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Brief Paper

Iterative learning control with initial rectifying action☆

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Abstract

This paper addresses the initial shift problem in iterative learning control with system relative degree. The tracking error caused by nonzero initial shift is detected when applying a conventional learning algorithm. Finite initial rectifying action is introduced in the learning algorithm and is shown effective in the improvement of tracking performance, in particular robustness with respect to variable initial shifts. The uniform convergence of the output trajectory to a desired one jointed smoothly with a specified transient trajectory from the starting position is ensured in the presence of fixed initial shift.

Keywords: Learning algorithms; Convergence; Robustness; Initial shift problem; Relative degree; Nonlinear systems

1. Introduction

The research on iterative learning control (ILC) started from the midst of the 1980s and most of early work focused on the convergence analysis of learning algorithms. The fundamental characteristics were examined for systems with direct transmission term (Sugie & Ono, 1991) and relative degree (Ahn, Choi, & Kim, 1993), respectively. ILC implementations involve perturbed initial conditions, as well as state disturbances and measurement noises. A number of efforts are also observed toward the robustness analysis. A common assumption in these analyses is that the initial condition at each cycle is reset to the desired one (Ahn et al., 1993; Arimoto, Kawamura, & Miyazaki, 1984; Hauser, 1987; Sugie & Ono, 1991), or within its neighborhood (Arimoto, Naniwa, & Suzuki, 1991; Bondi, Casalino, & Gambardella, 1988; Heinzinger, Fenwick, Paden, & Miyazaki, 1992; Saab, 1994). In case of perturbed initial conditions, boundedness of the tracking error is established and the error bound is shown to be proportional to the bound on initial condition errors.

Up to now, increasing attention has been devoted to relax the requirement (Lee & Bien, 1996; Park & Bien, 2000; Park, Bien, & Hwang, 1999; Porter & Mohamed, 1991; Wang & Cheah, 1998; Xu & Qu, 1998). In Lee and Bien (1996), for a PD-type ILC, the initial condition is required to keep the same for all cycles but different from the desired one or inside a neighborhood of any fixed point. The extended result to PID-type ILC and a general treatment can be found in Park and Bien (2000) and Park, Bien, and Hwang (1999). Benefiting from the relaxation, better tracking performance can be achieved in the face of perturbed initial conditions, where different input updates based on distinguishing the regions of the measured initial conditions result in smaller error bound. Learning algorithms in Lee and Bien (1996), Park and Bien (2000) and Park et al. (1999), however, are only applicable to systems with relative degree one. In Porter and Mohamed (1991), the problem was addressed for partially irregular LTI systems with rank-defective Markov parameters. Initial impulsive action is shown effective to totally eliminate the effect caused by fixed initial shift and enables zero-error trajectory tracking over entire tracking interval. However, the use of an impulsive action may be not practical. In Sun and Wang (1999), initial rectifying action was introduced as an alternative to avoid the impulsive action, also for partially irregular LTI systems. The initial rectifying action is finite and implementable and ensures

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uniform convergence of the system output to the desired trajectory, jointed smoothly with a transient trajectory from starting position. This paper is motivated by the work (Ahn et al., 1993) on ILC with system relative degree. An initial rectifying action is utilized to address the same problem based on the properties of well-defined relative degree nonlinear systems. The tracking error caused by fixed initial shift is detected when the conventional learning algorithm (Ahn et al., 1993) is applied. Initial rectifying action is introduced in the learning algorithm and is shown effective to guarantee the converged system output to achieve a desired trajectory with a smooth transient. The robustness with respect to variable initial shifts is addressed simultaneously. A numerical example is given to illustrate the defect of the conventional algorithm and the effectiveness of the proposed method.

2. Preliminaries

Consider the class of nonlinear continuous-time systems described by

$$\dot{x}(t) = f(x(t)) + B(x(t))u(t), \tag{1}$$

$$y(t) = g(x(t)), \tag{2}$$

where

$$x \in R^n,\ u = [u_1, \ldots, u_m]^{\mathrm{T}} \in R^m,$$

and

$$y = [y_1, \ldots, y_m]^{\mathrm{T}} \in R^m$$

denote the state, the control input, and the output of the system, respectively, $f(\cdot) \in R^n$, $B(\cdot) = [b_1(\cdot), \ldots, b_m(\cdot)] \in R^{n\times m}$, and $g(\cdot) = [g_1(\cdot), \ldots, g_m(\cdot)]^{\mathrm{T}} \in R^m$ are smooth in their domains of definition. According to Isidori (1995) and Nijmeijer and van der Schaft (1990), system (1) and (2) is said to have (vector) relative degree $\mu(=\{\mu_1, \ldots, \mu_m\})$ at a point x^0, if $(1 \leqslant q \leqslant m)$

(i) $L_{b_p} L^i_f g_q(x) = 0,\ 0 \leqslant i \leqslant \mu_q - 2,\ 1 \leqslant p \leqslant m$ and for all x in a neighborhood of x^0,
(ii) $L_{b_p} L_f^{\mu_q - 1} g_q(x^0) \neq 0$, for some $1 \leqslant p \leqslant m$.

