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Brief Paper

Sampled-data iterative learning control for nonlinear systems with arbitrary relative degree☆

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Abstract

In this paper, a sampled-data iterative learning control method is proposed for nonlinear systems without restriction on system relative degree. The learning algorithm does not require numerical differentiations of any order from the tracking error. A sufficient condition is derived to guarantee the convergence of the system output at each sampling instant to the desired trajectory. Numerical simulation is conducted to demonstrate the theoretical result.

Keywords: Learning control; Convergence; Relative degree; Sampled-data; Nonlinear systems

1. Introduction

Iterative learning control theory can be traced back to the midst of the 1980s. It attracted considerable attention largely due to applications in motion control of robotic manipulators. Many researchers focused their attention on ILC using differentiation, including D-type ILC and its variations. The explicit sufficient conditions guaranteeing the convergence and robustness have been derived (Arimoto, Kawamura & Miyazaki, 1984; Hauser, 1987; Moore, 1993; Bien & Xu, 1998; Chen & Wen, 1999; Sun & Huang, 1999). The fundamental characteristics of such learning algorithms have been examined for systems with direct transmission term (Sugie & Ono, 1991) and higher relative degree (Ahn, Choi & Kim, 1993). Especially, in Ahn et al. (1993), it was shown that ILC using differentiation can be applied to the systems with higher relative degree, in which the highest order of the error derivatives is equal to the relative degree. Numerical methods might be applied to obtain the error derivatives in the practical implementation of an ILC using differentiation. If the system output is contaminated with measurement noises, the numerical differentiation can be a source of severer noises.

In order to avoid the non causal operation, ILC without using differentiation was proposed by Kawamura, Miyazaki and Arimoto (1988), which is now referred to as P-type ILC. Several technical analyses of P-type ILC were presented for nonlinear continuous-time systems by imposing somewhat strict requirement on system dynamics, for example, the passivity property (Arimoto, Naniwa & Suzuki, 1991) and the boundedness of the input–output coupling matrix (Kuc, Lee & Nam, 1992; Saab, 1994; Chien & Liu, 1996). By these analysis techniques, it seems difficult to prove convergence of P-type ILC for nonlinear systems with higher relative degree. Recently, in Wang (2000), a fundamental concept was introduced in parallel to the P- and D-type ILCs, and a simple learning algorithm was proposed for easy implementation. It was shown that sampled-data ILC is one such particular form. Results have been developed for systems with relative degree one. In this paper, this approach is extended to the systems with any relative degree.

From the practical point of view, it is more convenient and direct to apply the digital control techniques for ILC design. Up to now, relative few studies dealt with sampled-data ILC problem (Tso & Ma, 1993; Cheah, Wang & Soh, 1994; Chien, 1997; Park, Bien & Hwang, 1998). In Tso and Ma (1993) and Cheah et al. (1994), the sampled-data learning algorithms based on the approximation of acceleration of the continuous cases were proposed for robotic manipulators. In Chien (1997), a sampled-data ILC was investigated for a class of nonlinear systems and a case study was also presented for the systems with zero

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input–output coupling matrix. It was shown that the learning processes can be done without using any differentiation of the output error. In Park et al. (1998), a learning algorithm adopting the holding mechanism is applied to LTI systems without restriction on relative degree. However, the existing results do not offer a way of converting the theories for nonlinear continuous-time systems with arbitrary relative degree to those for the sampled-data counterpart. Sampled-data ILC for nonlinear systems without restriction on system relative degree is still an open problem.

This paper aims to apply sampled-data iterative learning control methodology to a class of nonlinear systems with arbitrary relative degree. For this purpose, extended relative degree of nonlinear systems with a zero-order hold device is defined and a simple learning algorithm is developed using output measurements only. Convergence proof is provided by a new analysis technique instead of conventional λ-norm. It is shown that the proposed learning algorithm is applicable to a class of nonlinear continuous-time systems that most of the existing ILC theories fail to work, including D- and P-type ILCs.

