INT. J. CONTROL, 2000, VOL. 73, NO. 10, 890–901

On D-type and P-type ILC designs and anticipatory approach

DANWEI WANG†

Many schemes of iterative learning control (ILC) have been developed for continuous-time, non-linear dynamic systems to improve tracking performance. Two schemes, D-type and P-type, have been the bases for many ILC designs. Recently, the anticipatory ILC scheme has been introduced, on the basis of a different approach (Wang 1998a, 1999). In this paper, these basic schemes are compared from both analysis and implementation view points. The anticipatory ILC scheme is designed on the basis of a causal pair of the action taken and its resulting state variables. This approach has the anticipatory characteristics of the D-type ILC and the simplicity for implementation of P-type ILCs. A sampled-data ILC scheme is presented as another form of this anticipatory ILC scheme. Furthermore, control device saturation is taken into account and tracking error convergence results are established, with proofs. The convergence results are also provided in the presence of uncertainties, disturbances and measurement noises. Experimental results are presented to show the effectiveness of this scheme.

1. Introduction

Consider the continuous-time, non-linear dynamic systems described by the following state and output equations:

$$\dot{x}_i(t) = f(x_i(t), t) + B(x_i(t), t)u_i(t) \tag{1}$$

$$y_i(t) = g(x_i(t), t) \tag{2}$$

where the subscript i indicates the operation cycle, $x(t) \in R^n$ the state vector, $y(t) \in R^p$ the output vector and $u(t) \in R^r$ the input vector. The vector and matrix functions f, g and B are known to have only certain properties. Given a desired output trajectory $y_d(t)$ for a fixed operation period $\Gamma = [0, T]$, the aim is to find a desired feed-forward term $u_d(t)$ in an iterative manner, i.e. as $i \to \infty$, $u_i(t) \to u_d(t)$ and thus $y_i(t) \to y_d(t)$. In practice, convergence to a specified neighbourhood of the desired trajectories will be sufficient for most applications. The control $u_i(t)$ should be updated on the basis of the actions taken and its results produced in the previous operation cycle(s), i.e.

$$u_{i+1}(t) = u_i(t) + L(\cdot, e_j(\tau)) \tag{3}$$

where $j \leq i$, $\tau \in \Gamma$, $e_j(\tau) = y_d(\tau) - y_i(\tau)$ and $L(\cdot)$ is a function chosen by the designer.

In most existing ILCs, τ is set to t. In this paper, we consider an anticipatory iterative learning scheme of the form, with $\Delta > 0$ being a small number,

$$u_{i+1}(t) = u_i(t) + L(\cdot, e_i(t + \Delta)) \tag{4}$$

The rest of the paper is organized as follows. Section 2 reviews the basic D-type and P-type ILCs and lists some observations for comparative analysis. Section 3 describes the anticipatory ILC design in detail and provides convergence proof. Section 4 presents the sampled-data ILC design as another form of the anticipatory ILC scheme. Section 5 presents experimental results. Section 6 provides concluding remarks.

2. Revisiting D-type and P-type ILC

Most existing ILCs are of either D-type or P-type or their variations (Hauser 1987, Bien and Huh 1989, Arimoto 1990, Heinzinger *et al.* 1992, Kuc *et al.* 1992, Moore *et al.* 1992, Ahn *et al.* 1993, Saab 1994, Chien and Liu 1996, Cheah and Wang 1998b, Wang and Cheah 1998, Xu 1998, Xu and Zhu 1999). Here we revisit these two controllers for comparisons and to motivate the proposed anticipatory ILC approach.

2.1. *D-type ILC*

The basic form of D-type ILC is given as follows (Hauser 1987, Arimoto 1990, Heinzinger *et al.* 1992)

$$u_{i+1}(t) = u_i(t) + L(\cdot)(\dot{y}_d(t) - \dot{y}_i(t)) \tag{5}$$

Some observations of this ILC scheme can be obtained as follows.

(1) The right-hand side of the updating law (5) uses a causal pair of the action taken and the result produced $(u_i(t), \dot{y}_i(t))$. In the ith operation cycle, the input action $u_i(t)$ is used at time moment t and its directly produced result, seen from (1), is $\dot{x}_i(t)$ which is then transmitted to the output derivative $\dot{y}(t)$ using the differentiation of equation (2), i.e.

$$\dot{y}_i(t) = g_x(x_i(t), t)\dot{x}(t) + g_t(x_i(t), t) \tag{6}$$

All these transmissions are algebraic and occur at the same time moment t when the input action is applied. Based on (2) and (6), this pair is causal and algebraically related. The convergence

Received February 1999. Revised February 2000.
† School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798, Republic of Singapore. e-mail: edwwang@ntu.edu.sg

International Journal of Control ISSN 0020–7179 print/ISSN 1366–5820 online © 2000 Taylor & Francis Ltd
http://www.tandf.co.uk/journals

proofs of the D-type ILC and many advanced D-type-based ILC controllers are straightforward and the convergence ensures zero tracking errors in the absence of uncertainties and noises (Arimoto 1990, Heinzinger *et al.* 1992, Wang and Cheah 1998).

