(j), j)d_k(j) \\ &\quad + \rho(x_k(j), j)r_k(j), \end{aligned} \tag{4}$$

where $\mu'(x_k(j), j) = \mu(x_k(j), j) + \nu(x_k(j), n)K(x_d(j) - x_k(j))$. The learning control input $m_k(t)$ is added and updated according to a learning law so that the system response is identical to the behaviour of a specified discrete-time reference model as the action is repeated. This discrete-time reference model is specified by [11],

$$(x_d(j+1) - x(j+1)) = G_m(x_d(j) - x(j)) + H_m r(j), \tag{5}$$

where $G_m \in R^{n\times n}$, $H_m \in R^{n\times m}$ are matrices specifying the desired responses of the system. G_m and H_m should be selected such that the desired model (5) is asymptotically stable and has a unique solution over $j = 0, \ldots, N$. This model has desirable qualities in terms of speed of response, percentage of overshoot and robustness.

For the design of discrete-time MRLC system, the learning system is assumed to satisfy the following conditions from (A1) to (A8) [2], [8]:

(A1) Every operation ends in a finite time index number N.

(A2) A discrete-time reference model is given *a priori* as in equation (5).

(A3) The system dynamics is invertible such that for a given reference model described by equation (5), there exists a unique input $m_e(j) \in R^n$ corresponding to the solutions $x_e(j) \in R^n$ and $r_e(j) \in R^m$ of the reference model (5),

$$x_d(j+1) - x_e(j+1) = G_m(x_d(j) - x_e(j)) + H_m r_e(j), \tag{6}$$

where $r_e(j) = k_s(x_s(j) - x_e(j))$, $x_e(0) = x^0 \in R^n$ and x^0 is a constant vector.

(A4) Repeatability of the initial state may not be perfect but can be set as follows:

$$x_k(0) = x^0 + \sigma_k, \tag{7}$$

where $\|\sigma_k\| \leq \epsilon_{x0}$.

(A5) The dynamics fluctuation $d_k(j)$ is bounded by a constant ϵ_d.

(A6) The state and force variables are measurable but may be contaminated by measurement noises so that

$$\hat{x}_k(j) = x_k(j) + n_{1,k}(j), \quad \hat{r}_k(j) = r_k(j) + n_{2,k}(j), \tag{8}$$

where $n_{1,k}(j)$ and $n_{2,k}(j)$ are bounded by a constant ϵ_n.

(A7) $\mu(\cdot,\cdot)$, $\nu(\cdot,\cdot)$, $\rho(\cdot,\cdot)$ and $k_s(\cdot)$ are unknown but Lipschitzian functions of their arguments [2]. That is,

$$\begin{aligned}
\|\mu'(x_2(j), j) - \mu'(x_1(j), j)\| &\le c_\mu \|x_2(j) - x_1(j)\|, \\
\|\nu(x_2(j), j) - \nu(x_1(j), j)\| &\le c_\nu \|x_2(j) - x_1(j)\|, \\
\|\rho(x_2(j), j) - \rho(x_1(j), j)\| &\le c_\rho \|x_2(j) - x_1(j)\|, \\
\|k_s(x_s(j) - x_2(j)) - k_s(x_s(j) - x_2(j))\| &\le c_{ks} \|x_2(j) - x_1(j)\|,
\end{aligned}$$

$\forall j = 0, 1, \ldots, N$ where $c_\mu, c_\nu, c_\rho, c_s \in R^+$ are bounded Lipschitz constants.

(A8) The functions $\nu(\cdot,\cdot)$ and $\rho(\cdot,\cdot)$ are bounded on the interval $[0, N]$. □

The objective of discrete-time MRLC design is to develop a discrete-time iterative learning law such that the system response satisfies the behaviour of the specified discrete-time reference model (5) as the action is repeated. That is, as $k \to \infty$,

$$\hat{w}_k(j) \le e, \tag{9}$$

where

$$\hat{w}_k(j) = (x_d(j+1) - \hat{x}_k(j+1)) - G_m(x_d(j) - \hat{x}_k(j)) - H_m \hat{r}_k(j), \tag{10}$$

is defined as the discrete-time reference model error or impedance error, and e is a positive constant depending on the measurement noises, disturbances and repeatability of the system.

3. Discrete-Time Model Reference Learning Control

Taking into consideration the effect of measurement noise, the iterative discrete-time learning control input is proposed as,

$$m_{k+1}(j) = (1-\psi)m_k(j) + \psi m_0(j) + L(x_k(j))\hat{w}_k(j), \tag{11}$$

where L: $R^n \to R^{p\times n}$ is the learning gain, $\psi \in [0, 1)$ is the forgetting factor, $\hat{w}_k(j) \in R^n$ is contaminated by noise so that,

$$\begin{aligned}
\hat{w}_k(j) &= (x_d(j+1) - \hat{x}_k(j+1)) - G_m(x_d(j) - \hat{x}_k(j)) - H_m \hat{r}_k(j) \\
&= w_k(j) + \check{n}_k(j),
\end{aligned} \tag{12}$$

and $w_k(j) = (x_d(j+1) - x_k(j+1)) - G_m(x_d(j) - x_k(j)) - H_m r_k(j)$, $\check{n}_k(j) = -n_{1,k}(j+1) + G_m n_{1,k}(j) + H_m n_{2,k}(j)$, $\|\check{n}_k(j)\|_\infty \leq \check{b}_1 \epsilon_n$ and $\check{b}_1 = 1 + b_{Gm} + b_{Hm}$. The learning control gains $L(\cdot)$ has to be chosen carefully in this discrete-time setting to ensure desired convergence properties. The following norm measure is used to proof the convergence of the discrete-time MRLC system:

Definition. The α-norm for a function b: $j \in \Omega \to R$ with $\alpha \geq 1$ is defined as:

$$\|b\|_\alpha = \sup_{j\in\Omega} \frac{1}{\alpha^j} \|b(j)\|, \tag{13}$$

where $\Omega = [0, 1, 2, \ldots, N]$. The ∞-norm for a function b: $j \in \Omega \to R$ is defined as:

$$\|b\|_\infty = \sup_{j\in\Omega} \|b(j)\|, \tag{14}$$

where $\|\cdot\|$ is defined as the Euclidean norm. Note that the α-norm is equivalent to ∞-norm since,

$$\|b\|_\alpha \leq \|b\|_\infty \leq \alpha^N \|b\|_\alpha \tag{15}$$

□

The main result of the discrete-time Model Reference Learning Controller is given by the following theorem.