2. Problem formulation and preliminaries

A class of nonlinear continuous-time systems with linear input action is described by

$$\dot{x}(t) = f(x(t)) + B(x(t))u(t), \tag{1}$$

$$y(t) = g(x(t)), \tag{2}$$

where $x \in R^n$, $u \in R^r$, and $y \in R^m$ denote the state, the control input, and the output of the system respectively. The functions $f(\cdot) \in R^n$, $B(\cdot) = [b_1(\cdot), \ldots, b_r(\cdot)] \in R^{n\times r}$, and $g(\cdot) = [g_1(\cdot), \ldots, g_m(\cdot)]^{\mathrm{T}} \in R^m$ are smooth in their domain of definition, which are known about certain properties only. The system performs repeated operation and every operation ends in a finite time T, i.e. $t \in [0, T]$. For each fixed $x(0)$, S denotes a mapping from $(x(0), u(t), t\in[0,T])$ to $(x(t), t\in[0,T])$, and O a mapping from $(x(0), u(t), t\in[0,T])$ to $(y(t), t\in[0,T])$. In these notations, $x(\cdot) = S(x(0), u(\cdot))$ and $y(\cdot) = O(x(0), u(\cdot))$.

Due to applying digital control techniques, the operation interval $[0, T]$ is divided into N equal intervals with a chosen increment h. Then $h = T/N$ is the sampling period and $t = jh$, $0 \le j \le N$ are the sampling instants. The control input is generated using a zero-order hold device. Namely, the control input is taken piecewise constant between the sampling instants,

$$u(t) = u(jh), \quad t \in [jh, jh+h), \quad 0 \le j \le N-1. \tag{3}$$

The control problem to be solved in this paper is formulated as follows. Given a realizable trajectory $y_d(jh)$, $0 \le j \le N$ and a tolerance error bound $\varepsilon > 0$, find a control input $u(t)$ described by (3) so that the error between the output $y(t)$ of system (1) and (2) at the sampling instants and the desired trajectory $y_d(jh)$ is with the tolerance error bound, i.e. $\|y_d(jh) - y(jh)\| < \varepsilon$, $0 \le j \le N$, where $\|\cdot\|$ is the vector norm defined as $\|x\| = \max_{1\le i\le n}|x_i|$ for an n-dimensional vector $x = [x_1, \ldots, x_n]^{\mathrm{T}}$. Throughout the paper, for a matrix $A = \{a_{ij}\} \in R^{m\times n}$, the induced norm $\|A\| = \max_{1\le i\le m}\sum_{j=1}^{n}|a_{ij}|$ is used. For an appropriate initial condition, the realizability of a given trajectory $y_d(jh)$, $0 \le j \le N$ refers to that there exist a continuous-time trajectory $y_d(t)$, satisfying $y_d(jh) = y_d(t)|_{t=jh}$, and a control input described by (3) to generate the continuous-time trajectory.

To describe the input–output causal relationship of system (1)–(3), we introduce the following definition.

Definition 2.1. System (1)–(3) has extended relative degree $\{\eta_1, \ldots, \eta_m\}$ for $x(t) \in R^n$, $t \in [0, T]$, if, for $0 \le j \le N-1$,

$$\int_{jh}^{jh+h} L_{b_p} g_q(x(t_1))\,\mathrm{d}t_1 = 0,$$

$$\int_{jh}^{jh+h}\int_{jh}^{t_1}\cdots\int_{jh}^{t_i} L_{b_p} L_f^i g_q(x(t_{i+1}))\,\mathrm{d}t_{i+1}\ldots\mathrm{d}t_1 = 0,$$

