# Digital Repetitive Controlled Three-Phase PWM Rectifier

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***Abstract*—In this paper, a digital repetitive control (RC) strategy is proposed to achieve zero tracking error for constant-voltage constant-frequency (CVCF) pulse width modulation (PWM) converters. The proposed control scheme is of "plug-in" structure: a plug-in digital repetitive controller plus a conventional controller (e.g., PD controller). The design of the plug-in repetitive learning controller is systematically developed. The stability analysis of overall system is discussed. A repetitive controlled three-phase reversible PWM rectifier is given as an application example. Near unit power factor and constant output dc voltage are ensured under parameter uncertainties and load disturbances. Simulation and experimental results are provided to testify the effectiveness of the proposed control scheme.**

***Index Terms*—PWM converters, repetitive control.**

## I. INTRODUCTION

ONE BASIC requirement in control systems is the capability to regulate controlled variables to their reference inputs without steady-state errors against unknown and unmeasurable disturbances. Control systems with this property are called servo systems [1]. In servo system design, the internal model principle proposed in [2] plays an important role. According to the internal model principle [2]–[4], zero error tracking of any reference input, in steady-state, can be accomplished if a generator of the reference input is included in a stable closed-loop system. For example, a type 1 closed-loop system with an integrator $1/s$ [or $1/(z-1)$ in discrete time domain], i.e., the generator of unit step function, in the loop offers asymptotic tracking of a step input with zero steady state error. Repetitive control (RC) [1], [5], [6] is a special case of the internal model principle in control systems with periodic signals. In discrete time domain, RC ensures zero tracking error at least at sampling points. Moreover, integral control is a special case of repetitive control with unit period.

In fact, constant-voltage–constant-frequency (CVCF) power converters are servo systems with periodic sinusoidal and/or dc (step) reference commands. CVCF PWM power converters, such as dc-to-dc boost and buck converters, dc-to-ac inverters, and ac-to-dc rectifiers are widely employed in various power supplies. A good CVCF power supply should have the nominal constant output in the presence of disturbances and uncertainties, good dynamic response to disturbances, and remain stable under all operating conditions. Hence, repetitive control provides a zero tracking error solution for CVCF PWM converters. It was first applied to single-phase inverters with good preliminary results in [7], which attracts more research in [8]–[10] for single-phase inverter, [11] for 3-phase rectifier, and [12] for 3-phase inverter. In this case, the robustness analysis and design of repetitive controllers are neither systematic nor complete.

In this paper, a digital repetitive control scheme is proposed to achieve zero tracking error for CVCF PWM converters. It is of "plug-in" structure: a conventional feedback controller and a plug-in feedforward repetitive controller. The design of a digital repetitive control scheme is systematically developed with complete stability and robustness analysis. A digital repetitive controlled three-phase reversible PWM rectifier is illustrated in details. Simulation and experimental results are provided to testify the effectiveness of the proposed control scheme.

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## II. DIGITAL REPETITIVE LEARNING CONTROL

### A. Internal Model Principle

Exact compensation of certain types of deterministic disturbance and exact tracking of particular reference signals are two important issues in the control system design [4]. A particular disturbance and/or a reference signal of interest here are those that can be described as

$$D_g(s) = \frac{N_g(s)}{\Gamma_g(s)} \quad (1)$$

where $N_g(s)$ is the numerator polynomial; $\Gamma_g(s)$ is the denominator which represents the reference and/or disturbance generating polynomial.

For example, a constant signal can be described by the model $\dot{x}_d = 0$ with initial condition $x_d(0)$. And its corresponding Laplace transform is $x_d(0)/s$.

Fig. 1 shows a closed-loop control system, where $C(s)$ is the controller, $G_o(s)$ is the plant with its input $U(s)$, $D(s)$ is the disturbance, and $E(s)$ is the tracking error between the reference $R(s)$ and the output $Y(s)$. From Fig. 1, the tracking error $E(s)$ can be derived by

$$E(s) = \frac{1}{1 + C(s)G_o(s)} \left(R(s) - D(s)\right). \quad (2)$$

For steady state exact disturbance compensation and exact reference tracking of the closed-loop control system in Fig. 1, a sufficient condition is that the reference generating polynomial and the disturbance generating polynomial be included as part of the denominator of the controller $C(s)$. This is known as the *Internal Model Principle* (IMP) [4].

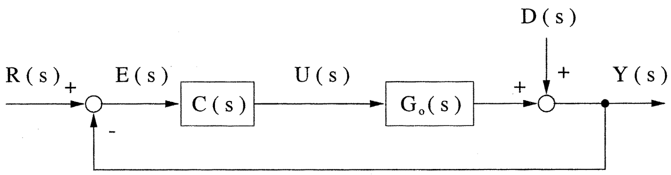



Fig. 1. Closed-loop control system.

In discrete time cases, the Internal Model Principle can be used to achieve some very interesting results, such as discrete repetitive control for system with periodic signals. A periodic signal $w(k)$ of $N$ samples can be modeled by

$$W(z) = \frac{z^{-N_1}}{1-z^{-N}}\,W_0(z) = \frac{z^{N_2}}{z^N-1}\,W_0(z) \tag{3}$$

where $W_0(z) = w(0) + w(1)z^{-1} + w(2)z^{-2} + \cdots + w(N-1)z^{-(N-1)}$; $N = N_1 + N_2$. Obviously, when $N = N_1 = 1$ and $N_2 = 0$, $W(z)$ is a step function generator. Thus, a dc signal can be regarded as a special case of periodic signals. Equation (3) is the unified generator of both periodic signals and dc signals. Using the Internal Model Principle, any $N$ period reference signal can be exactly tracked (at least at the sample points) by including $z^N - 1$ in the denominator of the controller. This idea is the basis of the digital repetitive control.