THEOREM *Consider the discrete-time Model Reference Learning Control systems described by equations (4), (2), (11) and (12) with bounded measurement noises, dynamic fluctuations and errors in initial condition that satisfy postulates (A4)–(A6). Let $L(\cdot)$ be any bounded learning gain that satisfies the condition:*

$$\| (1-\psi)I - L(x_k(j)) \cdot \nu(x_k(j), j) \| \leq \bar{p} < 1. \tag{16}$$

Then, the discrete-time reference model error generated by the control input $m_k(j)$ converges such that, for all $j = 0, 1, \ldots, N$,

$$\lim_{k\to\infty} \|w_k\|_\alpha \leq \check{c}_1 \psi \|m_e - m_0\|_\alpha + \check{c}_2 \epsilon_{x0} + \check{c}_3 \epsilon_d + \check{c}_4 \epsilon_n \triangleq e \tag{17}$$

where $\check{c}_1 \cdots \check{c}_4$ are constants to be defined. This bound e tends to zero when ψ, ϵ_{x0}, ϵ_d and ϵ_n tend to zero.

Proof: From equations (6) and (10), we have,

$$w_e(j) \triangleq (x_d(j+1) - x_e(j+1)) - G_m(x_d(j) - x_e(j)) - H_m r_e(j) = 0, \tag{18}$$

where

$$r_e(j) = k_s(x_s(j) - x_e(j)). \tag{19}$$

Hence, since $w_e(j) = 0$, from equations (12) and (18), we have,

$$\begin{aligned} \hat{w}_k(j) &= w_k(j) - w_e(j) + \check{n}_k(j) \\ &= \delta\bar{x}_k(j+1) - G_m\delta\bar{x}_k(j) + H_m\delta\bar{r}_k(j) + \check{n}_k(j), \end{aligned} \tag{20}$$

where

$$\delta\bar{r}_k(j) = k_s(x_s(j) - x_e(j)) - k_s(x_s(j) - x_k(j)), \tag{21}$$

and $\delta\bar{x}_k(j+1) = x_e(j+1) - x_k(j+1)$, $\delta\bar{x}_k(j) = x_e(j) - x_k(j)$ and $\delta\bar{r}_k(j) = r_e(j) - r_k(j)$. The desired state $x_e(j)$ exists but is unknown since $r_e(j)$ is unknown because $x_s(j)$ and $k_s(\cdot)$ are unknown. Here, the definitions of the desired state and force are for analysis and are not used in the control law. The following development follows a similar argument as in [15] to show the convergence of the reference model error $w_k(j)$ as the action is repeated. From equations (11), (20) and (4), we have,

$$\begin{aligned} &\delta\bar{m}_{k+1}(j) \\ &= [I - L(x_k(j)) \cdot \nu(x_k(j), j)]\delta\bar{m}_k(j) - L(x_k(j))\{(\mu'(x_e(j), j) - \mu'(x_k(j), j)) \\ &\quad + (\nu(x_e(j), j) - \nu(x_k(j), j))m_e(j) + \rho(x_k(j), j)\delta\bar{r}_k(j) \\ &\quad + (\rho(x_e(j), j) - \rho(x_k(j), j))r_e(j) - \nu(x_k(j), j)d(j) \\ &\quad - G_m\delta\bar{x}_k(j) + H_m\delta\bar{r}_k(j) + \check{n}_k(j)\}, \end{aligned} \tag{22}$$

where $\delta\bar{m}_{k+1}(j) = m_e(j) - m_{k+1}(j)$, $\delta\bar{m}_k(j) = m_e(j) - m_k(j)$ and $m_e(j)$ is described by

$$x_e(j+1) = \mu'(x_e(j), j) + \nu(x_e(j), j)m_e(j) + \rho(x_e(j), j)r_e(j). \tag{23}$$

Taking norm and using the bound, Lipschitz condition and equation (21) yields,

$$\begin{aligned} &\|\delta\bar{m}_{k+1}(j)\| \\ &\le \|(1-\psi)I - L(x_k(j)) \cdot \nu(x_k(j), j))\| \cdot \|\delta\bar{m}_k(j)\| + \psi\|\delta\bar{m}_0(j)\| \\ &\quad + \|L(x_k(j)\| \cdot \{\|\mu'(x_e(j), j) - \mu'(x_k(j), j)\| \\ &\quad + \|\nu(x_e(j), j) - \nu(x_k(j), j)\| \cdot \|m_e(j)\| \\ &\quad + \|\rho(x_k(j), j)\| \cdot \|k_s(x_s(j) - x_e(j)) - k_s(x_s(j) - x_k(j))\| \\ &\quad + \|\rho(x_e(j), j) - \rho(x_k(j), j)\| \cdot \|r_e(j)\| \\ &\quad + \|G_m\| \cdot \|\delta\bar{x}_k(j)\| + \|H_m\| \cdot \|k_s(x_s(j) - x_e(j)) - k_s(x_s(j) - x_k(j))\| \\ &\quad + \|\nu(x_k(j), j)\| \cdot \|d(j)\| + \|\check{n}(j)\|\} \\ &\le \bar{p}\|\delta\bar{m}_k(j)\| + \psi\|\delta\bar{m}_0(j)\| + b_L c_1\|\delta\bar{x}_k(j)\| + b_L\bar{b}_2\epsilon_d + b_L\check{b}_1\epsilon_n, \end{aligned} \tag{24}$$

where $\delta\bar{m}_0(j) = m_e(j) - m_0(j)$, $c_1 = c_\mu + c_\nu b_{me} + b_\rho b_{Ks} + c_\rho b_{re} + b_{Gm} + b_{Hm}c_{ks}$, b_L, $\bar{b}_2$, b_ρ, b_{Gm}, b_{Hm} are the norm bounds for $L(\cdot)$, $\nu(\cdot,\cdot)$, $\rho(\cdot,\cdot)$, G_m, H_m respectively,