(2) When the D-type ILC is used, the highest order derivative signals of a dynamic system are required. This requirement makes implementation difficult because the highest derivatives normally are not measurable and are very noisy from numerical differentiation. Most robots are equipped with only joint position sensors but not velocity and acceleration sensors. The acceleration signals have to be obtained by numerically differentiating the position measurements twice, and can contain severe noises. Furthermore, high noise levels reduce the effectiveness and accuracy. It is well known that the bounds of converged tracking errors are proportional to the noise levels (Hac 1990, Heinzinger *et al.* 1992, Oh *et al.* 1994, Wang and Cheah 1998). This implies that high-level noises in measurements can severely reduce the effectiveness of the D-type ILC in practice, despite the promises shown in theory and simulations.

2.2. *P-type ILC*

P-type ILCs are given as one of the following two basic forms (Arimoto 1990, Kuc *et al.* 1992, Saab 1994, Chien and Liu 1996), with and without a scalar forgetting factor $\gamma \in (0,1)$

$$u_{i+1}(t) = (1-\gamma)u_i(t) + \gamma u_0(t) + L(\cdot)(y_d(t) - y_i(t)) \tag{7}$$

or

$$u_{i+1}(t) = u_i(t) + L(\cdot)(y_d(t) - y_i(t)) \tag{8}$$

Some observations of the P-type updating law (7) or (8) can be made as follows.

(1) The pair $(u_i(t), y_i(t))$ on the right-hand side of the updating laws (7) and (8) is not a causal pair of the input action taken and its results produced in the ith operation cycle. In the ith operation, and when input action $u_i(t)$ is applied to dynamic equation (1), its produced results/effects cannot be seen from $x_i(t)$ at the same time moment t. Its produced results can only be seen later because of the system's dynamic nature. In other words, the state $x_i(t)$ is the result of the input actions applied before the time moment t but not of $u_i(t)$. Thus the learning laws do not capture the direction or trend of errors that occur in the previous operations. For example, when $y_d(t) - y_i(t) = 0$, the updating law (8) stops learning, but $\dot{y}_d(t) - \dot{y}_i(t)$ could be of any value.

(2) Up to now, the convergence results of P-type ILCs (7) and/or (8), found in Arimoto (1990), Kuc *et al.* (1992), Saab (1994) and Chien and Liu (1996) provide only limited success in showing theoretically the effectiveness of the P-type scheme for general non-linear dynamic continuous-time systems. As pointed out in Chien and Liu (1996), the form (8) is able to ensure the convergence of tracking errors only in the absence of uncertainties and/or disturbances. It is not robust to perturbation from the initial state or output errors. It is easily seen that, when $y_d(0) - y_i(0) \neq 0,\ \forall i \geq 0$, then (8) can result in $u_i(0) \to \infty$ as $i \to \infty$. To gain robustness against uncertainties, the forgetting factor is used in P-type learning (7) (Arimoto 1990, Saab 1994, Chien and Liu 1996). It has been shown in Arimoto (1990), Saab (1994) and Chien and Liu (1996) that the tracking errors will be bounded under various assumptions. Arimoto (1990) considers only the robotic systems. For general systems, the boundedness is established in Saab (1994) under a very strict and uncheckable assumption (A5). In Chien and Liu (1996), the bounds on the tracking errors, stated in Theorem 4.1 and defined in equations (3.22), (3.23) and (3.24), are inversely proportional to the forgetting factor. This implies that, the smaller the forgetting factor, the larger are the tracking error bounds. Thus, suppressing the tracking error bounds is in conflict with the aim of the forgetting factor. The forgetting factor γ in (7) should be small, ideally zero, to 'forget' the arbitrary initial guessed input $u_0(t)$. From (7), if the forgetting factor is non-zero, it is easy to see that the necessary condition for achieving $y_i(t) = y_d(t)$ and $u_{i+1}(t) = u_i(t) = u_d(t)$ simultaneously as $i \to \infty$ is $u_0(t) = u_d(t)$. Thus a selective learning scheme (Arimoto *et al.* 1991) is introduced to reduce the error $u_0(t) - u_d(t)$ by replacing $u_0(t)$ with a better $u_i(t)$ that is closer to $u_d(t)$, if such a $u_i(t)$ is found after certain number of operations.

(3) The P-type ILC does not require the highest order derivative signals in the dynamic system. Implementation requires only measurements of state variables, which are normally available and less noisy. When P-type ILCs are used together with D-type ILCs, so called PD-type ILCs, this has been shown to be effective in ensuring the convergence of the tracking errors (Heinzinger *et al.* 1992).

3. An Anticipatory ILC Scheme

We wish to propose an anticipatory scheme which should have the following features.

(1) Use a causal pair of the action taken and its result produced from the ith operation cycle to compute the action to be taken in the $(i+1)$th operation cycle.

(2) Capture the trend/direction information from the recorded errors in the previous operations but avoid using the highest order derivatives of the dynamic system.

(3) Keep the noise levels down in the measurement process for easy implementation.