Here μ_q is the minimum order of time derivative of the qth output to which a directly transmission is established from at least one component of the control input u, as follows:

$$y_q^{(j)} = L^j_f g_q(x), \quad 0 \leqslant j \leqslant \mu_q - 1, \tag{3}$$

$$y_q^{(\mu_q)} = L_f^{\mu_q} g_q(x) + [L_{b_1} L_f^{\mu_q-1} g_q(x), \ldots, L_{b_m} L_f^{\mu_q-1} g_q(x)]u. \tag{4}$$

Throughout this paper, the vector norm is defined as $\|a\| = \max_{1\leqslant i \leqslant n} |a_i|$ for an n-dimensional vector $a = [a_1, \ldots, a_n]^{\mathrm{T}}$ and the matrix norm as the induced norm by the vector norm, i.e., for a matrix $A = \{a_{ij}\} \in R^{m\times n}$, $\|A\| = \max_{1\leqslant i\leqslant m} \sum_{j=1}^n |a_{ij}|$. The λ-norm for a vector-valued function $b(t) \in R^n$ is defined as $\|b(\cdot)\|_\lambda = \sup_{t\in[0,T]} \{\mathrm{e}^{-\lambda t} \|b(t)\|\}$, $\lambda > 0$. The system undertaken is assumed to perform identical tasks repeatedly over a finite time interval, i.e., $t \in [0, T]$. Also the following properties are assumed.

(A1) The system dynamics described by (1) and (2) are invertible.
(A2) The system has relative degree $\mu(=\{\mu_1, \ldots, \mu_m\})$ for all $x(t),\ t \in [0, T]$.
(A3) The functions $f(\cdot)$, $B(\cdot)$, $g(\cdot)$, $L_f^{\mu_q} g_q(\cdot)$, $1 \leqslant q \leqslant m$, and $L_{b_p} L_f^{\mu_q-1} g_q(\cdot)$, $1 \leqslant p \leqslant m$, $1 \leqslant q \leqslant m$, are locally Lipschitz in $x(t)$ for $t \in [0, T]$, and l_f, l_b, l_g, l_1^q and $l_2^{p,q}$ denote the Lipschitz constants, respectively.
(A4) The operator $B(\cdot)$ is bounded for all $x(t),\ t \in [0, T]$.
(A5) For an appropriate x_0, all operations start within a neighborhood of x_0 in the sense that $\|x_0 - x_k(0)\| \leqslant c_{x_0}$ for a positive constant c_{x_0}, where $x_k(0)$ is the initial condition at the kth cycle.

3. Analysis

3.1. Conventional ILC and its tracking error

To deal with the system of relative degree μ, conventional D-type learning algorithm takes the form of (Ahn et al., 1993)

$$u_{k+1}(t) = u_k(t) + \Gamma(y_k(t))(y_d^{(\mu)}(t) - y_k^{(\mu)}(t)), \tag{5}$$

where k refers to the number of operation cycle and $y^{(\mu)} = [y_1^{(\mu_1)}, \ldots, y_m^{(\mu_m)}]^{\mathrm{T}}$. $\Gamma(\cdot) \in R^{m\times m}$ is the bounded learning gain and should be chosen such that

$$\|I - \Gamma(g(x))D(x)\| \leqslant \rho < 1 \tag{6}$$

with

$$D(x) = \begin{bmatrix} L_{b_1} L_f^{\mu_1-1} g_1(x), \ldots, L_{b_m} L_f^{\mu_1-1} g_1(x) \\ \vdots \\ L_{b_1} L_f^{\mu_m-1} g_m(x), \ldots, L_{b_m} L_f^{\mu_m-1} g_m(x) \end{bmatrix}.$$