### B. Problem Formulation

Consider a repetitive control system shown in Fig. 2. The transfer function $H(z)$ for the control system without plug-in repetitive controller can be written as

$$H(z) = \frac{G_c(z)G_s(z)}{1+G_c(z)G_s(z)} = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} \tag{4}$$

where $d$ is the known number of pure time step delays; $y(z)$ and $u(z)$ are the output and input of the plant $G_s(z)$, respectively; $d(z)$ is the disturbance with a known period of $N*T$ with $N = f/f_c$, $f$ being the frequency of $y_d(k)$, and $f_c = 1/T$ being the sampling frequency; $e(z)$, with $e(k) = y(k) - y_d(k)$, is the tracking error; $G_c(z)$ is a conventional feedback controller, e.g., PD controller.

Without loss of generality, it is assumed that (4) defines an asymptotically stable system: i.e., the roots of $A(z^{-1}) = 0$ are all inside the unit circle. This assumption implies that the closed-loop control system is asymptotically stable. It is also assumed that $B(z^{-1})$ and $1 - z^{-N}$ are coprime (i.e., $B(e^{\pm ij2\pi/N}) \neq 0$, $i = 1, 2, \ldots, [N/2]$, where $[N/2]$ denotes the largest integer $\leq N/2$). This assumption is necessary for the asymptotic convergence of $e(k)$ to zero.

Factorize $B(z^{-1})$ as

$$B(z^{-1}) = B^+(z^{-1})B^-(z^{-1}) \tag{5}$$

where $B^+(z^{-1})$ and $B^-(z^{-1})$ are the cancelable and uncancelable parts of $B(z^{-1})$, respectively. Thus $B^-(z^{-1})$ comprises roots on or outside the unit circle and undesirable roots which are inside the unit circle, and $B^+(z^{-1})$ comprises roots of $B(z^{-1})$ which are not in $B^-(z^{-1})$ [13].

The design objective is to find a robust zero error tracking control law $u(k)$ which assures the asymptotic stability of overall system and the asymptotic zero error tracking: $\lim_{k\to\infty} e(k) = 0$.

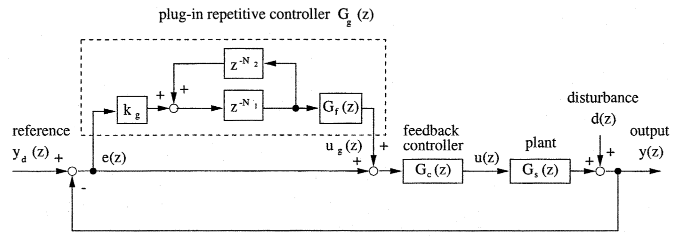



Fig. 2. "Plug-in" repetitive control system.

### C. Plug-In Repetitive Controller

*1) Controller Design:* Based on the Internal Model Principle, in order to achieve zero tracking error, the internal model for periodic signal is plugged into the control system as shown in Fig. 2 [6]. The plug-in part in Fig. 2 is called the "repetitive control." The transfer function of the plug-in repetitive controller $G_g(z)$ is

$$G_g(z) = \frac{k_g z^{-N_1}}{1-z^{-N}}\,G_f(z) = \frac{k_g z^{N_2}}{z^N-1}\,G_f(z) \tag{6}$$

where $k_g$ is the *control gain*; $G_f(z)$ is a low-pass filter.

For periodic ac reference inputs, $G_f(z)$ is usually chosen in the following form [6]:

$$G_f(z) = \frac{z^{-n_u}A(z^{-1})B^-(z)}{B^+(z^{-1})b} \tag{7}$$

where $N_2 = n_u + d$ with $d$ being the known number of pure time step delays; $B^-(z)$ is obtained from $B^-(z^{-1})$ with $z^{-1}$ replaced by $z$; $b$ is a scalar chosen so that $b \geq [B^-(1)]^2$; $n_u$ is the order of $B^-(z^{-1})$, and $z^{-n_u}$ makes the filter realizable. Equation (7) is a pole-zero cancellation or Zero-Phase compensation [14] as a filter design for $G_f(z)$.

For a dc reference input, its period $N$ is 1 with $N_1 = 1$ and $N_2 = 0$. $G_f(z)$ can be designed as (7). But in most cases, $G_f(z)$ for dc signal is simply chosen as follows $G_f(z) = 1$.

Therefore, the integral controller is also a special case of plug-in repetitive controllers (6). For a dc reference input, PID controller is one special case of the plug-in control scheme which includes a feedback PD controller and a plug-in integral controller.

*2) Stability Robustness:* In practice, model uncertainty exists in plants. Let $\Delta(z)$ be the modeling error for unmodeled dynamics. It is assumed that there exists a constant $\epsilon$ such that $\|\Delta(z)\| \leq \epsilon$. The relation between the true system transfer function $H_t(z)$ and the nominal system transfer function $H(z)$ can be written as

$$H_t(z) = H(z)(1+\Delta(z)) \tag{8}$$

where all poles of $H_t(z)$ are inside the unit circle.

From Fig. 2, the error transfer function for the overall system is

$$\begin{aligned} G_e(z) &= \frac{e(z)}{y_d(z)-d(z)} \\ &= \frac{1-z^{-N}}{1+G_c(z)G_s(z)} \cdot \frac{1}{1-z^{-N}(1-k_g z^{N_2}G_f(z)H_t(z))}. \end{aligned} \tag{9}$$

From (9), it can be concluded that the overall closed-loop system is stable if the following two conditions hold:

1) the roots of $1+G_c(z)G_s(z)=0$ are inside the unit circle;
2)

$$\|1-k_g z^{N_2} G_f(z) H_t(z)\| < 1, \quad \text{for all } z=e^{j\omega}, \quad \omega \in [0,\, \pi]. \tag{10}$$