From the dynamic state equation (1), the effects produced by $u_i(t)$ can be seen from $\dot{x}_i(t)$ at time moment t, or from state variable $x_i(t+\Delta)$ at time moment $t+\Delta$. The latter can be seen from

$$\begin{aligned} x_i(t+\Delta) &= x_i(t) + \int_t^{t+\Delta} \dot{x}_i(\tau)\, \mathrm{d}\tau \\ &= x_i(t) + \int_t^{t+\Delta} f(x_i(\tau),\tau) + B(x_i(\tau),\tau)u_i(\tau)\,, \mathrm{d}\tau \end{aligned} \tag{9}$$

This $x_i(t+\Delta)$ is carried over to $y_i(t+\Delta)$ in output equation (2). It is logical that the next input action is updated on the basis of the actions and their produced results in the previous operation cycle. Thus $\{u_i(t), y_i(t+\Delta)\}$ is a causal pair of dynamically related cause and effect in the ith operation. The iterative learning control scheme (4) is effect-driven. It has an anticipatory nature because $y_i(t+\Delta)$ is comparable to $\dot{y}_i(t)$ in capturing the trend/directional information. In particular, equation (4) can take a simpler form, with control device saturations being taken into account, as follows

$$\begin{aligned} v_{i+1}(t) &= u_i(t) + L(\cdot)[y_d(t+\Delta) - y_i(t+\Delta)] \\ u_{i+1}(t) &= sat(v_{i+1}(t)) \\ &\triangleq \begin{cases} \sigma & \text{if } v_{i+1}(t) > \sigma \\ v_{i+1}(t) & \text{if } \|v_{i+1}(t)\| \le \sigma \\ -\sigma & \text{if } v_{i+1}(t) < -\sigma \end{cases} \end{aligned} \tag{10}$$

where $\sigma > 0$ is the saturation constant. The design parameter Δ and the learning gain $L(\cdot)$ are to be chosen.

To state and prove the convergence of the ILC updating law (10), the following assumptions and properties are stated.

Assumption 1: *A desired output $y_d(t)$ being continuous for $t \in \Gamma$ is achievable with a unique input $u_d(t)$, for $t \in \Gamma$. This desired input $u_d(t)$ is bounded, i.e. $\|u_d(t)\| \le b_{u_d}$ for a positive constant $b_{u_d} \le \sigma_1 < \sigma$, and so is the corresponding state sequence $x_d(t)$, $t \in \Gamma$. At the desired state, equations (1)–(2) take the following form*

$$\dot{x}_d(t) = f(x_d(t),t) + B(x_d(t),t)u_d(t) \tag{11}$$

$$y_d(t) = g(x_d(t),t) \tag{12}$$

Assumption 2: *The output function $g(x(t),t)$ is continuous and differentiable in (x,t) with $g_x = \partial g/\partial x$ and $g_t = \partial g/\partial t$. The functions $f(x(t),t)$, $B(x(t),t)$, $g(x(t),t)$, $g_x(x(t),t)$ and $g_t(x(t),t)$ are globally uniformly Lipschitz in x on Γ. That is,*

$$\|\beta(x_1(t),t) - \beta(x_2(t),t)\| \le c_\beta \|x_1(t) - x_2(t)\|$$

for $t \in \Gamma$ and positive constants c_β for $\beta \in \{f, B, g, g_x, g_t\}$.

Assumption 3: *The functions $f(x(t),t)$, $B(x(t),t)$, $g(x(t),t)$, $g_x(x(t),t)$ and $g_t(x(t),t)$ are bounded in the sense of $\|\beta(x(t),t)\| \le b_\beta$ for $(x,t) \in R^p \times \Gamma$ and positive constants b_β, where $\beta \in \{f, B, g, g_x, g_t\}$.*

Assumption 4: *All operations start from the initial condition $x_i(0) = x_d(0)$ for all $i = 1,2,\ldots$.*

Note that Assumption 1 implies $v_d(t) = u_d(t)$ and that Assumption 4 is made for simplicity in presentation. Uncertainties in initial state, dynamic fluctuations, disturbances and measurement noises can be taken into account and convergence can be established to a similar line of proof. More discussions are given in Remark 4 and Theorem 2 later in this paper.

Theorem 1: *Applying the anticipatory iterative learning control (10) to the dynamic system (1)–(2). Under Assumptions 1–4, if the following inequality*

$$\left\| I - L(\cdot)\int_t^{t+\Delta} g_x(x(\mu),\mu)B(x(\mu),\mu)d\mu \right\| \le \rho < 1 \tag{13}$$

holds for all $x \in R^n$, $t \in \Gamma$ and Δ is chosen small enough, as operations repeat, $i \to \infty$, the control input error $u_d(t) - u_i(t)$ converges into a specified bound. Furthermore, the state and output tracking errors also converge into some specified bounds.

For convergence proof, the λ-norm is defined as follows.

Definition 1: The λ-norm for a function $b(t)$ is

$$\|b\|_\lambda \triangleq \sup_{t\in[0,T]} e^{-\lambda t}\|b(t)\| \tag{14}$$

where λ is a positive scalar.

Definition 2: The ∞-norm for a function $b(t)$ is

$$\|b\|_\infty \triangleq \sup_{t\in[0,T]} \|b(t)\| \tag{15}$$

Note that the λ-norm is equivalent to the ∞-norm because $\|b\|_\lambda \le \|b(t)\|_\infty \le \|b\|_\lambda e^{\lambda T}$.