Theorem 1. *Given a desired trajectory $y_d(t),\ t \in [0, T]$, let updating law (5) be applied to system (1) and (2) under assumptions (A1)–(A5). If at the beginning of each cycle $x_k(0) = x_0$, i.e., $c_{x_0} = 0$, (6) holds and the trajectory $y^*(t)$ is realizable, where $y^*(t) = [y_1^*(t), \ldots, y_m^*(t)]^{\mathrm{T}}$ and for $1 \leqslant q \leqslant m$,*

$$y_q^*(t) = y_{q,d}(t) - \sum_{j=0}^{\mu_q-1} \frac{t^j}{j!} (y_{q,d}^{(j)}(0) - y_{q,0}^{(j)}(0)), \quad t \in [0, T], \tag{7}$$

then the system output $y_k(t)$ *converges uniformly to* $y^*(t)$ *on* $[0,T]$ *as* $k\to\infty$.

Remark 3.1. Note that the system is called invertible if it is both right- and left-invertible. Clearly, for an appropriate x_0 and a realizable trajectory $y^*(t)$, assumption (A1) guarantees that there exists a unique control input $u^*(t)$ that will generate the trajectory (Nijmeijer and van der Schaft, 1990). Namely, $y^*(t)=g(x^*(t))$, $\dot{x}^*(t)=f(x^*(t))+B(x^*(t))u^*(t)$, $x^*(0)=x_0$, where $x^*(t)$ is the corresponding state.

Proof of Theorem 1. For simplicity, the time t is dropped in the following proof where confusion will not occur. Define $\hat{g}(x)=[L_f^{\mu_1}g_1(x),\ldots,L_f^{\mu_m}g_m(x)]^{\mathrm{T}}$. From (7), $y_{q,d}^{(\mu_q)}-y_{q,k}^{(\mu_q)}=y_q^{*(\mu_q)}-y_{q,k}^{(\mu_q)}$, $1\leqslant q\leqslant m$, which leads to

$$u^*-u_{k+1}=(I-\Gamma(g(x_k))D(x_k))(u^*-u_k)-\Gamma(g(x_k))$$
$$[\hat{g}(x^*)-\hat{g}(x_k)+(D(x^*)-D(x_k))u^*].$$

Taking norms and applying the bounds and the Lipschitz conditions give rise to $\|\hat{g}(x^*)-\hat{g}(x_k)\|\leqslant l_1\|\Delta x_k^*\|$, $\|D(x^*)-D(x_k)\|\leqslant l_2\|\Delta x_k^*\|$ and

$$\|\Delta u_{k+1}^*\|\leqslant\rho\|\Delta u_k^*\|+c_1\|\Delta x_k^*\|, \tag{8}$$

where $\Delta u_k^*=u^*-u_k$, $\Delta x_k^*=x^*-x_k$, $l_1=\max\{l_1^1,\ldots,l_1^m\}$, $l_2=\max\{l_2^{1,1}+\cdots+l_2^{m,1},\ldots,l_2^{1,m}+\cdots+l_2^{m,m}\}$, $c_1=c_\Gamma(l_1+l_2c_{u^*})$, c_Γ is the norm bound for $\Gamma(\cdot)$ and $c_{u^*}=\sup_{t\in[0,T]}\|u^*(t)\|$. In order to evaluate the state error on the right-hand side of (8), we integrate both sides of the state equations to obtain

$$\Delta x_k^*(t)=\int_0^t[f(x^*(s))-f(x_k(s))$$
$$+(B(x^*(s))-B(x_k(s)))u^*(s)$$
$$+B(x_k(s))\Delta u_k^*(s)]\,\mathrm{d}s.$$

Taking norms and using their properties produce

$$\|\Delta x_k^*(t)\|\leqslant\int_0^t(c_2\|\Delta x_k^*(s)\|+c_B\|\Delta u_k^*(s)\|)\,\mathrm{d}s, \tag{9}$$

where c_B is the norm bound for $B(\cdot)$ and $c_2=l_f+l_Bc_{u^*}$. Multiplying both sides of (9) by $\mathrm{e}^{-\lambda t}(\lambda>0)$ results in

$$\|\Delta x_k^*\|_\lambda\leqslant\frac{1-\mathrm{e}^{-\lambda T}}{\lambda}(c_2\|\Delta x_k^*\|_\lambda+c_B\|\Delta u_k^*\|_\lambda).$$