$$1 \le i \le \eta_q - 2,$$

where $1 \le p \le r$, $1 \le q \le m$ and the $m \times r$ matrix

$$\begin{bmatrix} \int_{jh}^{jh+h}\int_{jh}^{t_1}\cdots\int_{jh}^{t_{\eta_1-1}}[L_{b_1}L_f^{\eta_1-1}g_1(x(t_{\eta_1})), \ldots, L_{b_r}L_f^{\eta_1-1}g_1(x(t_{\eta_1}))]\,\mathrm{d}t_{\eta_1}\ldots\mathrm{d}t_1 \\ \vdots \\ \int_{jh}^{jh+h}\int_{jh}^{t_1}\cdots\int_{jh}^{t_{\eta_m-1}}[L_{b_1}L_f^{\eta_m-1}g_m(x(t_{\eta_m})), \ldots, L_{b_r}L_f^{\eta_m-1}g_m(x(t_{\eta_m}))]\,\mathrm{d}t_{\eta_m}\ldots\mathrm{d}t_1 \end{bmatrix}$$

is of full-column rank.

Remark 1. If system (1)–(3) has extended relative degree $\{\eta_1, \ldots, \eta_m\}$, the component of the system output $y(t) = [y_1(t), \ldots, y_m(t)]^{\mathrm{T}}$ at the sampling instant $jh + h$ is directly affected by at least one component of the control input at jh. Namely, for $0 \le j \le N-1$, $1 \le q \le m$,

$$\begin{aligned} y_q(jh+h) &= g_q(x(jh)) + hL_f g_q(x(jh)) + \cdots \\ &\quad + \frac{h^{\eta_q-1}}{(\eta_q-1)!}L_f^{\eta_q-1}g_q(x(jh)) \\ &\quad + \int_{jh}^{jh+h}\int_{jh}^{t_1}\cdots\int_{jh}^{t_{\eta_q-1}} L_f^{\eta_q}g_q(x(t_{\eta_q}))\,\mathrm{d}t_{\eta_q}\ldots\mathrm{d}t_1 \end{aligned}$$

$$+\int_{jh}^{jh+h}\int_{jh}^{t_1}\cdots\int_{jh}^{t_{\eta_q-1}}[L_{b_1}L_f^{\eta_q-1}g_q(x(t_{\eta_q})),\ldots,$$
$$L_{b_r}L_f^{\eta_q-1}g_q(x(t_{\eta_q}))]\,\mathrm{d}t_{\eta_q}\ldots\mathrm{d}t_1\,u(jh). \quad (4)$$

It indicates that the output $y(jh+h)$ is due to the control action $u(jh)$. Thus, $\{u(jh), y_q(jh+h), 1\le q\le m\}$ is one causal pair.

Remark 2. Definition 2.1 allows that $L_{b_p}L_f^{\eta_q-1}g_q(x)=0$, $1\le p\le r$, and/or $L_{b_p}L_f^{\eta_q-1}g_q(x)=0$, $1\le q\le m$ for some states, in contrast to the nonzero requirement in Ahn et al. (1993) and Isidori (1995). And the number of outputs can be greater than the number of inputs.

Also the following properties for system (1)–(3) are assumed:

(A1) The mappings S and O are one to one.
(A2) The system has extended relative degree $\{\eta_1,\ldots,\eta_m\}$ for all $x(t)\in R^n$, $t\in[0,T]$.
(A3) The functions $f(\cdot), B(\cdot)$, $L_f^i g_q(\cdot)$, $0\le i\le\eta_q$, $1\le q\le m$ and $L_{b_p}L_f^{\eta_q-1}g_q(\cdot)$, $1\le p\le r$, $1\le q\le m$ are uniformly globally Lipschitz in x on the interval $[0,T]$. The Lipishitz constants are denoted by l_f, l_b, l_{fg} and l_{bfg}, respectively.
(A4) The operator $B(\cdot)$ is bounded on R^n.