$b_{re} = \|r_e\|_\infty$, $b_{me} = \|m_e\|_\infty$, and $c_\mu, c_\nu, c_\rho, c_{ks}$ are the lipschitz constants for $\mu(\cdot,\cdot)$, $\nu(\cdot,\cdot)$, $\rho(\cdot,\cdot)$, $k_s(\cdot)$ respectively. Similarly, from equations (4) and (21), we have,

$$\|\delta\bar{x}_k(j+1)\| \le c_2\|\delta\bar{x}_k(j)\| + \bar{b}_2\|\delta\bar{m}_k(j)\| + \bar{b}_2\epsilon_d, \tag{25}$$

where $c_2 = c_\mu + c_\nu b_{me} + b_\rho c_{ks} + c_\rho b_{re}$. For $j = 0$, since $\delta\bar{x}_k(0) = -\sigma_k$ as stated in assumption (A4), we have from equation (25),

$$\|\delta\bar{x}_k(1)\| \le c_2\|\delta\bar{x}_k(0)\| + \bar{b}_2(\|\delta\bar{m}_k(0)\| + \epsilon_d). \tag{26}$$

For $j = 1$, we have,

$$\begin{aligned}\|\delta\bar{x}_k(2)\| &\le c_2\|\delta\bar{x}_k(1)\| + \bar{b}_2(\|\delta\bar{m}_k(1)\| + \epsilon_d)\\ &\le c_2^2\|\delta\bar{x}_k(0)\| + \bar{b}_2\sum_{i=0}^{1} c_2^{1-i}(\|\delta\bar{m}_k(i)\| + \epsilon_d).\end{aligned} \tag{27}$$

(28)

Therefore, by induction,

$$\begin{aligned}\|\delta\bar{x}_k(j)\| &\le c_2\|\delta\bar{x}_k(j-1)\| + \bar{b}_2(\|\delta\bar{m}_k(j-1)\| + \epsilon_d)\\ &\le c_2^j\|\delta\bar{x}_k(0)\| + \bar{b}_2\sum_{i=0}^{j-1} c_2^{j-1-i}(\|\delta\bar{m}_k(i)\| + \epsilon_d).\end{aligned} \tag{29}$$

Substitute equation (29) into equation (24) yields,

$$\begin{aligned}\|\delta\bar{m}_{k+1}(j)\| \le \bar{p}\ &\|\delta\bar{m}_k(j)\| + \psi\|\delta\bar{m}_0(j)\| + b_L c_1 c_2^j\|\delta\bar{x}_k(0)\|\\ &+ b_L c_1\bar{b}_2\sum_{i=0}^{j-1} c_2^{j-1-i}(\|\delta\bar{m}_k(i)\| + \epsilon_d) + b_L\bar{b}_1\epsilon_d + b_L\check{b}_1\epsilon_n.\end{aligned} \tag{30}$$

Multiplying both sides of equation (30) by $\frac{1}{\alpha^j}$ and define $c \triangleq \max\{b_L c_1\bar{b}_2, c_2\}$, we obtain,

$$\begin{aligned}\frac{1}{\alpha^j}\|\delta\bar{m}_{k+1}(j)\| \le\ &\bar{p}\frac{1}{\alpha^j}\|\delta\bar{m}_k(j)\| + \psi\frac{1}{\alpha^j}\|\delta\bar{m}_0(j)\|\\ &+ \frac{c}{\alpha}\sum_{i=0}^{j-1}\left(\frac{c}{\alpha}\right)^{j-1-i}\left(\frac{1}{\alpha^i}\|\delta\bar{m}_k(i)\| + \frac{1}{\alpha^i}\epsilon_d\right)\\ &+ b_L c_1\left(\frac{c_2}{\alpha}\right)^j\epsilon_{x0} + \frac{1}{\alpha^j}(b_L\bar{b}_2\epsilon_d + b_L\check{b}_1\epsilon_n).\end{aligned} \tag{31}$$

Therefore,

$$\|\delta\bar{m}_{k+1}\|_\alpha \le \hat{p}\|\delta\bar{m}_k\|_\alpha + \epsilon \tag{32}$$

where $\hat{p} = \bar{p} + \frac{c(1-(\frac{c}{\alpha})^N)}{\alpha - c}$, $\alpha > c$ and $\epsilon = \psi\|\delta\bar{m}_0\|_\alpha + b_L c_1\epsilon_{x0} + b_L\check{b}_1\epsilon_n + c_3\epsilon_d$, and

$c_3 = b_L \bar{b}_2 + c(\frac{1-(\frac{c}{\alpha})^N}{\alpha - c})$. Since $\bar{p}$ is less than 1, choose $\alpha > \max\{c, 1\}$ and large enough so that,

$$0 \leq \hat{p} < 1,$$

then equation (32) converges such that,

$$\lim_{k\to\infty} \|\delta\bar{m}_k\|_\alpha \leq \frac{\epsilon}{1-\hat{p}}, \tag{33}$$

for all $j = 0, 1, \ldots, N$. Similarly, taking α-norm on equation (29) yields,

$$\|\delta\bar{x}_k\|_\alpha \leq c_4\|\delta\bar{m}_k\|_\alpha + \epsilon_{x0} + c_4\epsilon_d, \tag{34}$$

where $c_4 = \frac{\bar{b}_2(1-(\frac{c_2}{\alpha})^N)}{\alpha - c_2}$. From equations (20), (21) and (25), we have,

$$\|w_k\| \leq (c_2 + b_{Gm} + c_{ks}b_{Hm})\|\delta\bar{x}_k\| + \bar{b}_2\|\delta\bar{m}_k\| + \bar{b}_2\epsilon_d. \tag{35}$$