For convenience of presentation, we define the following shorthand notations

$$\delta y_i(t) \triangleq y_\mathrm{d}(t) - y_i(t);\quad \delta x_i(t) \triangleq x_\mathrm{d}(t) - x_i(t)$$
$$\delta u_i(t) \triangleq u_\mathrm{d}(t) - u_i(t);\quad \delta v_i(t) \triangleq u_\mathrm{d}(t) - v_i(t)$$
$$g_\mathrm{d}(t) \triangleq g(x_\mathrm{d}(t),t);\quad g_i(t) \triangleq g(x_i(t),t)$$
$$\delta g_i(t) \triangleq g_\mathrm{d}(t) - g_i(t);\quad \dot g_\mathrm{d}(t) \triangleq (\mathrm{d}/\mathrm{d}t)g(x_\mathrm{d}(t),t)$$
$$\dot g_i(t) \triangleq (\mathrm{d}/\mathrm{d}t)g(x_i(t),t);\quad f_\mathrm{d}(t) \triangleq f(x_\mathrm{d}(t),t)$$
$$f_i(t) \triangleq f(x_i(t),t);\quad B_\mathrm{d}(t) \triangleq B(x_\mathrm{d}(t),t)$$
$$B_i(t) \triangleq B(x_i(t),t);\quad g_{xi}(t) \triangleq (\partial/\partial x)g(x_i(t),t)$$
$$g_{x\mathrm{d}}(t) \triangleq (\partial/\partial x)g(x_\mathrm{d}(t),t);\quad \delta g_{xi}(t) \triangleq g_{x\mathrm{d}}(t) - g_{xi}(t)$$
$$g_{ti}(t) \triangleq (\partial/\partial t)g(x_i(t),t);\quad g_{t\mathrm{d}}(t) \triangleq (\partial/\partial t)g(x_\mathrm{d}(t),t)$$
$$\delta g_{ti}(t) \triangleq g_{t\mathrm{d}}(t) - g_{ti}(t)$$
$$\delta(g_{xi}f_i)(t) \triangleq g_{x\mathrm{d}}(t)f_\mathrm{d}(t) - g_{xi}(t)f_i(t)$$
$$\delta(g_{xi}B_i)(t) \triangleq g_{x\mathrm{d}}(t)B_\mathrm{d}(t) - g_{xi}(t)B_i(t).$$

Proof of Theorem 1: Taking the difference between (12) and (2) at time $t+\Delta$ yields

$$\begin{aligned}\delta y_i(t+\Delta) &= \delta y_i(t) + \int_t^{t+\Delta}[\dot g_\mathrm{d}(\tau) - \dot g_i(\tau)]\,\mathrm{d}\tau\\ &= \delta g_i(t) + \int_t^{t+\Delta}[\delta(g_{xi}f_i)(\tau) + \delta(g_{xi}B_i)(\tau)u_\mathrm{d}(\tau)\\ &\quad + \delta g_{ti}(\tau)]\,\mathrm{d}\tau\\ &\quad + \int_t^{t+\Delta} g_{xi}(\tau)B_i(\tau)\delta u_i(\tau)\,\mathrm{d}\tau\\ &= \int_t^{t+\Delta} g_{xi}(\tau)B_i(\tau)\,\mathrm{d}\tau\delta u_i(t)\\ &\quad + \delta g_i(t) + \int_t^{t+\Delta}[\delta(g_{xi}f_i)(\tau)\\ &\quad + \delta(g_{xi}B_i)(\tau)u_\mathrm{d}(\tau) + \delta g_{ti}(\tau)]\,\mathrm{d}\tau\\ &\quad + \int_t^{t+\Delta} g_{xi}(\tau)B_i(\tau)[\delta u_i(\tau) - \delta u_i(t)]\,\mathrm{d}\tau \end{aligned}\tag{16}$$

From the anticipatory law (10), we have the following input error equation

$$\begin{aligned}\delta v_{i+1}(t) &= \delta u_i(t) - L(\cdot)\delta y_i(t+\Delta)\\ &= \left[I - L(\cdot)\int_t^{t+\Delta} g_{x_i}(\tau)B_i(\tau)\,\mathrm{d}\tau\right]\delta u_i(t) - L(\cdot)\delta g_i(t)\\ &\quad - L(\cdot)\int_t^{t+\Delta}[\delta(g_{xi}f_i)(\tau) + \delta(g_{xi}B_i)(\tau)u_\mathrm{d}(\tau)\\ &\quad + \delta g_{ti}(\tau)]\,\mathrm{d}\tau - L(\cdot)\int_t^{t+\Delta} g_{xi}(\tau)B_i(\tau)[\delta u_i(\tau)\\ &\quad - \delta u_i(t)]\,\mathrm{d}\tau \end{aligned}\tag{17}$$

Taking norms on both sides and using (13)

$$\begin{aligned}\|\delta v_{i+1}(t)\| &\le \rho\|\delta u_i(t)\| + b_L c_g\|\delta x_i(t)\|\\ &\quad + b_L(c_{g_xf} + b_{u\mathrm{d}}c_{g_xB} + c_{g_t})\int_t^{t+\Delta}\|\delta x_i(\tau)\|\,\mathrm{d}\tau\\ &\quad + b_L b_{g_x} b_B\int_t^{t+\Delta}\|\delta u_i(\tau) - \delta u_i(t)\|\,\mathrm{d}\tau \end{aligned}\tag{18}$$

where $c_{g_xf} = b_{g_x}c_f + b_f c_{g_x}$, $c_{g_xB} = b_{g_x}c_B + b_B c_{g_x}$.