In view of (5)–(8), it can be derived that

$$G_f(z)H_t(z) = \frac{z^{-N_2} B^-(z) B^-(z^{-1})}{b} + \Delta(z). \tag{11}$$

Then, (10) yields

$$0 < k_g < \frac{2}{\max \|B^-(z)B^-(z^{-1})/b + \Delta(z)\|} \le \frac{2}{1+\epsilon}. \tag{12}$$

Obviously, if the overall closed-loop system shown in Fig. 2 is asymptotically stable and the angular frequency $\omega$ of the reference input $y_d(t)$ and the disturbance $d(t)$ approaches $\omega_m = 2\pi m f$, $m = 0, 1, 2, \ldots, M$ [$M = N/2$ for even $N$ and $M = (N-1)/2$ for odd $N$], then $z^{-N} \to 1$, $\lim_{\omega \to \omega_m} \|G_e(j\omega)\| = 0$, and thus

$$\lim_{\omega \to \omega_m} \|e(j\omega)\| = 0. \tag{13}$$

Equation (13) indicates that, if the frequency of the reference input or disturbance is less than half of the sampling frequency, zero steady-state tracking errors for both dc and ac reference inputs are ensured using the plug-in controller $G_g(z)$ [10], even in the presence of modeling uncertainty.

### *D. Modified Repetitive Controller*

For the repetitive control system, the larger the control gain $k_g$ is, the faster the tracking error $e(z)$ converges. If the stability region for $k_g$ can be expanded beyond $(0,\, 2/(1+\epsilon))$, it will be more flexible to design system performance. In order to achieve such a goal, as shown in Fig. 3(a), a low-pass filter $Q(z,\, z^{-1})$ is added into $G_g(z)$ as follows [13]:

$$G_g(z) = \frac{k_g Q(z,\, z^{-1}) z^{-N_1}}{1 - Q(z,\, z^{-1}) z^{-N}} G_f(z) \tag{14}$$

where, with coefficients $\alpha_i$ ($i = 0, 1, \ldots, m$; $m = 0, 1, 2, \ldots$) to be designed

$$Q(z,\, z^{-1}) = \frac{\sum_{i=0}^{m} \alpha_i z^i + \sum_{i=1}^{m} \alpha_i z^{-i}}{2\sum_{i=1}^{m} \alpha_i + \alpha_0}. \tag{15}$$

And correspondingly, (10) is modified as follows [15]:

$$\|1-k_g z^{N_2} G_f(z) H_t(z)\| < \left\| \frac{1}{Q(z,\, z^{-1})} \right\| \quad \text{for all } z=e^{j\omega}, \quad 0 \le \omega \le \pi. \tag{16}$$

Note that $Q(z,\, z^{-1})$ is a moving average filter that has zero phase shift. Fig. 3(b) shows that the right-hand side of (14) for three choices of $Q(z,\, z^{-1})$ [16] and $1 \le \|1/Q(z,\, z^{-1})\|$. The stability region is substantially larger when $Q(z,\, z^{-1})$ is used.

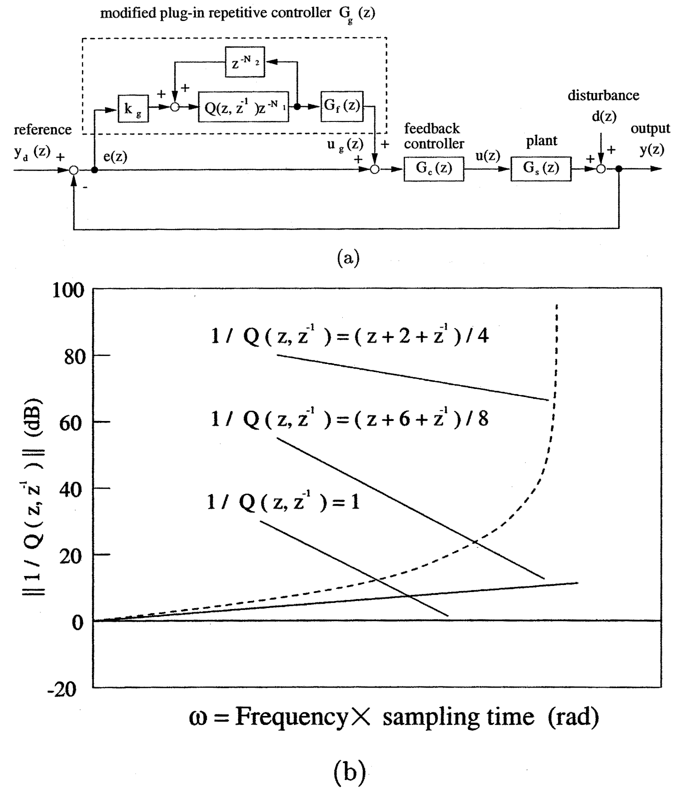



Fig. 3. Modified repetitive control system. (a) Modified repetitive controller. (b) Stability bounds for modified repetitive controller.

A first order filter $Q(z,\, z^{-1}) = (z+2+z^{-1})/4$ is generally sufficient in most cases. The repetitive controller with $Q(z,\, z^{-1})$ is named the "modified" repetitive controller. However, since high frequency periodic disturbances are not perfectly canceled by this controller, a trade-off is made between tracking precision and system robustness in this case [17].

### *E. Discussion*

In the proposed scheme, the plug-in repetitive controller and the feedback controller are complementary. The conventional feedback controller offers fast response and robustness. However, a feedback controller has no memory and will repeat, in all subsequent cycles, any imperfection existing in the performance of the feedback controller. Whereas, the plug-in repetitive controller is essentially an intelligent controller using previous information stored in memory, and ensures steady-state zero error tracking by repetitive learning. Because the plug-in controller adjusts its output once per period, it will take longer time to force the tracking error converge gradually. The proposed plug-in control scheme comprises of the advantages of both feedback controllers and plug-in repetitive controllers: the fast dynamic response of feedback controllers and the high precision tracking ability of repetitive controllers.