Given a realizable trajectory $y_d(jh)$, $0\le j\le N$ for system (1)–(3) with extended relative degree $\{\eta_1,\ldots,\eta_m\}$, there exists a continuous-time trajectory $y_d(t)=[y_{1,d}(t),\ldots,y_{m,d}(t)]^{\mathrm{T}}$, $t\in[0,T]$ where $y_{q,d}(t)$ is η_q times continuously differentiable for $1\le q\le m$. (A1) implies that there exists a unique control input $u_d(t)$, satisfying $u_d(t)=u_d(jh)$, $t\in[jh,jh+h)$, $0\le j\le N-1$, which drives the system output to follow the continuous-time trajectory so that

$$y_d(t)=g(x_d(t)), \quad (5)$$

$$\dot{x}_d(t)=f(x_d(t))+B(x_d(t))u_d(t), \quad (6)$$

where $x_d(t)$ is the resultant state.

3. Sampled-data ILC and convergence analysis

To the learning control design, the next input action is updated based on the action and its produced result in the previous operation cycle. In view of (4), $\{u(jh), y(jh+h)\}$ is a pair of algebraically related cause and effect. This observation is lent to the updating law

$$u_{k+1}(jh)=u_k(jh)+\Phi_k(jh)e_k(jh+h), \quad 0\le j\le N-1, \quad (7)$$

where k indicates the number of operation cycle and $e_k(jh+h)=y_d(jh+h)-y_k(jh+h)$ is the output error, or the tracking error, and $\Phi_k(jh)\in R^{r\times m}$ is the learning gain matrix piecewise continuous and bounded for $0\le j\le N-1$ and for all k. The sampled-data iterative learning control system is illustrated in Fig. 1.

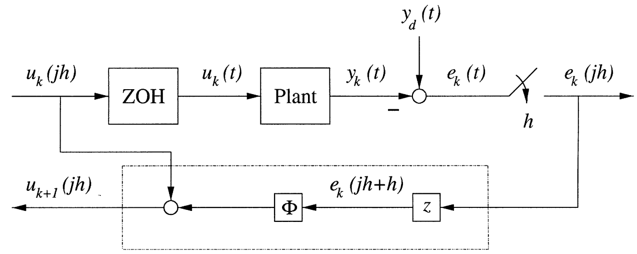

Fig. 1. Sampled-data iterative learning control system.

We need the following lemma to aid the presentation of our main result.

Lemma 3.1. *Let $\{a_k\}$ be a real sequence defined as*

$$a_k\le\rho a_{k-1}+b_k, \quad 0\le\rho<1,$$

where b_k is a specified real sequence. Then $\lim_{k\to\infty}b_k=b_\infty$ implies that $\limsup_{k\to\infty}a_k\le b_\infty/(1-\rho)$.

Proof. Defining $s_k=a_k-b_\infty/(1-\rho)$ and $t_k=b_k-b_\infty$ leads to

$$s_k\le\rho s_{k-1}+t_k,$$

where t_k converges to zero as $k\to\infty$. Therefore, there exists a positive integer K, for any given $\varepsilon>0$, such that $|t_k|<\varepsilon$ as $k>K$. Then, for $k>K$,

$$s_k<\rho^k s_0+\frac{1-\rho^k}{1-\rho}\varepsilon.$$

Since $0\le\rho<1$ and ε can be arbitrary value, $\limsup_{k\to\infty}s_k\le 0$. The lemma follows. □

When the updating law (7) is applied to system (1)–(3) which has extended relative degrees $\{\eta_1,\ldots,\eta_m\}$, the convergence result is summarized in the following theorem.

Theorem 3.1. *Given a realizable trajectory $y_d(jh)$, $0\le j\le N$ with the sampling period h being small enough, let system (1)–(3) satisfy assumptions (A1)–(A4), and the updating law (7) be applied. The control input error $u_d(jh)-u_k(jh)$ for $0\le j\le N-1$, the state error $x_d(jh)-x_k(jh)$ and the output error $y_d(jh)-y_k(jh)$ for $0\le j\le N$ converge to zero as $k\to\infty$, if $x_k(0)=x_d(0)$ for all k, and the learning gain matrix $\Phi_k(jh)$ is designed such that, for $0\le j\le N-1$ and for all k,*