The saturation feature in (10) leads to the following two inequalities

$$\int_t^{t+\Delta}\|\delta u_i(\tau) - \delta u_i(t)\|\,\mathrm{d}\tau \le 4\sigma\Delta \tag{19}$$

and

$$\|\delta u_i(t)\| \le \|\delta v_i(t)\| \tag{20}$$

Using these two inequalities in equation (18) yields

$$\begin{aligned}\|\delta v_{i+1}(t)\| &\le \rho\|\delta v_i(t)\| + b_L c_g\|\delta x_i(t)\|\\ &\quad + b_L(c_{g_xf} + b_{u\mathrm{d}}c_{g_xB} + c_{g_t})\int_t^{t+\Delta}\|\delta x_i(\tau)\|\,\mathrm{d}\tau\\ &\quad + 4b_L b_{g_x} b_B\sigma\Delta \end{aligned}\tag{21}$$

Multiplying both sides by $\mathrm{e}^{-\lambda t}$, with λ being a positive constant, we have, in λ-norm,

$$\|\delta v_{i+1}\|_\lambda \le \rho\|\delta v_i\|_\lambda + \frac{p(\mathrm{e}^{\lambda\Delta} - 1)}{\lambda}\|\delta x_i\|_\lambda + 4b_L b_{g_x} b_B\sigma\Delta \tag{22}$$

where $p \triangleq b_L(c_g + c_{g_xf} + b_{u\mathrm{d}}c_{g_xB} + c_{g_t})$ and the following inequality has been used

$$\begin{aligned}\int_t^{t+\Delta}\mathrm{e}^{-\lambda t}\|\delta x_i(\tau)\|\,\mathrm{d}\tau &= \int_t^{t+\Delta}\mathrm{e}^{\lambda(\tau-t)}\|\delta x_i(\tau)\|\mathrm{e}^{-\lambda\tau}\,\mathrm{d}\tau\\ &\le \|x_i\|_\lambda\int_t^{t+\Delta}\mathrm{e}^{\lambda(\tau-t)}\,\mathrm{d}\tau\\ &= \frac{(\mathrm{e}^{\lambda\Delta} - 1)}{\lambda}\|x_i\|_\lambda \end{aligned}\tag{23}$$

Applying the Bellman–Gronwall inequality (Flett 1980) to equation (1) produces

$$\|\delta x_i(t)\| \le b_B\int_0^t \mathrm{e}^{h_1(t-\tau)}\|\delta u_i(\tau)\|\,\mathrm{d}\tau \tag{24}$$

where $h_1 = c_f + b_{u\mathrm{d}}c_B$. Multiply both sides of (24) by $\mathrm{e}^{-\lambda t}$, with $\lambda > h_1$. Equation (24) becomes, in λ-norm,

$$
\begin{aligned}
\|\delta x_i\|_\lambda &\le b_B \int_0^t \mathrm{e}^{(\lambda-h_1)(\tau-t)} \|\delta u_i(\tau)\| \mathrm{e}^{-\lambda\tau}\,\mathrm{d}\tau \\
&\le b_B \|\delta u_i\|_\lambda \int_0^t \mathrm{e}^{(\lambda-h_1)(\tau-t)}\,\mathrm{d}\tau \\
&\le \frac{b_B\left(1-\mathrm{e}^{-(\lambda-h_1)T}\right)}{\lambda-h_1}\|\delta u_i\|_\lambda \\
&\le \frac{b_B\left(1-\mathrm{e}^{-(\lambda-h_1)T}\right)}{\lambda-h_1}\|\delta v_i\|_\lambda \qquad (25)
\end{aligned}
$$

Substituting this inequality into equation (22), we obtain

$$
\begin{aligned}
\|\delta v_{i+1}\|_\lambda \le{}& \left[\rho + \frac{p\left(\mathrm{e}^{\lambda\Delta}-1\right)}{\lambda}\cdot\frac{b_B\left(1-\mathrm{e}^{-(\lambda-h_1)T}\right)}{\lambda-h_1}\right]\|\delta v_i\|_\lambda \\
&+ 4 b_L b_{g_x} b_B \sigma \Delta \qquad (26)
\end{aligned}
$$

Define

$$
\bar\rho = \rho + \frac{p\left(\mathrm{e}^{\lambda\Delta}-1\right)}{\lambda}\cdot\frac{b_B\left(1-\mathrm{e}^{-(\lambda-h_1)T}\right)}{\lambda-h_1} \qquad (27)
$$

In view of the fact that $\rho < 1$, it is possible to choose λ sufficiently large to ensure

$$
\rho + \frac{p b_B\left(1-\mathrm{e}^{-(\lambda-h_1)T}\right)}{\lambda-h_1} < 1 \qquad (28)
$$

For any such fixed λ, it is possible to choose Δ small enough such that

$$
\frac{\left(\mathrm{e}^{\lambda\Delta}-1\right)}{\lambda} < 1 \qquad (29)
$$

because $\left(\mathrm{e}^{\lambda\Delta}-1\right)\to 0$ as $\Delta\to 0$. Combining the above two inequalities implies that $\bar\rho < 1$ and that equation (26) is a contraction. We have

$$
\lim_{i\to\infty}\sup \|\delta v_i\|_\lambda \le \frac{4 b_L b_{g_x} b_B \sigma}{1-\bar\rho}\Delta \qquad (30)
$$