Since reference inputs for CVCF PWM converters are constant dc signals or CVCF sinusoidal ac signals, the plug-in digital repetitive plus a conventional feedback controller (e.g., PD controller) constructs the robust zero error tracking (RZET) control scheme for CVCF PWM converters. Since PD controller is simple, popular and effective for most control systems, it is

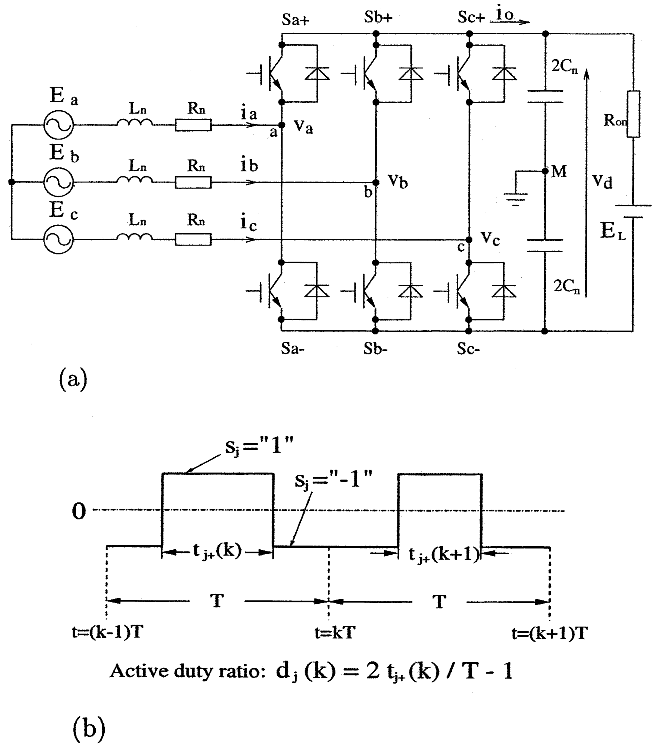



Fig. 4. PWM rectifier and waveform of switching function. (a) Three-phase reversible PWM rectifier with nominal parameters. (b) PWM switching waveform.

usually adopted as the conventional feedback controller in our proposed "plug-in" control scheme. And for the dc reference input, our proposed control scheme is PID control.

In the following section, the digital repetitive control scheme is applied to a three-phase reversible PWM rectifier to testify its validity and effectiveness. The PWM rectifier is a typical application example for digital repetitive control, because it requires high-precision periodic ac current and constant dc voltage tracking.

## III. Repetitive Controlled Three-Phase Reversible PWM Rectifier

The dynamics of the three-phase reversible PWM rectifier (as shown in Fig. 4) can be described as

$$\begin{pmatrix}\dot{i}_a\\ \dot{i}_b\\ \dot{i}_c\end{pmatrix} = \begin{pmatrix}-\dfrac{R_n}{L_n} & 0 & 0\\ 0 & -\dfrac{R_n}{L_n} & 0\\ 0 & 0 & -\dfrac{R_n}{L_n}\end{pmatrix}\begin{pmatrix}i_a\\ i_b\\ i_c\end{pmatrix} + \begin{pmatrix}\dfrac{1}{L_n}(E_a - v_a)\\ \dfrac{1}{L_n}(E_b - v_b)\\ \dfrac{1}{L_n}(E_c - v_c)\end{pmatrix} \quad (17)$$

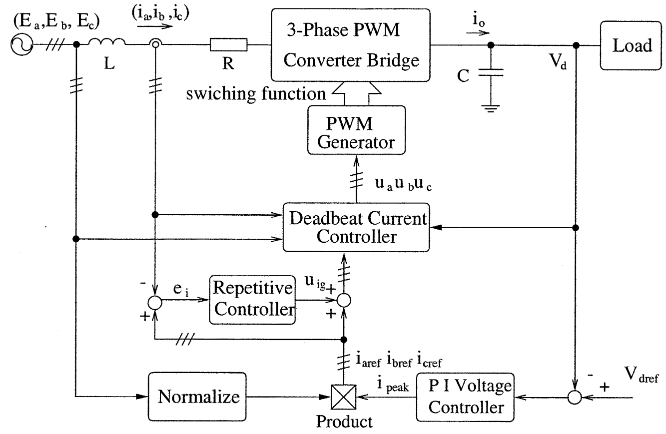



Fig. 5. Repetitive controlled three-phase reversible rectifier system.

and

$$\dot{v}_d = -\frac{1}{C_n R_{on}} v_d + \frac{1}{C} i_o + \frac{1}{C_n R_{on}} E_L \quad (18)$$

where the neutral point M of dc bus is referred as the ground; phase currents $i_a$, $i_b$, $i_c$ and dc bus voltage $v_d$ are the state variables; $v_a, v_b, v_c$ are the PWM modulated input voltages at port $a$, $b$, $c$; $i_o$ is the PWM modulated input current; $E_a$, $E_b$, $E_c$ are the known three-phase sinusoidal voltages, and $E_a + E_b + E_c = 0$; $L_n, C_n, R_n$ and $R_{on}$ are the nominal values of the components; $E_L$ is the $emf$ of the load.

The output equation can be expressed as

$$y = [i_a \quad i_b \quad i_c \quad v_d]^T. \quad (19)$$

On the instantaneous basis, the PWM modulated control inputs $v_j(t)$ ($j = a, b, c$) and $i_o$ can be written as

$$v_j(t) = \tfrac{1}{2} v_d(t) s_j \quad (20)$$

$$i_o = i_a s_a + i_b s_b + i_c s_c \quad (21)$$

where $s_j$ ($j = a, b, c$) are switching functions. For a two-level PWM modulator, the switching functions $s_j$ ($j = a, b, c$) are defined as: $s_j = +1$, when the switch $S_{j+}$ is on and the switch $S_{j-}$ is off; $s_j = -1$, when the switch $S_{j-}$ is on and the switch $S_{j+}$ is off. As shown in Fig. 4, each PWM switching waveform at port $j$ ($j = a, b, c$) is a pulse of magnitude "+1" with width being $t_{j+}(k)$ in the sampling interval $T$ and the active duty ratio $d_j(k)$ ($j = a, b, c$) ($-1 \le d_j(k) \le 1$) being $d_j(k) = (t_{j+}(k) - (T - t_{j+}(k)))/T = ((2t_{j+}(k))/T) - 1$.