A $\lambda>0$ is chosen such that $1-c_2(1-\mathrm{e}^{-\lambda T})/\lambda>0$, which implies

$$\|\Delta x_k^*\|_\lambda\leqslant\frac{c_B(1-\mathrm{e}^{-\lambda T})/\lambda}{1-c_2(1-\mathrm{e}^{-\lambda T})/\lambda}\|\Delta u_k^*\|_\lambda. \tag{10}$$

Then multiplying both sides of (8) by $\mathrm{e}^{-\lambda t}$ and using (10) yield

$$\|\Delta u_{k+1}^*\|_\lambda\leqslant\left(\rho+\frac{c_1c_B(1-\mathrm{e}^{-\lambda T})/\lambda}{1-c_2(1-\mathrm{e}^{-\lambda T})/\lambda}\right)\|\Delta u_k^*\|_\lambda. \tag{11}$$

Since $0\leqslant\rho<1$, it is possible to find a sufficiently large λ such that $\bar{\rho}=\rho+\frac{c_1c_B(1-\mathrm{e}^{-\lambda T})/\lambda}{1-c_2(1-\mathrm{e}^{-\lambda T})/\lambda}<1$. Then, (11) is a contraction in $\|\Delta u_k^*\|_\lambda$. As the iterations increase, we obtain $\|\Delta u_k^*\|_\lambda\to0$ so that $u_k\to u^*$ uniformly on $[0,T]$ as $k\to\infty$. It follows from (10) that $x_k\to x^*$ uniformly on $[0,T]$ as $k\to\infty$. Furthermore, by the assumption on $g(\cdot)$ in (A3), $y_k\to y^*$ uniformly on $[0,T]$ as $k\to\infty$. This completes the proof. $\square$

Remark 3.2. The above theorem shows that the converged output follows $y^*(t)$ which deviates from $y_d(t)$ by $\sum_{j=0}^{\mu_q-1}(t^j/j!)(y_{q,d}^{(j)}(0)-y_{q,0}^{(j)}(0))$, $1\leqslant q\leqslant m$, $t\in[0,T]$. Thus, uniform convergence of (5) is guaranteed only when $y_{q,0}^{(j)}(0)=y_{q,d}^{(j)}(0)$, $0\leqslant j\leqslant\mu_q-1$, $1\leqslant q\leqslant m$. At the kth cycle, the output trajectory $y_{q,k}(t)$, $1\leqslant q\leqslant m$, depends on $y_{q,k}^{(j)}(0)$. However, (7) indicates that the limit trajectory $y_q^*(t)$ only relies on $y_{q,0}^{(j)}(0)$ as k tends to infinity. That is due to the aligned initial condition, $x_k(0)=x_0$, which implies $y_{q,k}^{(j)}(0)=y_{q,0}^{(j)}(0)$, $0\leqslant j\leqslant\mu_q-1$, $1\leqslant q\leqslant m$.

3.2. Initial rectifying action

To overcome the deviated convergence shown in Theorem 1, an initial rectifying action is introduced into (5) in the form of

$$u_{k+1}(t)$$
$$=u_k(t)+\Gamma(y_k(t))(y_d^{(\mu)}(t)-y_k^{(\mu)}(t))-\Gamma(y_k(t))$$
$$\begin{bmatrix}\displaystyle\sum_{j=0}^{\mu_1-1}\left(\frac{t^j}{j!}\int_t^h\theta_{\mu_1,h}(s)\,\mathrm{d}s\right)^{(\mu_1)}(y_{1,d}^{(j)}(0)-y_{1,0}^{(j)}(0))\\ \vdots\\ \displaystyle\sum_{j=0}^{\mu_m-1}\left(\frac{t^j}{j!}\int_t^h\theta_{\mu_m,h}(s)\,\mathrm{d}s\right)^{(\mu_m)}(y_{m,d}^{(j)}(0)-y_{m,0}^{(j)}(0))\end{bmatrix}, \tag{12}$$

where $\theta_{\mu_q,h}:[0,T]\to R$, $1\leqslant q\leqslant m$, satisfies

$$\int_0^h\theta_{\mu_q,h}(s)\,\mathrm{d}s=1,$$

$$\left.\left(\frac{t^j}{j!}\int_t^h\theta_{\mu_q,h}(\tau)\,\mathrm{d}\tau\right)^{(\mu_q)}\right|_{t=0}=\begin{cases}1, & \mu_q=j,\\ 0, & \mu_q\neq j.\end{cases}$$

One candidate of such function is given as

$$\theta_{\mu_q,h}(t)=\begin{cases}\dfrac{1}{h^{2\mu_q+1}}\dfrac{(2\mu_q+1)!}{\mu_q!^2}t^{\mu_q}(h-t)^{\mu_q}, & t\in[0,h],\\ 0, & t\in(h,T].\end{cases} \tag{13}$$