Using (10) and noting that $u_\mathrm{d}(t) = v_\mathrm{d}(t)$ and $\|u_\mathrm{d}(t)\| \le b_{u\mathrm{d}} \le \sigma_1 < \sigma$, we have

$$
\lim_{i\to\infty}\sup \|\delta u_i\|_\lambda \le \frac{4 b_L b_{g_x} b_B \sigma}{1-\bar\rho}\Delta \qquad (31)
$$

Using (30) in equation (25) yields

$$
\lim_{i\to\infty}\sup \|\delta x_i\|_\lambda \le \frac{b_B\left(1-\mathrm{e}^{-(\lambda-h_1)T}\right)}{\lambda-h_1}\cdot\frac{4 b_L b_{g_x} b_B \sigma}{1-\bar\rho}\Delta \qquad (32)
$$

Finally, using equations (2) and (12) produces

$$
\lim_{i\to\infty}\sup \|\delta y_i\|_\lambda \le \frac{c_g b_B\left(1-\mathrm{e}^{-(\lambda-h_1)T}\right)}{\lambda-h_1}\cdot\frac{4 b_L b_{g_x} b_B \sigma}{1-\bar\rho}\Delta \qquad (33)
$$

This completes the proof. □

Remark 1: The tracking error bounds in (30), (32) and (33) are proportional to the time shift Δ. The anticipatory scheme requires $\Delta > 0$ but it can be small enough to specify the bounds as required.

Remark 2: Due to the shift by the time interval Δ, the learning law (4) or (10) cannot be used for $t > T-\Delta$. During this final time interval $(T-\Delta, T)$, the final output measurement $y_i(T)$ can be used in the learning law. That is, for $t \in (T-\Delta, T)$, the first equation in (14) takes the form

$$
v_{i+1}(t) = u_i(t) + L(\cdot)\left(y_\mathrm{d}(T) - y_i(T)\right) \qquad (34)
$$

Remark 3: This learning law (10) is different from the differential approximation of the D-type ILC (5) which normally takes the form of (Tso and Ma 1993, Cheah *et al.* 1994)

$$
u_{i+1}(t) = u_i(t) + L(\cdot)\frac{\left(e_i(t+\Delta)\right) - e_i(t)}{h} \qquad (35)
$$

with h being the sampling time interval, and is basically a numerical differentiation that is the source of severe noise in practical implementations. This learning law (10) is different from the P-type ILC (7) because this anticipatory ILC scheme requires $\Delta > 0$. If $\Delta = 0$ is chosen, (13) is not satisfied and the resulting controller from (10) is not an anticipatory ILC.

Remark 4: In practical applications, control systems involve state resetting uncertainties, measurement noises and disturbances. In Theorem 1, convergence is established in the absence of these uncertainties, measurement noises and disturbances. However, robustness of this anticipatory iterative learning law can be easily established, as stated in the following theorem.

Consider the following dynamic system with disturbances, measurement noises and state resetting uncertainties

$$
\dot x_i(t) = f(x_i(t), t) + B(x_i(t), t)u_i(t) + \eta_i(t) \qquad (36)
$$

$$
y_i(t) = g(x_i(t), t) + \xi_i(t) \qquad (37)
$$

where $\eta_i(t)$ and $\xi_i(t)$ are the state disturbance and output measurement noise, respectively. Assumption 4 is restated as the following Assumption 4$'$:

Assumption 4$'$: *The initial state resetting error* $\|x_\mathrm{d}(0) - x_i(0)\| \le b_{x0}$, $\forall i$ *and state disturbance* $\|\eta_i(t)\| \le b_\eta$, *output measurement noise* $\|\xi_i(t)\| \le b_\xi$ $\forall i$ *and* $\forall t \in \Gamma = [0, T]$ *for some positive constants* b_{x0}, b_η, b_ξ.

Theorem 2: *Applying the anticipatory iterative learning control* (10) *to the dynamic system* (36)–(37). *Under assumptions 1–3 and* 4$'$, *if the inequality* (13) *holds for all* $x \in R^n$, $t \in \Gamma$ *and* Δ *is chosen small enough, as operations repeat,* $i \to \infty$, *the tracking errors converge into the following error bounds*

$$\limsup_{i\to\infty} \|\delta u_i\|_\lambda \le \frac{4b_L b_{g_x} b_B \sigma}{1-\bar\rho}\Delta + \mu_u(b_{x0}, b_\eta, b_\xi) \tag{38}$$

$$\limsup_{i\to\infty} \|\delta x_i\|_\lambda \le \frac{b_B(1-e^{-(\lambda-h_1)T})}{\lambda-h_1}\cdot\frac{4b_L b_{g_x} b_B \sigma}{1-\bar\rho}\Delta + \mu_x(b_{x0}, b_\eta, b_\xi) \tag{39}$$

$$\limsup_{i\to\infty} \|\delta y_i\|_\lambda \le \frac{c_g b_B(1-e^{-(\lambda-h_1)T})}{\lambda-h_1}\cdot\frac{4b_L b_{g_x} b_B \sigma}{1-\bar\rho}\Delta + \mu_y(b_{x0}, b_\eta, b_\xi) \tag{40}$$

where $\mu_u(b_{x0}, b_\eta, b_\xi)$, $\mu_x(b_{x0}, b_\eta, b_\xi)$ and $\mu_y(b_{x0}, b_\eta, b_\xi)$ are some constants proportional to constants b_{x0}, b_η, and b_ξ, linearly and independently.