The objective for the PWM rectifier system with a RC controller is to achieve unit power factor, low current harmonics and constant output dc voltage.

### A. Controller Design

As shown in Fig. 5, a control scheme of double-loop structure is proposed for the PWM rectifier: an inner ac current loop; and an outer dc voltage loop.

*1) Current-Loop Controller:* The corresponding sampled-data state equations for (17) can be expressed as

$$\begin{pmatrix} i_a(k+1) \\ i_b(k+1) \\ i_c(k+1) \end{pmatrix} = \begin{pmatrix} \dfrac{b_1-b_2}{b_1} & 0 & 0 \\ 0 & \dfrac{b_1-b_2}{b_1} & 0 \\ 0 & 0 & \dfrac{b_1-b_2}{b_1} \end{pmatrix} \begin{pmatrix} i_a(k) \\ i_b(k) \\ i_c(k) \end{pmatrix} + \begin{pmatrix} \dfrac{1}{b_1} E_a(k) - \dfrac{v_d(k)}{2} \dfrac{u_a(k)}{b_1} \\ \dfrac{1}{b_1} E_b(k) - \dfrac{v_d(k)}{2} \dfrac{u_b(k)}{b_1} \\ \dfrac{1}{b_1} E_c(k) - \dfrac{v_d(k)}{2} \dfrac{u_c(k)}{b_1} \end{pmatrix} \quad (22)$$

where $T$ is the sampling period; $u_j(k) = d_j(k) = ((2t_{j+}(k) - T))/T$ ($j = a, b, c$); $b_1 = L_n/T$; $b_2 = R_n$; $L_n$ and $R_n$ are the nominal value of $L$ and $R$, respectively.

It is clear that (22) can be treated as three independent phase subsystems. The current-loop control scheme for each phase subsystem is shown in Fig. 6. From (22), the sampled-data equation for each phase subsystem with nominal parameters is

$$i_j(k+1) = \frac{b_1-b_2}{b_1} i_j(k) + \frac{1}{b_1} E_j(k) - \frac{v_d(k)}{2} \frac{1}{b_1} u_j(k). \quad (23)$$

Then the nominal transfer function $G_s(z)$ for each phase subsystem is

$$G_s(z) = \frac{i_j(z)}{u_j(z)} = -\frac{v_d(z)}{2} \frac{1}{b_1 z - b_1 + b_2}. \quad (24)$$

If the current controller is chosen for the plant (23) as

$$u_j(k) = \frac{2}{v_d(k)} \left[E_j(k) - b_1 i_{jref}(k) + (b_1 - b_2) i_j(k)\right] \quad (25)$$

$i_j(k+1) = i_{jref}(k)$ is obtained. Then the transfer function for each phase current-loop control system without repetitive controller is $H(z) = z^{-1}$. Equation (25) is a *deadbeat controller* which is referred as the *predictive controller* in [18], [19]. The deadbeat controller offers fast response with only one sampling period $T$ delay.

However, the deadbeat current controller is based on an accurate nominal model of the reversible PWM rectifier. In practice, there are uncertainties in the converter parameters, such as $\Delta L = L - L_n$, $\Delta C = C - C_n$ and $\Delta R = R - R_n$. Therefore, even with phase angles feedforward compensation, zero tracking phase error can not be achieved. Based on the repetitive control theory in Section II, a repetitive controller is proposed as follows for the reversible rectifier to overcome the uncertainties and improve the current tracking

$$G_{gc}(z) = \frac{k_g z^{-N+1} Q(z, z^{-1})}{1 - Q(z, z^{-1}) z^{-N}} \quad (26)$$

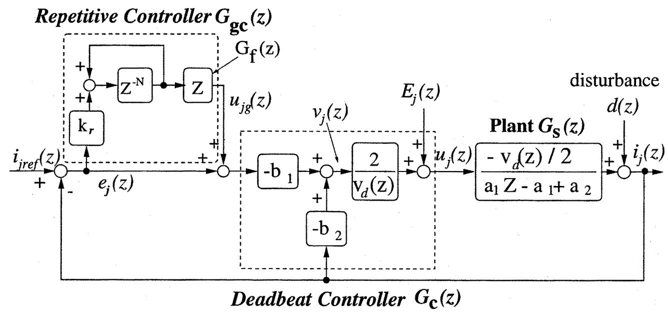



Fig. 6. Current-loop control scheme.

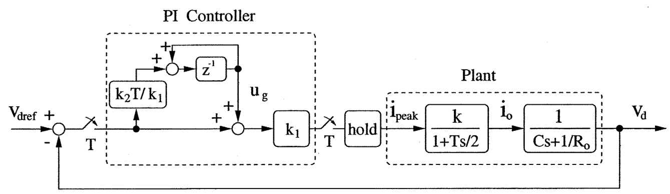



Fig. 7. Voltage-loop control scheme.