Theorem 2. *Given a desired trajectory* $y_d(t)$, $t\in[0,T]$, *let updating law* (12) *be applied to system* (1) and (2) *under assumptions* (A1)–(A5). *If the learning gain* $\Gamma(\cdot)$ *is chosen such that* (6) *holds and the trajectory* $y^*(t)$ *is realizable, where* $y^*(t)=[y_1^*(t),\dots,y_m^*(t)]^{\mathrm T}$ *and for* $1\leqslant q\leqslant m$,

$$y_q^*(t)=y_{q,d}(t)-\sum_{j=0}^{\mu_q-1}\left(\frac{t^j}{j!}\int_t^h \theta_{\mu_q,h}(s)\,\mathrm ds\right)(y_{q,d}^{(j)}(0)-y_{q,0}^{(j)}(0)),\quad t\in[0,T], \tag{14}$$

then the asymptotic bound of the error $y^*(t)-y_k(t)$ *is proportional to* c_{x_0} *on* $[0,T]$ *as* $k\to\infty$.

Proof. As in the proof of Theorem 1, for an appropriate x_0, let u^* be the control input which generates the trajectory y^* and x^* is the corresponding state. From (14), $y_{q,d}^{(\mu_q)}-y_{q,k}^{(\mu_q)}=y_q^{*(\mu_q)}-y_{q,k}^{(\mu_q)}$, $1\leqslant q\leqslant m$, which leads to

$$\Delta u_{k+1}^*=(I-\Gamma(g(x_k))D(x_k))\Delta u_k^*-\Gamma(y_k)[\hat g(x^*)-\hat g(x_k)+(D(x^*)-D(x_k))u^*].$$

Taking norms and applying the bounds and the Lipschitz conditions give rise to

$$\|\Delta u_{k+1}^*\|\leqslant\rho\|\Delta u_k^*\|+c_1\|\Delta x_k^*\|, \tag{15}$$

where $c_1=c_\Gamma(l_1+l_2c_{u^*})$, $l_1=\max\{l_1^1,\dots,l_1^m\}$ and $l_2=\max\{l_2^{1,1}+\cdots+l_2^{m,1},\dots,l_2^{1,m}+\cdots+l_2^{m,m}\}$. For evaluating the state error in (15), we integrate the state equations to obtain

$$\Delta x_k^*(t)=\int_0^t [f(x^*(s))-f(x_k(s))+(B(x^*(s))-B(x_k(s)))u^*(s)+B(x_k(s))\Delta u_k^*(s)]\,\mathrm ds+x_0-x_k(0).$$

Parallel to (10), a $\lambda>0$ is chosen such that $1-c_2(1-\mathrm e^{-\lambda T})/\lambda>0$ and $c_2=l_f+l_Bc_{u^*}$, which ensures

$$\|\Delta x_k^*\|_\lambda\leqslant\frac{c_B(1-\mathrm e^{-\lambda T})/\lambda}{1-c_2(1-\mathrm e^{-\lambda T})/\lambda}\|\Delta u_k^*\|_\lambda+\frac{1}{1-c_2(1-\mathrm e^{-\lambda T})/\lambda}c_{x_0}. \tag{16}$$

Then taking the λ-norm for both sides of (15) and using (16) result in

$$\|\Delta u_{k+1}^*\|_\lambda\leqslant\left(\rho+\frac{c_1c_B(1-\mathrm e^{-\lambda T})\lambda}{1-c_2(1-\mathrm e^{-\lambda T})/\lambda}\right)\|\Delta u_k^*\|_\lambda+\frac{c_1}{1-c_2(1-\mathrm e^{-\lambda T})/\lambda}c_{x_0}. \tag{17}$$

Since $0\leqslant\rho<1$, it is possible to find a sufficiently large λ such that $\bar\rho=\rho+\frac{c_1c_B(1-\mathrm e^{-\lambda T})/\lambda}{1-c_2(1-\mathrm e^{-\lambda T})/\lambda}<1$. Then, (17) is a contraction in $\|\Delta u_k^*\|_\lambda$. When the iterations increase,

$$\limsup_{k\to\infty}\|\Delta u_k^*\|_\lambda\leqslant\frac{1}{1-\bar\rho}\frac{c_1}{1-c_2(1-\mathrm e^{-\lambda T})/\lambda}c_{x_0}.$$