The proof of this result follows the same lines of that for Theorem 1 and thus is omitted.

Remark 5: The first terms in convergence error bounds (38), (39) and (40) can be specified by the design parameter Δ whereas the second terms cannot. Thus, in practical applications, the first terms in these bounds are not dominant when Δ is chosen small enough.

4. Sampled-data ILC approach

This proposed anticipatory scheme can be easily implemented using a sampled-data approach (Wang 1995, Zhang 1996, Chien 1998, Longman 1998). Here we propose a sampled-data ILC taking into account the saturation of the control device. Fix a sampling interval $\Delta = h = T/N$ and choose

$$u_i(t) = u_i(k\Delta) \tag{41}$$

for $k \in \mathcal{N} = \{0, 1, 2, \ldots, N\}$ and $t \in [k\Delta, (k+1)\Delta)$. The sampled-data anticipatory ILC takes the following form, with consideration of control device saturation

$$v_{i+1}(k\Delta) = u_i(k\Delta) + L(\cdot)[y_d((k+1)\Delta) - y_i((k+1)\Delta)]$$

$$u_{i+1}(k\Delta) = sat(v_{i+1}(k\Delta))$$

$$\triangleq \begin{cases} \sigma & \text{if } v_{i+1}(k\Delta) > \sigma \\ v_{i+1}(k\Delta) & \text{if } \|v_{i+1}(k\Delta)\| \le \sigma \\ -\sigma & \text{if } v_{i+1}(k\Delta) < -\sigma \end{cases} \tag{42}$$

where $\sigma > 0$ is the saturation constant.

Remark 6: The right-hand side of the updating law (42) uses the causal pair $\{u_i(k\Delta), y_i((k+1)\Delta)\}$ which are the input action taken during the sampling period $t \in [k\Delta, (k+1)\Delta)$ and its result produced at the end of the same period, $t = (k+1)\Delta$.

Remark 7: After each operation cycle, this updating law will be used N times to compute N values of u_{i+1} for the next operation cycle. This off-line computation is not demanding and time saving.

The design of (41) and (42) implies that the input remains constant for the whole sampling period, and its produced result is taken from the end of the sampling period. We can state the convergence results using (42) at the sampling instances as follows.

Theorem 3: *Consider the system* (1) *and* (2) *with the ILC* (41) *and* (42). *Under Assumptions* 1–4, *if the learning gain $L(\cdot)$ is chosen such that the following inequality*

$$\left\| I - L(\cdot)\int_{k\Delta}^{(k+1)\Delta} g_x(x(\tau),\tau)B(x(\tau),\tau)\,d\tau \right\| \le \rho < 1 \tag{43}$$

holds for all $(x,k) \in R^p \times \mathcal{N}$ and Δ is small enough, as $i \to \infty$, $\|\delta u_i\|_\lambda \to 0$, $\|\delta x_i\|_\lambda \to 0$, $\|\delta y_i\|_\lambda \to 0$ at the sampling time instances.

Two norms of a positive real function $q : \mathcal{N} \to R$ are defined for the proof of Theorem 3:

Definition 2: The α-norm is defined as $\|q(\cdot)\|_\alpha = \sup_{k\in\mathcal{N}} q(k)\alpha^k$ with $0 < \alpha \le 1$.

Definition 3: The ∞-norm is defined as $\|q(\cdot)\|_\infty = \sup_{k\in\mathcal{N}} q(k)$.

Note that these two norms are equivalent by noting that $\|q(k)\|_\alpha \le \|q(k)\|_\infty \le (1/\alpha)^n \|q(k)\|_\alpha$.

Proof of Theorem 3: In the following proof, $z(k) \triangleq (k\Delta)$, for $z \in \{x, y, u, v, f, B, g, g_x, g_t\}$. Taking the difference between (2) at the desired state and (2) at the ith operation cycle yields, at sampling instant $t = (k+1)\Delta$,

$$\begin{aligned}
\delta y_i(k+1) &= g(x_d(k+1), k+1) - g(x_i(k+1), k+1) \\
&= g(x_d(k), k) - g(x_i(k), k) \\
&\quad + \int_{k\Delta}^{(k+1)\Delta} [\dot g(x_d(\tau),\tau) - \dot g(x_i(\tau),\tau)]\,d\tau \\
&= \delta g(x_i(k), k) + \int_{k\Delta}^{(k+1)\Delta} [g_{x_d}\dot x_d(\tau) + g_{td}(\tau) \\
&\quad - g_{x_i}\dot x_i(\tau) - g_{ti}(\tau)]\,d\tau \\
&= \int_{k\Delta}^{(k+1)\Delta} [g_x(x_i(\tau),\tau)B(x_i(\tau),\tau)]\,d\tau (u_d(k) \\
&\quad - u_i(k)) + \delta g(x_i(k), k) \\
&\quad + \int_{k\Delta}^{(k+1)\Delta} [(g_x(x_d(\tau),\tau)f(x_d(\tau),\tau) \\
&\quad - g_x(x_i(\tau),\tau)f(x_i(\tau),\tau)) \\
&\quad + (g_x(x_d(\tau),\tau)B(x_d(\tau),\tau) \\
&\quad - g_x(x_i(\tau),\tau)B(x_i(\tau),\tau))u_d(k) \\
&\quad + (g_t(x_d(\tau),\tau) - g_t(x_i(\tau),\tau))]\,d\tau
\end{aligned} \tag{44}$$

In the above equation and the following development, the continuous-time t or τ is dropped and functions are represented in short, i.e. $z(x_i(t), t)$ being denoted as z_i and $z(x_\mathrm{d}(t), t)$ as z_d for $z \in \{f, B, g, g_x, g_t\}$, when and where confusion does not occur. We also use δz_i to denote $z_\mathrm{d} - z_i$ for $z \in \{x, y, u, v, f, B, g, g_x, g_t\}$.