Therefore, taking α-norm on equation (35) and substitute equations (34) and (33) into it, we have,

$$\begin{aligned}\lim_{k\to\infty} \|w_k\|_\alpha &\leq c_5 \lim_{k\to\infty} \|\delta\bar{m}_k\|_\alpha + c_6\epsilon_{x0} + c_5\epsilon_d \\ &\leq \check{c}_1\psi\|\delta\bar{m}_0\|_\alpha + \check{c}_2\epsilon_{x0} + \check{c}_3\epsilon_d + \check{c}_4\epsilon_n \triangleq e \end{aligned}\tag{36}$$

uniformly for all $j = 0, 1, \ldots, N$, where $c_5 = c_6c_4 + \bar{b}_2$, $c_6 = c_2 + b_{Gm} + c_{ks}b_{Hm}$, $\check{c}_1 = \frac{c_5}{1-\hat{p}}$, $\check{c}_2 = \check{c}_1 b_L c_1 + c_6$, $\check{c}_3 = \check{c}_1 c_3 + c_5$ and $\check{c}_3 = \check{c}_1 b_L \bar{b}_1$. ■

Remark 1. Equation (17) or (36) shows the dependence of the bound of the discrete-time reference model error $w_k(j)$ on the bounds of the initial state error, dynamics fluctuations, measurement noises, ψ and the error between the desired learning control input $m_e(j)$ and the initial bias term $m_0(j)$. If ϵ_{x0}, ϵ_d, ϵ_n and ψ are zero, the bound of the impedance error also tends to zero.

Remark 2. The main advantage of the discrete-time MRLC approach is that a discrete-time reference model (5) can be specified instead of the continuous time reference model. This is important since most modern industrial controllers are implemented in discrete-time. In addition, the discrete-time MRLC method does not require the measurement of the state derivative as in [1], [7], [4] as seen from equations (11) and (12).

Remark 3. The discrete-time MRLC scheme can be applied to both contact task where $r_k(j) \neq 0$ and non contact task where $r_k(j) = 0$. If the force variable $r_k(j) = 0$, the reference model is specified as:

$$(x_d(j+1) - x_k(j+1)) = G_m(x_d(j) - x_k(j)). \tag{37}$$

Therefore, an advantage of MRLC approach is that a single learning controller can be implemented without the need to switch the learning controller from non contact to and from contact tasks as needed in most of the learning controller designs in the literature. This feature is demonstrated and verified experimentally on a SCARA robot in the next section. □

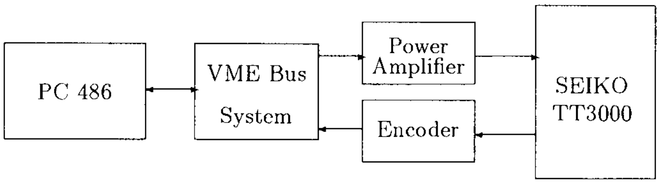

Figure 1. Block diagram of the experimental system.

4. Experimental Results

In this section, the discrete-time Model Reference Learning Controller is applied to the impedance control of robotic manipulators. Experimental results are presented to illustrate the performance of the proposed controller.

The experimental system consist of the industrial manipulator SEIKO TT3000, supervisory computer using PC 486, VME bus-based system, power amplifiers and sensors systems as shown in Figure 1. At the top of the system hierarchy is the supervisory computer and at the lower level are the multiprocessors using VME bus-based system. The supervisory computer is mainly used for task planning and high-level programming. The lower level is used for real time data collection and control. This VME bus-based system consists of the host computer MVME 147 and the target computer MVME104. MVME 147 is a MC68030 based system with 4MB DRAM and 25 MHz system clock and MVME104 is a MC68010 based system with 512K bytes RAM and 10 MHz system clock. The MVME104 is also responsible for input, output operations using encoder input ports and digital to analog converters. The robot used in this experiment is the industrial robot SEIKO TT3000. This robot is the Selective Compliance Assembly Robot Arm (SCARA) type manipulator with three degrees of freedom as illustrated in the schematic diagram of Figure 2. The first joint is a prismatic joint, the second and third joints are revolute joints. The dynamics model of this manipulator [10] with three joints, taking into consideration of the contact force f, can be expressed as in state space form as:

$$\begin{bmatrix} \dot{\theta}_k(t) \\ \ddot{\theta}_k(t) \end{bmatrix} = \begin{bmatrix} \dot{\theta}_k(t) \\ -M^{-1}(\theta_k(t))V(\theta_k(t), \dot{\theta}_k(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}(\theta_k(t)) \end{bmatrix} u_k(t) - \begin{bmatrix} 0 \\ M^{-1}(\theta_k(t)) \end{bmatrix} f_k(t). \tag{38}$$

where

$$M(\theta) = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}, \quad V(\theta, \dot{\theta}) = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, \quad \theta = \begin{bmatrix} z_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$
$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \qquad f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}, \tag{39}$$

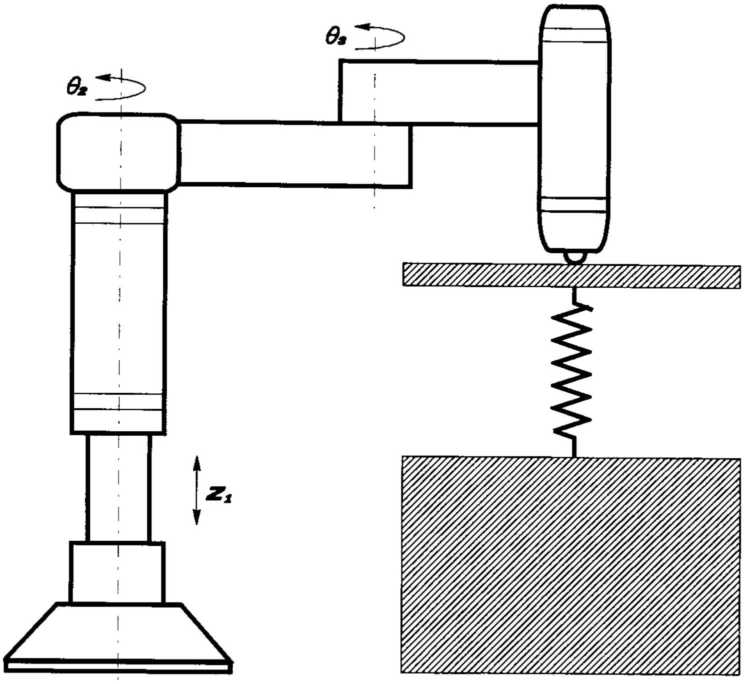

Figure 2. A SCARA robot.