The results for Δx_k^* and y^*-y_k can be derived by using (16) and the assumption on $g(\cdot)$ in (A3). The theorem follows. □

Remark 3.3. Theorem 2 implies that the system output converges to the trajectory $y^*(t)$ for all $t\in[0,T]$ as c_{x_0} tends to zero. From (14), $y^*(t)=y_d(t)$, $t\in(h,T]$. Uniform convergence of the system output to the desired trajectory $y_d(t)$ is thus ensured on $(h,T]$, while the converged output trajectory on $[0,h]$ is governed by the initial rectifying action. The transition trajectory from initial position to the desired trajectory is a smooth and joins the desired trajectory at $t=h$ moment which can be specified. When (5) is applied, the asymptotic bound of the error between $y_d(t)$ and $y_k(t)$ is already known to be proportional to a positive constant $c_{x_{d0}}$, which is the bound on the error between $x_k(0)$ and $x_d(0)$, the desired initial condition. It would be very large when $x_k(0)$ is reset in the neighborhood of x_0 and $\|x_0-x_d(0)\|\gg c_{x_0}$. Theorem 2 shows that the proposed initial rectifying action leads to the bound proportional to c_{x_0} after $t\geqslant h$. It is thus largely reduced. Therefore, the initial rectifying action helps to improve tracking performance. If the 0th cycle initial error $y_{q,d}^{(j)}(0)-y_{q,0}^{(j)}(0)$, $0\leqslant j\leqslant\mu_q-1$, $1\leqslant q\leqslant m$, in (12), is replaced with the kth cycle initial error $y_{q,d}^{(j)}(0)-y_{q,k}^{(j)}(0)$, a new updating law is achieved in the form of

$$u_{k+1}(t)=u_k(t)+\Gamma(y_k(t))(y_d^{(\mu)}(t)-y_k^{(\mu)}(t))-\Gamma(y_k(t))\begin{bmatrix}\sum_{j=0}^{\mu_1-1}\left(\frac{t^j}{j!}\int_t^h\theta_{\mu_1,h}(s)\,\mathrm ds\right)^{(\mu_1)}(y_{1,d}^{(j)}(0)-y_{1,k}^{(j)}(0))\\ \vdots\\ \sum_{j=0}^{\mu_m-1}\left(\frac{t^j}{j!}\int_t^h\theta_{\mu_m,h}(s)\,\mathrm ds\right)^{(\mu_m)}(y_{m,d}^{(j)}(0)-y_{m,k}^{(j)}(0))\end{bmatrix}. \tag{18}$$

From (14),

$$y_{q,d}^{(\mu_q)}-y_{q,k}^{(\mu_q)}=y_k^{*(\mu_q)}-y_{q,k}^{(\mu_q)}+\sum_{j=0}^{\mu_q-1}\left(\frac{t^j}{j!}\int_t^h\theta_{\mu_q,h}(s)\,\mathrm ds\right)^{(\mu_q)}(L_f^jg_q(x_0)-L_f^jg_q(x_k(0))).$$

The same result as Theorem 2 can be obtained. Obviously, if different input updates in (18) are made in the manner like that in Lee and Bien (1996), based on distinguishing the regions of measured initial conditions, better tracking performance in the face of perturbed initial conditions would be achieved.