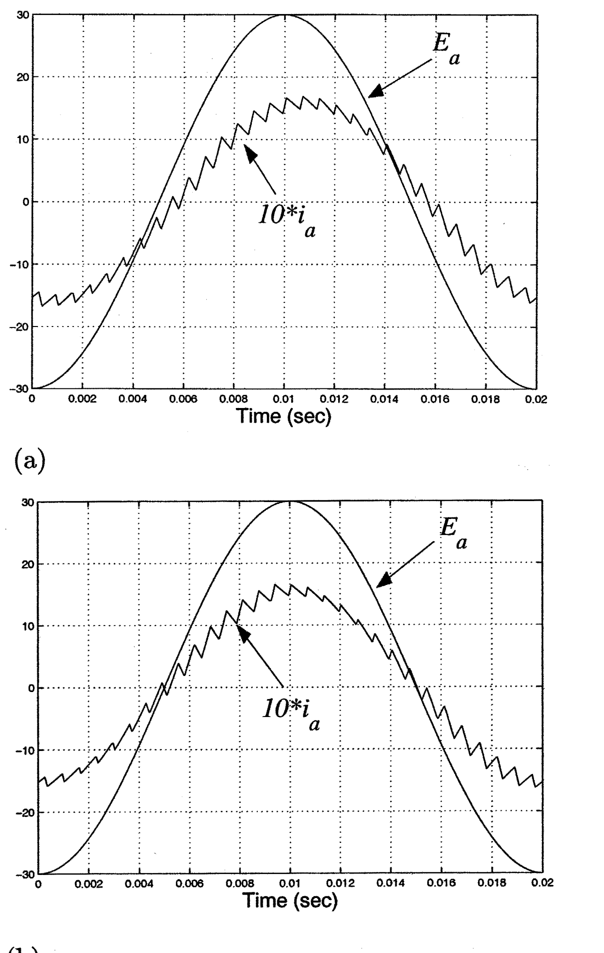



Fig. 8. Simulation steady-state ac-side response under load 100 $\Omega$ (Vertical: 10 V/div for $E_a$, 10 A/div for $i_a$). (a) Deadbeat controlled $E_a$ and $i_a$. (b) Deadbeat plus repetitive controlled $E_a$ and $i_a$.

where $N = f_c/f$; $f$ is the frequency of $E_j(t)$ ($j = a, b, c$); $f_c = 1/T$ is the sampling frequency; $G_f = 1/H(z) = z$. In order to enhance the robustness, $Q(z, z^{-1})$ can chosen as

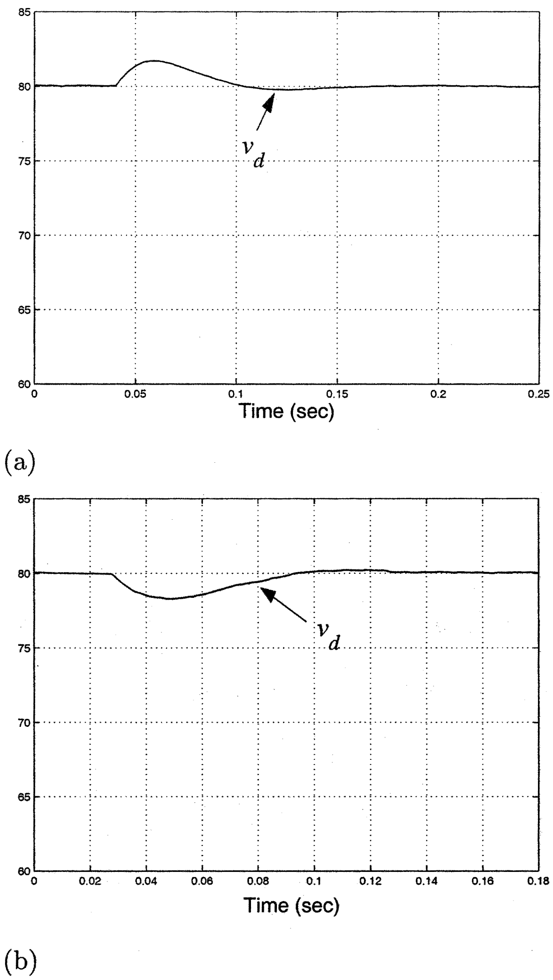



Fig. 9. Simulated transient response under load change: (a) dc bus voltage $v_d(t)$ under load change from 50 Ω to 100 Ω (Vertical: 5 V/div) and (b) dc bus voltage $v_d(t)$ under load change from 100 Ω to 50 Ω (Vertical: 5 V/div).

$\alpha_1 z + \alpha_0 + \alpha_1 z^{-1}$. If $Q(z, z^{-1}) = 1$, in sampled-data form, the repetitive controller can be expressed as $u_{ig}(k) = u_r(k - N) + k_g e(k - N + 1)$, which is the same as the anticipatory learning control law [20].

In practice, converter parameters are $L = L_n + \Delta L$, $R = R_n + \Delta R$. Therefore, the transfer function $G_s(z)$ for each actual subsystem is, with $a_1 = L/T$, $a_2 = R$

$$G_s(z) = -\frac{v_d(z)}{2}\,\frac{1}{a_1 z - a_1 + a_2}. \tag{27}$$

Therefore, as shown in Fig. 6, when a deadbeat controller (25) is designed for the plant (27), the transfer function $H(z)$ for the subsystem without repetitive controller is

$$H_t(z) = \frac{b_1}{a_1 z - (a_1 - b_1) + (a_2 - b_2)}. \tag{28}$$

When $L = L_n$ and $R = R_n$, a deadbeat response $H_t(z) = z^{-1}$ is achieved. According to the stability analysis in Section II, the overall system is stable if $\|((a_1 - b_1) - (a_2 - b_2))/a_1\| < 1$ and $\|1 - k_g z H_t(z)\| < 1$.

*2) Voltage-Loop Controller:* From (21) and Fig. 5, we have $i_o = i_a s_a + i_b s_b + i_c s_c = i_{peak}(\sin\theta_1 s_a + \sin\theta_2 s_b + \sin\theta_3 s_c)$.