and

$$
\begin{aligned}
m_{11} &= m_1 + m_2 + m_3, & m_{12} &= 0,\\
m_{13} &= 0, & m_{22} &= (m_2 + m_3)a_2^2 + m_3 a_3^2 + 2m_3 a_2 a_3 \cos(\theta_3),\\
m_{23} &= m_3 a_3^2 + m_3 a_2 a_3 \cos(\theta_3), & m_{33} &= m_3 a_3^2,\\
v_1 &= m_{11} g, & v_2 &= -m_3 a_2 a_3 \sin(\theta_3)(\dot{\theta}_3^2 + 2\dot{\theta}_2\dot{\theta}_3),\\
v_3 &= m_3 a_2 a_3 \sin(\theta_3)\dot{\theta}_2^2,
\end{aligned} \tag{40}
$$

and m_1, m_2 and m_3 are the masses of link one, two and three respectively in kilograms, a_2 and a_3 are the length of link two and three respectively in meters and g is the constant acceleration due to gravity in meter per second per second. Three encoders are employed for position measurement of each joint and a differentiator is used to estimate the velocity from the position measurements. The pulse per revolution for encoder two and three are 600 and 800 respectively. For the prismatic joint one, one pulse corresponds to a displacement of 0.01044 mm. To measure the contact force, a force sensor made by Lord Corporation is mounted on the end-effector of the robot. It is assumed that the state space formulation of the robot dynamic model described by equations (38) is discretized as in the following form of equation (4):

$$x_k(j+1) = \bar{\mu}(x_k(j)) + \bar{v}(x_k(j))u_k(j) + \bar{\rho}(x_k(j))r_k(j), \tag{41}$$

where $x_k(j) = [\theta_k^T(j)\ \dot{\theta}_k^T(j)]^T \in R^6$ denotes the state variables, $\bar{\mu}(\cdot) \in R^6$, $\bar{\nu}(\cdot) \in R^{6\times3}$ and $\bar{\rho}(\cdot) \in R^{6\times3}$ are nonlinear system dynamic terms, $r_k(j) = f_k(j) \in R^3$ and $u_k(j) \in R^3$ denotes the control input. For example, Euler's approximation gives,

$$\begin{bmatrix} \theta_k(j+1) \\ \dot{\theta}_k(j+1) \end{bmatrix} = \begin{bmatrix} \theta_k(j) + \Delta\dot{\theta}_k(j) \\ \dot{\theta}_k(j) - \Delta M^{-1}(\theta_k(j))V(\theta_k(j), \dot{\theta}_k(j)) \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta M^{-1}(\theta_k(j)) \end{bmatrix} u_k(j) - \begin{bmatrix} 0 \\ \Delta M^{-1}(\theta_k(j)) \end{bmatrix} f_k(j), \tag{42}$$

where Δ is the sampling time interval. Hence, the nonlinear terms $\bar{\mu}(\cdot)$, $\bar{\nu}(\cdot)$ and $\bar{\rho}(\cdot)$ can be given as,

$$\begin{aligned} \bar{\mu}(x_k(j)) &= \begin{bmatrix} \theta_k(j) + \Delta\dot{\theta}_k(j) \\ \dot{\theta}_k(j) - \Delta M^{-1}(\theta_k(j))V(\theta_k(j), \dot{\theta}_k(j)) \end{bmatrix}, \\ \bar{\nu}(x_k(j)) &= -\bar{\rho}(x_k(j)) = \begin{bmatrix} 0 \\ \Delta M^{-1}(\theta_k(j)) \end{bmatrix}. \end{aligned} \tag{43}$$

From equation (3), suppose that a feedback control law has been designed for stability of the closed-loop system as follows:

$$u_k(j) = K_p(\theta_d(j) - \theta_k(j)) + K_v(\dot{\theta}_d(j) - \dot{\theta}_k(j)) + m_k(j), \tag{44}$$

where $K_p \in R^{3\times3}$ and $K_v \in R^{3\times3}$ are the feedback gains and $m_k(t) \in R^3$ is the learning control input. It is assumed that $\bar{\mu}(\cdot)$, $\bar{\nu}(\cdot)$ and $\bar{\rho}(\cdot)$ are local lipschitz continuous which satisfy assumption (A7) and $\bar{\nu}(\cdot)$, $\bar{\rho}(\cdot)$ satisfies assumption (A8) in the finite workspace and finite operation time interval.

In impedance control [9] of robotic manipulators, the control objective is specified explicitly by a reference model (or target impedance) rather than a desired trajectory. Impedance control does not attempt to track motion and force trajectories but rather to regulate the mechanical impedance specified by a target model at the robot end-effector [9]:

$$M_m(\ddot{\theta}_d - \ddot{\theta}) + C_m(\dot{\theta}_d - \dot{\theta}) + K_m(\theta_d - \theta) = -f, \tag{45}$$

where M_m, C_m and $K_m \in R^{3\times3}$ are matrices which specify the desired dynamic relationship between the reference position error $(\theta_d - \theta)$ and the external force f. From equation (45), the target impedance can be expressed in discrete-time as,

$$\begin{bmatrix} \theta_d(j+1) - \theta_k(j+1) \\ \dot{\theta}_k(j+1) - \dot{\theta}_d(j+1) \end{bmatrix} = G_m \begin{bmatrix} \theta_d(j) - \theta_k(j) \\ \dot{\theta}_k(j) - \dot{\theta}_d(j) \end{bmatrix} + H_m f_k(j), \tag{46}$$

where $G_m \in R^{6\times6}$, $H_m \in R^{6\times3}$ are matrices specifying the desired responses of the system. G_m, H_m should be selected so that the reference model is asymptotically stable and has a unique solution over $j = 0, \ldots, N$. From equation (11), the discrete-time learning impedance control law is given as,

$$m_{k+1}(j) = m_k(j) + L(x_k(j))\hat{w}_k(j), \tag{47}$$

$$\hat{w}_k(j) = (x_d(j+1) - \hat{x}_k(j+1)) - G_m(x_d(j) - \hat{x}_k(j)) - H_m\hat{r}_k(j), \tag{48}$$