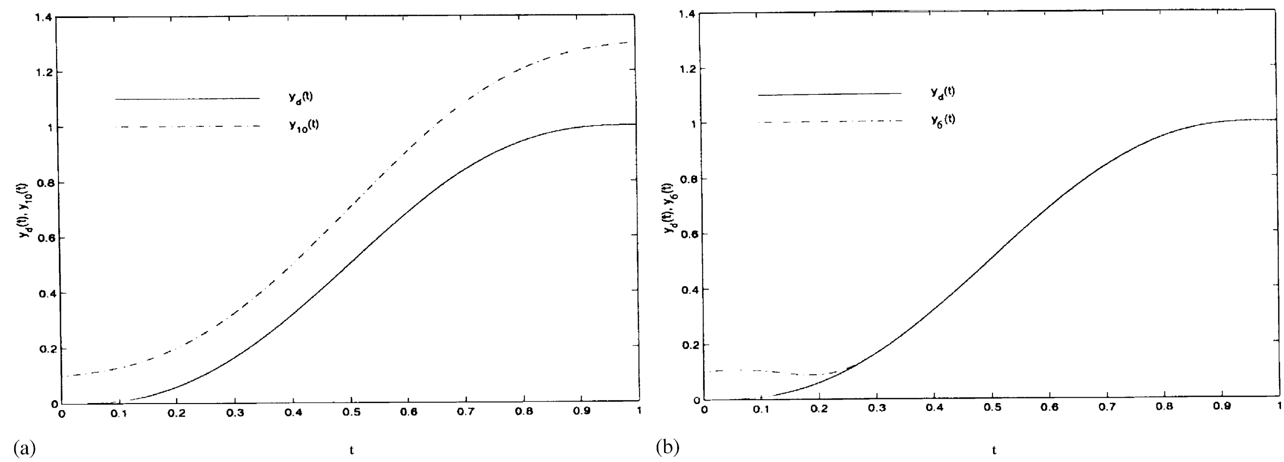

Fig. 1. Comparison of convergence performance in the presence of fixed initial shift: (a) Using conventional ILC (5); (b) Using proposed ILC (19).

4. Example

Consider the following nonlinear continuous-time system

$$\dot{x}_1(t) = x_2(t),$$

$$\dot{x}_2(t) = \sin(x_3(t)) + u(t),$$

$$\dot{x}_3(t) = \cos(x_1(t) + x_2(t))u(t),$$

$$y(t) = x_1(t).$$

The system has relative degree two. Let the desired trajectory be given to be $y_d(t) = 6t^5 - 15t^4 + 10t^3,\ t \in [0,1]$. Simulations are conducted for two ILCs: (I) using conventional learning algorithm (5) with $\Gamma(y_k(t)) = 0.7$; and (II) using learning algorithm with initial rectifying action (12), $h = 0.3$ and $\Gamma(y_k(t)) = 0.7$. For both ILCs, the initial control input is chosen as $u_0(t) = 0,\ t \in [0,1]$. For this case, convergence performance of the learning algorithms is examined in the presence of fixed initial shift where the initial condition at each iteration is reset to $x_k(0) = [x_{1,k}(0), x_{2,k}(0), x_{3,k}(0)]^{\mathrm{T}} = [0.1, 0.2, 0.3]^{\mathrm{T}}$, and thus $y_k(0) = 0.1,\ \dot{y}_k(0) = 0.2$. According to Theorem 1, the converged trajectory by applying (5) is $\lim_{k\to\infty} y_k(t) = y_d(t) - (y_d(0) - y_0(0)) - t(\dot{y}_d(0) - \dot{y}_0(0))$. Fig. 1(a) shows the output trajectory at the tenth iteration where the output trajectory tracks the desired one with the lasting tracking error. The ILC with initial rectifying action (12) can be used to eliminate the error and it is given as

$$\begin{aligned} u_{k+1}(t) = {} & u_k(t) + \Gamma(y_k(t))(\ddot{y}_d(t) - \ddot{y}_k(t)) \\ & + \Gamma(y_k(t))[\dot{\theta}_{2,h}(t)(y_d(0) - y_0(0)) \\ & + (t\dot{\theta}_{2,h}(t) + 2\theta_{2,h}(t))(\dot{y}_d(0) - \dot{y}_0(0))], \end{aligned} \qquad (19)$$

where

$$\theta_{2,h}(t) = \begin{cases} \dfrac{30}{h^5}\, t^2(h-t)^2, & t \in [0,h], \\ 0, & t \in (h,T]. \end{cases}$$

From Theorem 2, the converged trajectory is $\lim_{k\to\infty} y_k(t) = y_d(t) - \int_t^h \theta_{2,h}(s)\,\mathrm{d}s(y_d(0) - y_0(0)) - t\int_t^h \theta_{2,h}(s)\,\mathrm{d}s\,(\dot{y}_d(0) - \dot{y}_0(0))$. Define performance index $J_k = \sup_{t\in[0.3,1]} \|y_d(t) - y_k(t)\|$. The iteration stops if $J_k < 0.001$. This performance requirement is achieved at the sixth cycle. Fig. 1(b) shows the output trajectory at the sixth cycle, which uniformly converges to the desired trajectory over the interval [0.3, 1]. To examine the robustness in the presence of variable initial shifts, let the initial conditions be reset to $x_{1,k}(0) = 0.1 + 0.01$randn, $x_{2,k}(0) = 0.2 + 0.01$randn and $x_{3,k}(0) = 0.3 + 0.01$randn. The randn is a generator of random scalar with normal distribution (mean = 0 and variance = 1) but bounded on the interval $[-1, 1]$. Repetition is done until $k = 100$. Figs. 2 shows the tracking results produced by applying (5) and (19), respectively. Clearly, better performance is achieved by the ILC with initial rectifying action.