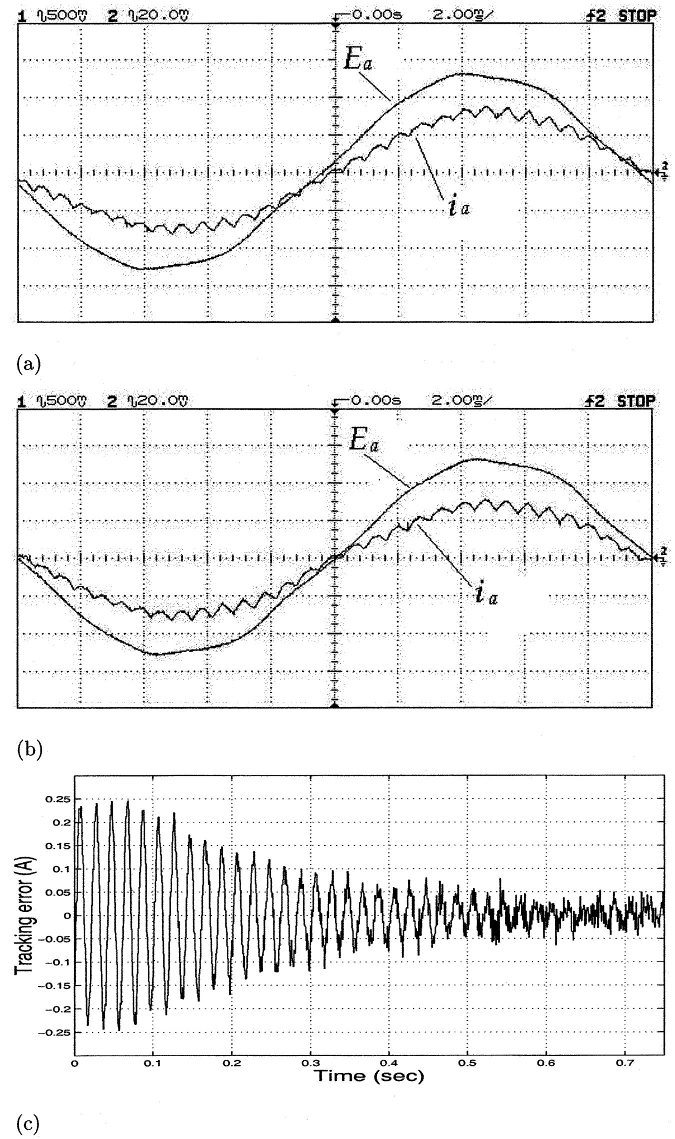



Fig. 10. Experimental ac-side current response under load 100 Ω (Voltage: 12 V/div; Current: 1 A/div). (a) Deadbeat controlled $E_a$ and $i_a$. (b) Deadbeat plus repetitive controlled $E_a$ and $i_a$. (c) Transient current tracking error $e(t) = i_{aref} - i_a$ when the repetitive controller is plugged in.

The transfer function from $i_{peak}$ to $i_o$ can be approximately treated as

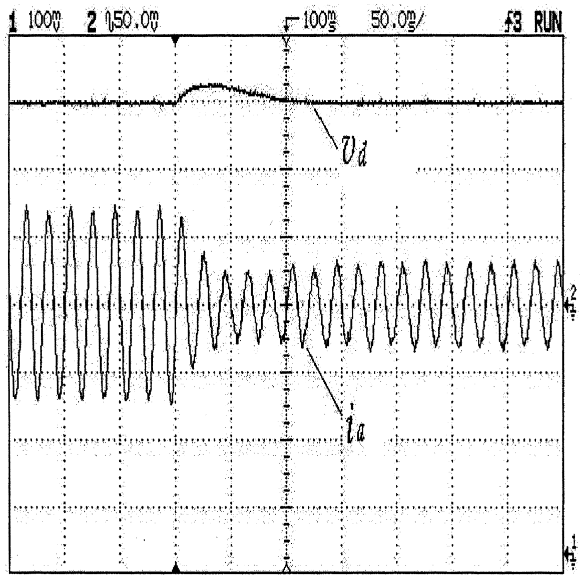
$$\frac{i_o(s)}{i_{peak}(s)} = \frac{k}{1 + \frac{T}{2}s} \tag{29}$$

where $-3 \le k \le 3$. From (18), the transfer function from $i_o$ to $v_d$ can be obtained as

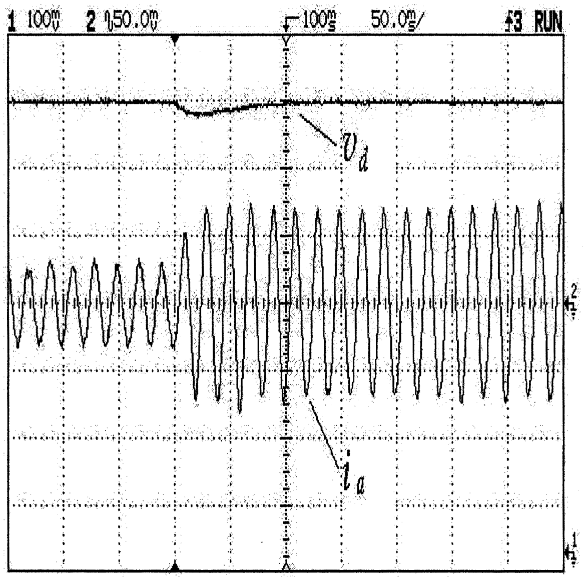
$$\frac{v_d(s)}{i_o(s)} = \frac{1}{Cs + 1/R_o}. \tag{30}$$

If the pole in (29) is far away from the origin compared to that in (30), i.e., $-(2/T) \gg 1/CR_o$, the transfer function from $i_{peak}$ to $v_d$ can be simplified as

$$\frac{v_d(s)}{i_{peak}(s)} = \frac{1}{Cs + 1/R_o}. \tag{31}$$




(a)




(b)

Fig. 11. Experimental response under sudden load change (Voltage: 12 V/div; Current: 2.5 A/div): (a) dc bus voltage $v_d(t)$ and current $i_a(t)$ under load change from 50 $\Omega$ to 100 $\Omega$ and (b) dc bus voltage $v_d(t)$ and current $i_a(t)$ under load change from 100 $\Omega$ to 50 $\Omega$.