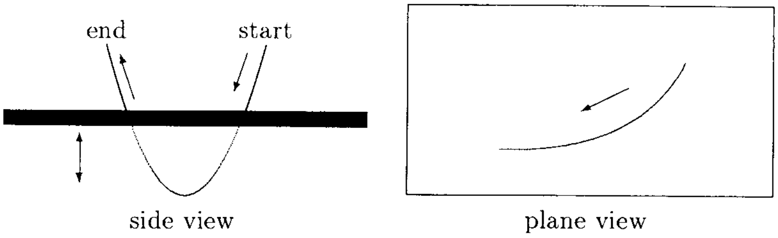

Figure 3. End-effector path.

and ψ is chosen as zero, L: $R^6 \to R^{3\times 6}$ is the learning gain. Similarly, if the discrete-time is discretized using Euler's approximation, then

$$G_m = \begin{bmatrix} I_n & \Delta I_n \\ -\Delta M_m^{-1} K_m & I_n - \Delta M_m^{-1} C_m \end{bmatrix}, \quad H_m = \begin{bmatrix} 0 \\ -\Delta M_m^{-1} \end{bmatrix}. \tag{49}$$

From the definition of $\bar{v}(x(j))$ and with the learning gain $L(x(j))$ partitioned as $[L_1(x(j)), L_2(x(j))]$ where $L_1(x(j)) \in R^{3\times 3}$ and $L_2(x(j)) \in R^{3\times 3}$, equation (16) is equivalent to,

$$\|I_n - L_2(x_k(j)) \cdot \Delta \cdot M^{-1}(\theta_k(j))\| \leq \bar{p} < 1. \tag{50}$$

Therefore, $L_1(\cdot)$ can be chosen as zero and the iterative learning control law (47) can be rewritten as,

$$m_{k+1}(j) = m_k(j) + L_2(\theta_k(j))\hat{w}_{2,k}(j), \tag{51}$$

where

$$\begin{aligned} \hat{w}_{2,k}(j) &= (\dot{\theta}_d(j+1) - \hat{\dot{\theta}}_k(j+1)) + (\Delta M_m^{-1} C_m - I)(\dot{\theta}_d(j) - \hat{\dot{\theta}}_k(j)) \\ &\quad + \Delta M_m^{-1} K_m(\theta_d(j) - \hat{\theta}_k(j)) + \Delta M_m^{-1} \hat{f}_k(j). \end{aligned} \tag{52}$$

The learning gain $L_2(\cdot)$ should be chosen to satisfy equation (50) while the reference model matrices are caculated independently.

To effectively verify the proposed MRLC laws, the end-effector was set to follow the path which involved free motion tracking, transition from free motion to contact motion, contact motion on the constraint plane with compliance, transition from contact motion to free motion, and finally free motion tracking again as illustrated in Figure 3. In this experiment, a steel ball is attached to the force sensor and hence the frictional force along the constraint plane is negligible. In another words, $f = [f_1(t), 0, 0]^T$. Mathematically, the task is specified by the target impedance (45) as follows:

$$M_m = \begin{bmatrix} 50 & & \\ & 40 & \\ & & 40 \end{bmatrix} \quad C_m = \begin{bmatrix} 200 & & \\ & 200 & \\ & & 200 \end{bmatrix} \quad K_m = \begin{bmatrix} 800 & & \\ & 1000 & \\ & & 1000 \end{bmatrix},$$

and the reference trajectories $\theta_d(t) = [z_{1d}^T(t), \theta_{2d}^T(t), \theta_{3d}^T(t)]^T$ are described by,

$$z_{1d}(t) = \begin{cases} -0.045(\frac{6f_s^5}{1500^5}t^5 - \frac{15f_s^4}{1500^4}t^4 + \frac{10f_s^3}{1500^3}t^3) & \text{for } 0 \le t < \frac{1500}{f_s} \\ -0.045 + 0.045(\frac{6f_s^5}{1500^5}t^5 - \frac{15f_s^4}{1500^4}t^4 + \frac{10f_s^3}{1500^3}t^3) & \text{for } \frac{1500}{f_s} \le t < \frac{3000}{f_s} \\ 0 & \text{for } \frac{3000}{f_s} \le t \le \frac{3600}{f_s} \end{cases}$$

$$\theta_{2d}(t) = \begin{cases} 2.017 + 0.6(\frac{6f_s^5}{3000^5}t^5 - \frac{15f_s^4}{3000^4}t^4 + \frac{10f_s^3}{3000^3}t^3) & \text{for } 0 \le t < \frac{3000}{f_s} \\ 2.617 & \text{for } \frac{3000}{f_s} \le t \le \frac{3600}{f_s} \end{cases}$$

$$\theta_{3d}(t) = \begin{cases} 1.885 - 0.5(\frac{6f_s^5}{3000^5}t^5 - \frac{15f_s^4}{3000^4}t^4 + \frac{10f_s^3}{3000^3}t^3) & \text{for } 0 \le t < \frac{3000}{f_s} \\ 1.385 & \text{for } \frac{3000}{f_s} \le t \le \frac{3600}{f_s} \end{cases} \quad (53)$$

Here, $z_{1d}(t)$ is specified in meter, $\theta_{2d}(t)$ and $\theta_{3d}(t)$ are specified in radian. The sampling time Δ of the discrete-time learning control system was $\frac{5}{f_s}$ sec where $f_s = 244Hz$ and the period T of the whole operation was $\frac{3600}{f_s}$ sec. Using Euler's approximation, the discrete-time impedance learning control law described by equations (44), (51) and (52) were applied to the robotic system with the controller gains set as follows:

$$K_p = \begin{bmatrix} 10 & & \\ & 10 & \\ & & 250 \end{bmatrix}, \quad K_v = \begin{bmatrix} 12 & & \\ & 12 & \\ & & 100 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 80 & & \\ & 80 & \\ & & 4000 \end{bmatrix}. \quad (54)$$