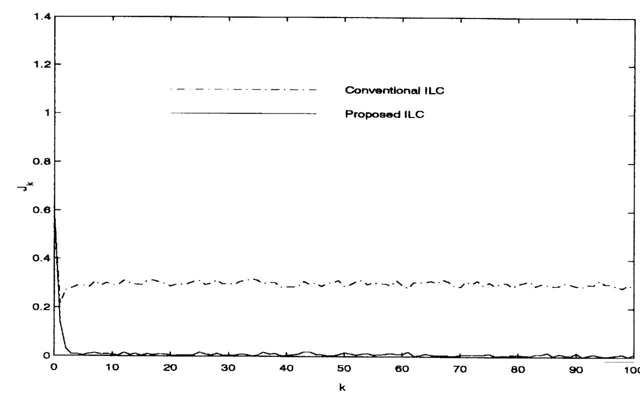

Fig. 2. Robustness performance comparison by using updating laws (5) and (19) in the presence of random initial shifts.

5. Conclusion

For trajectory tracking of a class of nonlinear systems with well-defined relative degree, the defect in tracking performance exists due to initial shift when the conventional learning algorithm is applied. The learning algorithm with the proposed initial action is able to rectify this defect in tracking performance and achieves uniform convergence of the output trajectories to the desired one with specified smooth transience. The results also show that the robustness performance of the conventional learning algorithm with respect to variable initial shifts can be improved by the initial rectifying action.

References

Ahn, H.-S., Choi, C. -H., & Kim, K. -B. (1993). Iterative learning control for a class of nonlinear systems. *Automatica*, *29*(6), 1575–1578.

Arimoto, S., Kawamura, S., & Miyazaki, F. (1984). Bettering operation of robots by learning. *Journal of Robotic Systems*, *1*(2), 123–140.

Arimoto, S., Naniwa, T., & Suzuki, H. (1991). Selective learning with a forgetting factor for robotic motion control. *Proceedings of the 1991 IEEE international conference on robotics and automation*, Sacramento, CA, pp. 728–733.

Bondi, P., Casalino, G., & Gambardella, L. (1988). On the iterative learning control theory for robotic manipulators. *IEEE Journal of Robotics and Automation*, *4*(1), 14–22.

Hauser, J. E. (1987). Learning control for a class of nonlinear systems. *Proceedings of the 26th IEEE conference on decision and control*, Los Angeles, CA, pp. 859–860.

Heinzinger, G., Fenwick, D., Paden, B., & Miyazaki, F. (1992). Stability of learning control with disturbances and uncertain initial conditions. *IEEE Transactions on Automatic Control*, *37*(1), 110–114.

Isidori, A. (1995). *Nonlinear control systems*. Berlin: Springer.

Lee, H.-S., & Bien, Z. (1996). Study on robustness of iterative learning control with non-zero initial error. *International Journal of Control*, *64*(3), 345–359.

Nijmeijer, H., & van der Schaft, A. J. (1990). *Nonlinear dynamical control systems*. New York: Springer.

Park, K.-H., & Bien, Z. (2000). A generalized iterative learning controller against initial state error. *International Journal of Control*, *73*(10), 871–881.

Park, K.-H., Bien, Z., & Hwang, D. -H. (1999). A study on the robustness of a PID-type iterative learning controller against initial state error. *International Journal of Systems Science*, *30*(1), 49–59.

Porter, B., & Mohamed, S. S. (1991). Iterative learning control of partially irregular multivariable plants with initial impulsive action. *International Journal of Systems Science*, *22*(3), 447–454.

Saab, S. S. (1994). On the P-type learning control. *IEEE Transactions on Automatic Control*, *39*(11), 2298–2302.

Sugie, T., & Ono, T. (1991). An iterative learning control law for dynamical systems. *Automatica*, *27*(4), 729–732.

Sun, M., & Wang, D. (1999). Iterative learning control for linear systems with initial rectifying action. *International Journal of Control*, submitted for publication.

Wang, D., & Cheah, C. C. (1998). An iterative learning control scheme for impedance control of robotic manipulators. *International Journal of Robotics Research*, *17*(10), 1091–1104.

Xu, J.-X., & Qu, Z. (1998). Robust iterative learning control for a class of nonlinear systems. *Automatica*, *34*(8), 983–988.

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