Moreover, according to the principle of energy balance [11], the steady-state $i_{peak}$ can be approximately derived as

$$i_{peak} = \frac{2}{R_o}\frac{v_{dref}^2}{E_{apeak} + E_{bpeak} + E_{cpeak}} \tag{32}$$

where $E_{apeak}$, $E_{apeak}$ and $E_{apeak}$ are the peak values of $E_a$, $E_b$ and $E_c$, respectively.

Since $i_o = v_d/R_o$, steady-state $k$ can calculated as

$$k = \frac{v_{dref}}{i_{peak}R_o}. \tag{33}$$

Because $v_{dref}$ is a dc signal, a zero tracking controller—PI controller is employed as

$$G_v(z) = k_1 + k_2\frac{T}{z-1} \tag{34}$$

where gains $k_1$ and $k_2$ are designed to ensure a stable system with a satisfactory dynamic response.

The voltage-loop control subsystem is shown in Fig. 7. It is easy to analyze the stability of the closed voltage-loop control system using *Routh–Hurwitz* Criterion.

## IV. Simulation and Experiment

A three-phase reversible PWM rectifier system with repetitive learning controller, shown in Fig. 5, has been implemented with an experiment setup.

For both simulation and experiment, system parameters are selected as follows: $L_n = 15$ mH; $L = 19$ mH; $R_n = 0.5\ \Omega$; $R = 1.0\ \Omega$; $C = 4700\ \mu$F; $v_{dref} = 80$ V; three-phase sinusoidal voltages $E_j\ (j = a, b, c)$ are 50 Hz 30 V (peak); $f_c = 1/T = 1.5$ KHz; $N = f_c/f = 30$; $R_{on} = 100\ \Omega$, $E_L = 0$, $\alpha_0 = 0.95$, $\alpha_1 = 0.025$.

With these parameter values, the pole of $H_t(z)$ is obtained as 0.19 (inside the unit circle) and thus the system is stable. The maximum value of $\|zH_t(z)\|$ in frequency domain is 0.9786. According to the stability condition $\|1 - k_g zH_t(z)\| < 1$ for repetitive control design, the system with repetitive controller is stable if $k_g \in (0, 2.04)$. We choose $k_g = 0.2$.

Since $-(2/T) = 3000 \gg 1/CRon = 2.1$, (31) can be approximately written as $v_d(s)/i_{peak}(s) = 1/(0.0047s + 0.01)$. From (32) and (33), the steady-state $k$ is equal to 0.56. And the transfer function from $V_{dref}$ to $V_d$ can be approximately expressed as

$$\frac{v_d(s)}{v_{dref}(s)} = \frac{0.56k_1 s + 0.56k_2}{0.0047s^2 + (0.01 + 0.56k_1)s + 0.56k_2}. \tag{35}$$

Based on the *Routh–Hurwitz* criterion, the system of (35) is stable if $0.01 + 0.56k_1 > 0$ and $0.56k_2 > 0$. Equation (35) is a second-order system. When the damping ratio of (35) is equal to 0.4, we have $k_1 = 0.5$ and $k_2 = 50$.

Fig. 8 shows the $a$-phase simulation steady-state current response of the reversible PWM rectifier with/without the repetitive current controller. As shown in Fig. 8(a), the zero crossing points of both phase voltage $E_a(t)$ and phase current $i_a(t)$ show that there is a lag between $E_a(t)$ and $i_a(t)$ for the deadbeat controlled rectifier. Fig. 8(b) shows that the repetitive controller force the phase displacement between $E_a(t)$ and $i_a(t)$ approaches zero successfully. Therefore, the power factor approaches unity. The response of other phase currents are similar to that of the $a$-phase subsystem.

Fig. 9 shows the simulation dc bus voltage response with sudden load changes between 50 $\Omega$ and 100 $\Omega$. Both cases show that $v_d(t)$ returns to the reference value 80 V after about 120 ms. With PI controller, $v_d(t)$ can track the reference dc voltage with near zero error.

Fig. 10 shows the $a$-phase experimental current response of the reversible PWM rectifier with/without the plug-in repetitive current controller. Obviously, Fig. 10(a) and (b) show that repetitive controller reduces the phase displacement between $E_a(t)$ and $i_a(t)$ to almost zero. And Fig. 10(c) shows that the peak of current tracking error is reduced from 0.25 A to about 0.04 A after 0.7 s. Therefore, the power factor approaches unity. The response of other phase currents are similar to that of the $a$-phase subsystem.

Fig. 11 shows one experimental system response with sudden load changes between 50 $\Omega$ and 100 $\Omega$. In both cases, after about 110 ms, $v_d(t)$ returns to the reference value 80 V. With PI controller, $v_d(t)$ can track the reference dc voltage with near zero error.

## V. Conclusion

In this paper, a digital repetitive control scheme is proposed for CVCF PWM converters to achieve zero error tracking. As an example, the proposed digital repetitive control scheme is applied to a three-phase PWM rectifier. Simulation and experiment results show that the tracking errors in ac current loop and dc voltage loop are quickly eliminated by the proposed digital control scheme. The digital repetitive controlled PWM rectifier has a near unit power factor and constant dc output voltage under disturbances and parameter uncertainties while maintaining good response characteristics. Simulation and experimental results verify the effectiveness of the proposed control scheme.

The developments and results for a three-phase rectifier in this paper, a single-phase inverter in [10], and a three-phase inverter in [12] show that the proposed control scheme is a robust zero tracking error control scheme for CVCF PWM converters. The proposed digital repetitive control scheme provides a simple and high-performance control solution for CVCF PWM converters.

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