The discrete-time impedance error was calculated as,

$$\begin{aligned} M_m \hat{w}_{2.k}(j) &= M_m(\dot{\theta}_d(j+1) - \hat{\dot{\theta}}_k(j+1)) + (\Delta C_m - M_m)(\dot{\theta}_d(j) - \hat{\dot{\theta}}_k(j)) \\ &\quad + \Delta K_m(\theta_d(j) - \hat{\theta}_k(j)) + \Delta \hat{f}_k(j). \end{aligned} \quad (55)$$

The learning procedures started with equations (51) and (52). Here the state and force variables which enter calculations represent the measurements from experimental setup. The experimental results of the impedance errors, the reference trajectory errors $(\theta_d - \hat{\theta}_k)$ and the contact force $\hat{f}_1$ are shown in Figure 4 to Figure 10. In the first trial, i.e., $k = 0$, m_0 was set to zero and hence the controller is a PD feedback law with no learning control. The learning process contained the following steps which was done off-line after each operation.

(1) Calculate $\hat{w}_{2.k}(t)$ from equation (52) using sampled reference trajectory $\theta_d(t)$ from the reference model and the motion and force, $\hat{\theta}_k(t)$ and $\hat{f}_k(t)$, respectively, from measurements.

(2) Calculate the learning control input $m_{k+1}(t)$ for the next trial from equation (51) using the calculated $\hat{w}_{2.k}(t)$ and the learning control input $m_k(t)$ of the current operation.

(3) Repeat the action using the calculated learning control input $m_{k+1}(t)$.

As the operation repeated, the impedance errors decreased as shown in Figures 4 to 6. From Figures 7 to 9, the results also showed that the reference trajectory errors decreased when

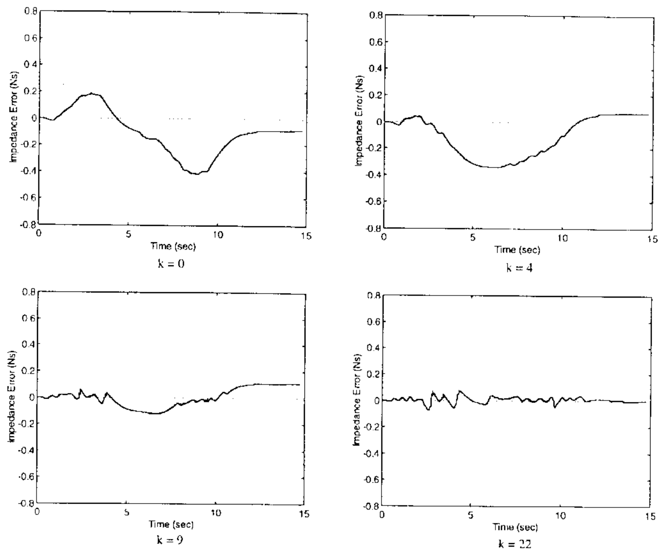

Figure 4. The impedance error of joint one.

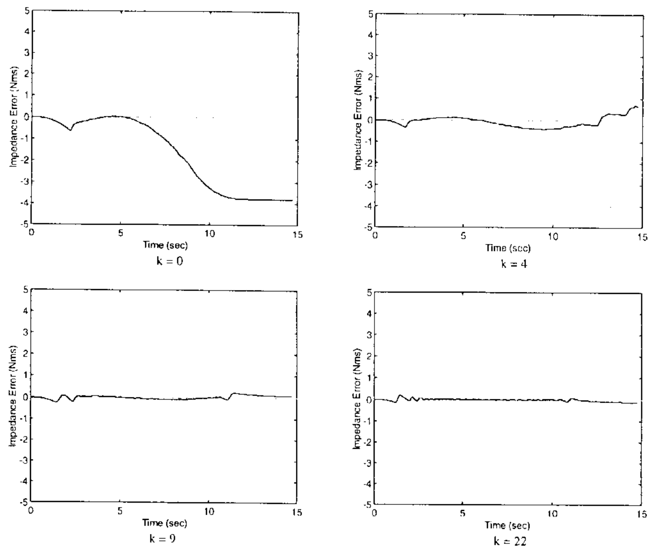

Figure 5. The impedance error of joint two.

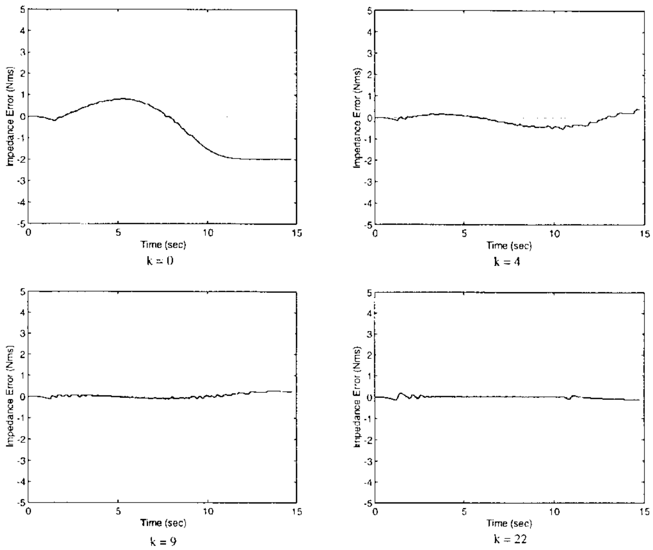

Figure 6. The impedance error of joint three.