$$\|I-\Phi_k(jh)D_k(jh)\|\le\rho<1, \quad (8)$$

where

$$D_k(jh) = \begin{bmatrix} \int_{jh}^{jh+h}\int_{jh}^{t_1}\cdots\int_{jh}^{t_{\eta_1-1}}[L_{b_1}L_f^{\eta_1-1}g_1(x_k(t_{\eta_1})),\ldots,L_{b_r}L_f^{\eta_1-1}g_1(x_k(t_{\eta_1}))]\,\mathrm{d}t_{\eta_1}\ldots\mathrm{d}t_1 \\ \vdots \\ \int_{jh}^{jh+h}\int_{jh}^{t_1}\cdots\int_{jh}^{t_{\eta_m-1}}[L_{b_1}L_f^{\eta_m-1}g_m(x_k(t_{\eta_m})),\ldots,L_{b_r}L_f^{\eta_m-1}g_m(x_k(t_{\eta_m}))]\,\mathrm{d}t_{\eta_m}\ldots\mathrm{d}t_1 \end{bmatrix}.$$

Proof. Define $\Delta u_k(\cdot) = u_d(\cdot)$ and $\Delta x_k(\cdot) = x_d(\cdot) - x_k(\cdot)$. It follows from (4) to (7) that

$$\Delta u_{k+1}(jh) = (I - \Phi_k(jh)D_k(jh))\Delta u_k(jh) - \Phi_k(jh)(\omega_k(jh) + v_k(jh)),$$

where

$$\omega_k(jh) = [\omega_{1,k}(jh),\ldots,\omega_{m,k}(jh)]^{\mathrm{T}},$$

$$v_k(jh) = [v_{1,k}(jh),\ldots,v_{m,k}(jh)]^{\mathrm{T}},$$

and

$$\omega_{q,k}(jh) = g_q(x_d(jh)) - g_q(x_k(jh)) + h[L_f g_q(x_d(jh)) - L_f g_q(x_k(jh))] + \ldots + \frac{h^{\eta_q-1}}{(\eta_q-1)!}[L_f^{\eta_q-1}g_q(x_d(jh)) - L_f^{\eta_q-1}g_q(x_k(jh))],$$

$$v_{q,k}(jh) = \int_{jh}^{jh+h}\int_{jh}^{t_1}\cdots\int_{jh}^{t_{\eta_q-1}}(L_f^{\eta_q}g_q(x_d(t_{\eta_q})) - L_f^{\eta_q}g_q(x_k(t_{\eta_q})))\,\mathrm{d}t_{\eta_q}\ldots\mathrm{d}t_1 + \int_{jh}^{jh+h}\int_{jh}^{t_1}\cdots\int_{jh}^{t_{\eta_q-1}}[L_{b_1}L_f^{\eta_q-1}g_q(x_d(t_{\eta_q})) - L_{b_1}L_f^{\eta_q-1}g_q(x_k(t_{\eta_q})),\ldots,L_{b_r}L_f^{\eta_q-1}g_q(x_d(t_{\eta_q})) - L_{b_r}L_f^{\eta_q-1}g_q(x_k(t_{\eta_q}))]\,\mathrm{d}t_{\eta_q}\ldots\mathrm{d}t_1\, u_d(jh),$$

where $1 \le q \le m$. Taking the norms and applying the bounds and the Lipschitz conditions, we have that

$$\|\Delta u_{k+1}(jh)\| \le \rho\|\Delta u_k(jh)\| + c_\Phi(\|\omega_k(jh)\| + \|v_k(jh)\|) \quad (9)$$

and

$$\|\omega_k(jh)\| \le c_1\|\Delta x_k(jh)\|$$

$$\|v_k(jh)\| \le c_2\left\|\begin{bmatrix} \int_{jh}^{jh+h}\int_{jh}^{t_1}\cdots\int_{jh}^{t_{\eta_1-1}}\|\Delta x_k(t_{\eta_1})\|\,\mathrm{d}t_{\eta_1}\cdots\mathrm{d}t_1 \\ \vdots \\ \int_{jh}^{jh+h}\int_{jh}^{t_1}\cdots\int_{jh}^{t_{\eta_m-1}}\|\Delta x_k(t_{\eta_m})\|\,\mathrm{d}t_{\eta_m}\cdots\mathrm{d}t_1 \end{bmatrix}\right\|,$$

where c_Φ is the norm bound for $\Phi_k(jh)$; $c_1 = \max\{1 + h/1! + \cdots + h^{\eta_i-1}/(\eta_i-1)!,\ i = 1,\ldots,m\}l_{fg}$, $c_2 = l_{fg} + rl_{bfg}c_{ud}$ and $c_{ud} = \sup_{0\le j\le N-1}\|u_d(jh)\|$.

Integrating state equations (1) and (6) for $t\in[jh, jh+h]$ and applying Bellman–Gronwall lemma yield

$$\|\Delta x_k(t)\| \le \|\Delta x_k(jh)\|\mathrm{e}^{c_3(t-jh)} + \int_{jh}^{t}\mathrm{e}^{c_3(t-s)}c_B\|\Delta u_k(s)\|\,\mathrm{d}s \le c_4\|\Delta x_k(jh)\| + (\mathrm{e}^{c_3h}-1)c_5\|\Delta u_k(jh)\|, \quad (10)$$

where $c_3 = l_f + l_B c_{ud}$, c_B is the norm bound for $B(x_k(t))$, $c_4 = \mathrm{e}^{c_3h}$ and $c_5 = c_B/c_3$, which gives rise to

$$\|\Delta x_k(jh)\| \le c_4\|\Delta x_k(jh-h)\| + (\mathrm{e}^{c_3h}-1)c_5\|\Delta u_k(jh-h)\|, \quad 1 \le j \le N.$$

Since $x_k(0) = x_d(0)$ for all k, then

$$\|\Delta x_k(jh)\| \le \sum_{i=0}^{j-1}(\mathrm{e}^{c_3h}-1)c_4^{j-1-i}c_5\|\Delta u_k(ih)\|, \quad 1 \le j \le N. \quad (11)$$

Substituting (10) into (9) produces

$$\|\Delta u_{k+1}(jh)\| \le \hat{\rho}\|\Delta u_k(jh)\| + c_6\|\Delta x_k(jh)\|, \quad (12)$$

where

$$\hat{\rho} = \rho + c_h c_\Phi c_2(\mathrm{e}^{c_3h}-1)c_5,\ c_6 = c_\Phi c_1 + c_h c_\Phi c_2 c_4,$$

and $c_h = \max\{h^{\eta_1}/\eta_1!,\ldots,h^{\eta_m}/\eta_m!\}$. Then substituting (11) into (12) and defining $c_7 = (\mathrm{e}^{c_3h}-1)\max\{1, c_4,\ldots, c_4^N\}c_5c_6$, we get

$$\|\Delta u_{k+1}(jh)\| \le \hat{\rho}\|\Delta u_k(jh)\| + c_7\sum_{i=0}^{j-1}\|\Delta u_k(ih)\|, \quad 1 \le j \le N-1. \quad (13)$$

Now, the estimation of the control input errors can be made. For the first sampling instant $j = 0$, it follows from (12) that, by using $x_k(0) = x_d(0)$ for all k,

$$\|\Delta u_{k+1}(0)\| \le \hat{\rho}\|\Delta u_k(0)\|. \quad (14)$$

Since $0 \le \rho < 1$, it is possible to choose the sampling period h small enough such that $\hat{\rho} < 1$. Applying Lemma 3.1 to (14) gives

$$\lim_{k\to\infty}\|\Delta u_k(0)\| = 0. \quad (15)$$

For the second sampling instant $j = 1$, (13) reduces to

$$\|\Delta u_{k+1}(h)\| \le \hat{\rho}\|\Delta u_k(h)\| + c_7\|\Delta u_k(0)\|. \quad (16)$$