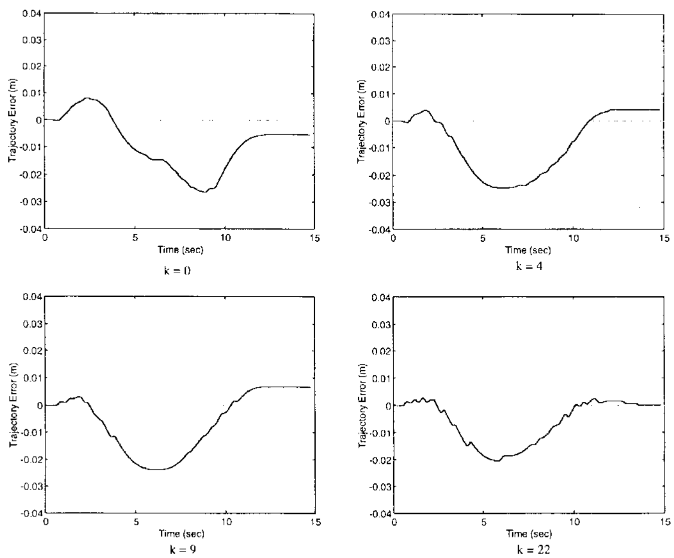

Figure 7. The reference trajectory error of joint one.

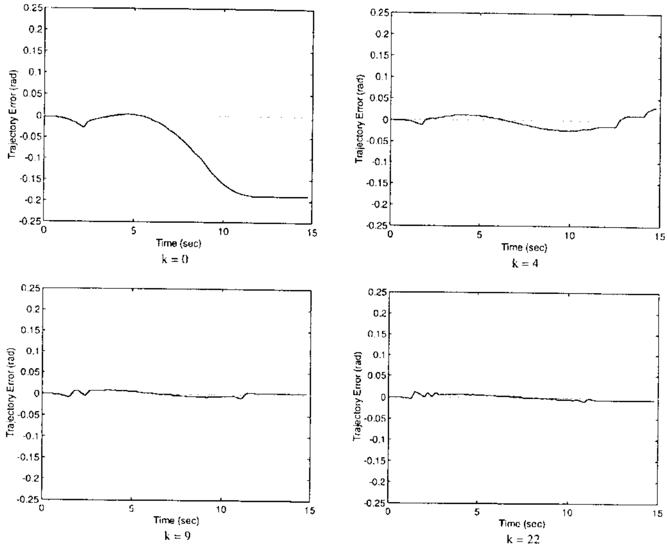

Figure 8. The reference trajectory error of joint two.

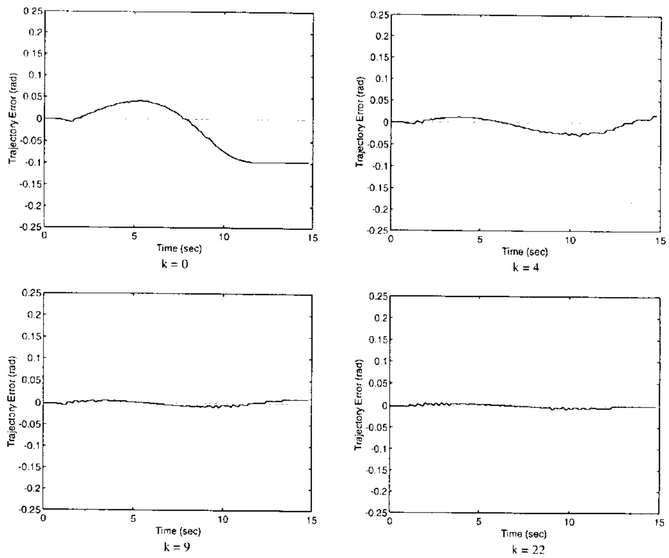

Figure 9. The reference trajectory error of joint three.

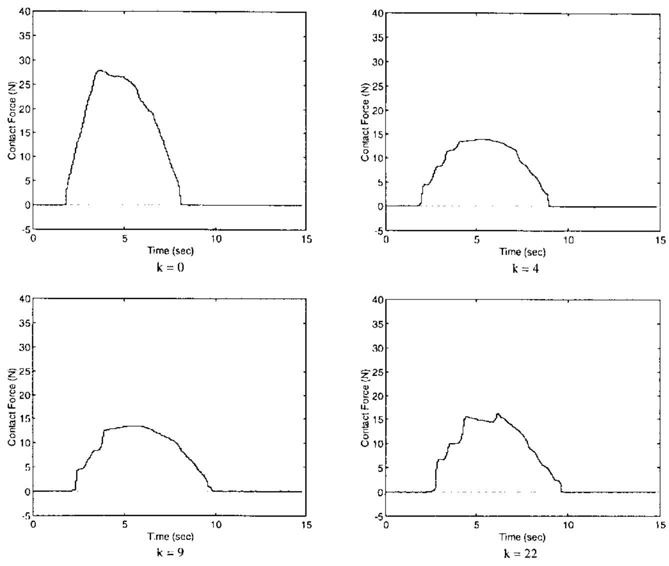

Figure 10. The contact force.

the impedance errors decreased. Note that these errors converged though the contact points from free motion to and from contact motion were changing at every iteration as shown by the contact force in Figure 10. The experimental results illustrate the validity of the theory presented in section 3 and show that the discrete-time Model Reference Learning Controller reduces the impedance error or reference model error tremendously. These results also illustrate the superiority of learning control as compared to no learning control on the first trial.

5. Conclusion

A discrete time Model Reference Learning Control algorithm has been developed for a class of discrete-time nonlinear system. Convergence of the reference model error can be achieved by suitable selection of the learning controller gain. Sufficient condition is derived for selection of learning gain to guarantee convergence of the reference model error for learning controller design. Robustness of the Model Reference Learning Controllers can be ensured even in the presence of dynamics fluctuations, output measurement noises and errors in initial conditions. It has been shown that the reference model error converges to bound depending on the bound of the dynamics fluctuation, output measurement noise and error in initial condition. This bound tends to zero in the absence of these disturbances. The proposed discrete-time MRLC scheme is implemented on an industrial robot SEIKO TT3000. A single learning controller was implemented without the need to switch the learning controller from non contact to and from contact tasks as needed in most of the learning controller designs in the literature. Theoretical analysis and experimental results show that the learning controller is able to make the system response converges to that specified by the discrete-time reference model as the action is repeated.

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