Applying Lemma 3.1 to (16) results in

$$\limsup_{k\to\infty}\|\Delta u_k(h)\| \le \frac{c_7}{1-\hat{\rho}}\lim_{k\to\infty}\|\Delta u_k(0)\|$$

and furthermore, (15) implies

$$\lim_{k\to\infty}\|\Delta u_k(h)\| = 0.$$

Now, by induction, assume that at sampling instants $j = 0,1,\ldots,l-1$, we obtain

$$\lim_{k\to\infty} \|\Delta u_k(jh)\| = 0.$$

For the sampling instant $j = l$, applying Lemma 3.1 to (13), we see that

$$\limsup_{k\to\infty} \|\Delta u_k(lh)\| \le \frac{c_7}{1-\hat{\rho}} \lim_{k\to\infty} \sum_{i=0}^{l-1} \|\Delta u_k(ih)\|$$

which leads to

$$\lim_{k\to\infty} \|\Delta u_k(lh)\| = 0.$$

Therefore, the result for $\Delta u_k(jh)$ is shown for each $0 \le j \le N-1$. Then it is easy to obtain the results for $\Delta x_k(jh)$ by using (11) and for $e_k(jh)$ by using the assumption that $g(\cdot)$ is Lipschitz in x. This complete the proof. □

Remark 3. The updating law (7) utilizes the output measurements available at sampling instants and the control input requires to be updated at N instants only. Thus, the proposed ILC method demands less computation and memory. In the practical implementation, input saturation can be used to keep the input from wandering excessively in the transient stage of learning. Convergence of the updating law (7) installed with input saturation, however, follows the similar lines to those for the proof of Theorem 3.1.

4. An illustration example

Consider a simple nonlinear continuous-time systems described by

$$\dot{x}_1(t) = x_2(t),$$

$$\dot{x}_2(t) = 0.1\sin(x_2(t)) + (1 + 0.1\cos(x_1(t)))u(t),$$

$$y(t) = x_1(t),$$

where $t \in [0,1]$. The sampling period is chosen as $h = 0.1$ second and thus $N = 10$. Let the desired trajectory take the value at each instant as

$$y_d(jh) = 4(jh)^3 - 3(jh)^4, \quad 0 \le j \le 10.$$

Both relative degree of the continuous-time system and extended relative degree of the system via ZOH holding mechanism are two. The updating law (7) is used with the learning gain $\phi = 70$ that satisfies (8). The initial conditions at each iteration are set to $x_{i,k}(0) = 0$, $i = 1,2$, matching the desired initial conditions, and the initial control is chosen as $u_0(t) = 0$, $t \in [0,1]$. The input saturation function is adopted to keep the input from wandering excessively. Define the performance index $J_k = \max_{0 \le j \le 10} |y_d(jh) - y_k(jh)|$. The iteration stops when the tracking index $J_k < 0.001$. Via simulation, the tracking performance is achieved at the 47th iteration. Figs. 2 and 3 show the tracking errors and the resultant input trajectory. It is clearly shown that the output trajectory converges to the desired trajectory at all sampling instants, and that the proposed ILC approach is effective.

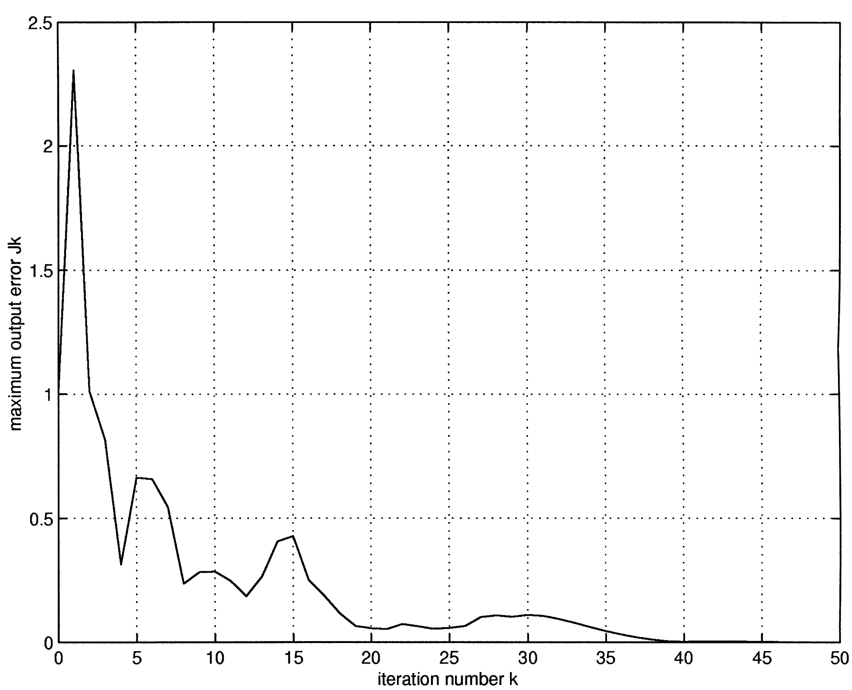

Fig. 2. Tracking errors J_k.

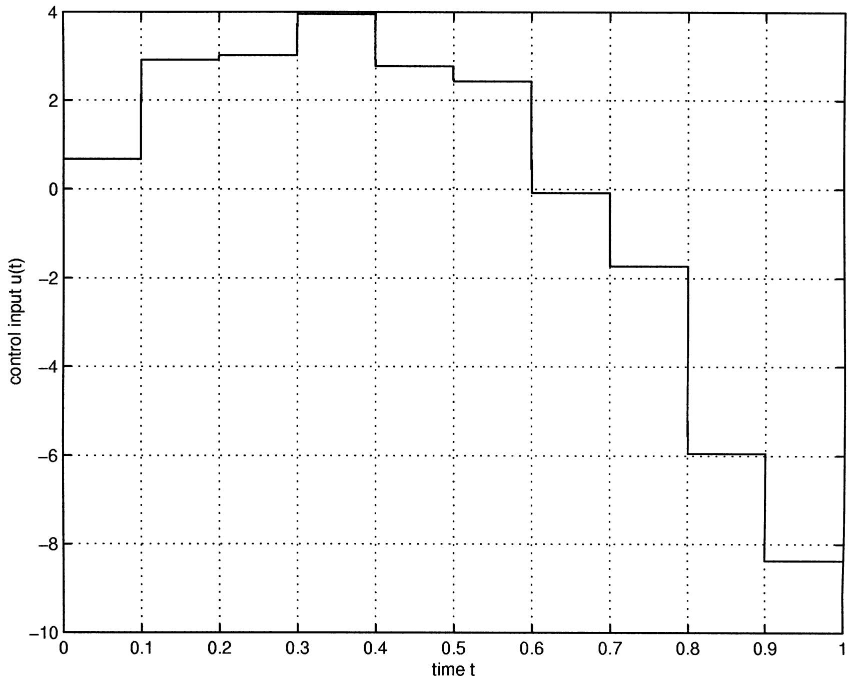

Fig. 3. Input profile $u(t)$ at the 47th iteration.

5. Conclusion

In this paper, the concept of the extended relative degree is introduced to describe the input–output causality of a class of nonlinear systems where a zero-order hold device is used. A sampled-data ILC is proposed which updates the control input using the tracking error one step ahead. If the sampling period is set to be small enough and certain condition is imposed on the learning gain matrix, the system output at each sampling instant has been shown to converge to the desired trajectory. The convergence is proved by new analysis techniques instead of conventional λ-norm. The proposed control method does not require the extended relative degree explicitly and thus is applicable to nonlinear continuous-time systems with arbitrary relative degree.

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