Substituting the above equation into (42) and using $u_\mathrm{d}(k)$ on both sides,

$$\begin{aligned}\delta v_{i+1}(k) &= \delta u_i(k) - L(\cdot)\delta y_i(k+1)\\ &= \left[I - L(\cdot)\int_{k\Delta}^{(k+1)\Delta} g_x(x_i(\tau),\tau)B(x_i(\tau),\tau)\,\mathrm{d}\tau\right]\\ &\quad\times \delta u_i(k) - L(\cdot)\delta g_i(k)\\ &\quad - L(\cdot)\int_{k\Delta}^{(k+1)\Delta}[\delta(g_{x_i}f_i)(\tau)\\ &\quad + \delta(g_{x_i}B_i)(\tau)u_\mathrm{d}(\tau) + \delta g_{ti}]\,\mathrm{d}\tau \end{aligned} \tag{45}$$

Taking norms on both sides of the above equation yields

$$\begin{aligned}\|\delta v_{i+1}(k)\| &\le \rho\|\delta u_i(k)\| + b_L c_g\|\delta x_i(k)\|\\ &\quad + b_L h_2\int_{k\Delta}^{(k+1)\Delta}\|\delta x_i(\tau)\|\,\mathrm{d}\tau\end{aligned} \tag{46}$$

where, noting inequality (43),

$$\left\| I - L(\cdot)\int_{k\Delta}^{(k+1)\Delta} g_x(x_i(\tau),\tau)B(x_i(\tau),\tau)\,\mathrm{d}\tau \right\| \le \rho < 1$$

and some constants are defined as: $h_1 = c_f + b_{u_\mathrm{d}}c_B$ and $h_2 = b_{g_x}h_1 + c_{g_x}(b_f + b_B b_{u_\mathrm{d}}) + c_{g_t}$. On the other hand, from equation (1), for $t \in \Gamma$,

$$\delta x_i(t) = \int_0^t [\delta f_i + \delta B_i u_\mathrm{d} + B_i \delta u_i]\,\mathrm{d}\tau$$

Taking norms on both sides of the above equation, we get

$$\|\delta x_i(t)\| \le \int_0^t [h_1\|\delta x_i(\tau)\| + b_B\|\delta u_i(\tau)\|]\,\mathrm{d}\tau$$

Using a general form of the Bellman–Gronwall inequality (Flett 1980), the above equation becomes

$$\|\delta x_i(t)\| \le \int_0^t \mathrm{e}^{h_1(t-\tau)} b_B\|\delta u_i(\tau)\|\,\mathrm{d}\tau$$

For $t \le (k+1)\Delta$, the above inequality becomes

$$\|\delta x_i(t)\| \le b_B \mathrm{e}^{\Delta h_1(k+1)}\int_0^{(k+1)\Delta} \mathrm{e}^{-h_1\tau}\|\delta u_i(\tau)\|\,\mathrm{d}\tau$$

Note that the time interval $[0, (k+1)\Delta)$ consists of $k+1$ sampling intervals $[j\Delta, (j+1)\Delta)$ for $j = 0, 1, \ldots, k$, and the control inputs are constants in each of these sampling intervals. We have

$$\begin{aligned}\|\delta x_i(t)\| &\le b_B \mathrm{e}^{\Delta h_1(k+1)}\sum_{j=0}^{k}\|\delta u_i(j)\|\int_{j\Delta}^{(j+1)\Delta}\mathrm{e}^{-h_1\tau}\mathrm{d}\tau\\ &\le b_B\frac{\mathrm{e}^{\Delta h_1}-1}{h_1}\sum_{j=0}^{k}\|\delta u_i(j)\|\mathrm{e}^{\Delta h_1(k-j)}\end{aligned} \tag{47}$$

Using inequality (47) in equation (46) for $t \in [k\Delta, (k+1)\Delta)$, we obtain

$$\begin{aligned}\|\delta v_{i+1}(k)\| &\le \rho\|\delta u_i(k)\| + (b_L b_{g_x} + \Delta h_2)b_B\frac{\mathrm{e}^{\Delta h_1}-1}{h_1}\\ &\quad\times\sum_{j=0}^{k}\|\delta u_i(j)\|\mathrm{e}^{\Delta h_1(k-j)}\end{aligned}$$

Multipling both sides of above equation by α^k, with $0 < \alpha < \mathrm{e}^{\Delta h_1}$,

$$\begin{aligned}\|\delta v_{i+1}(k)\|\alpha^k &\le \rho\|\delta u_i(k)\|\alpha^k + (b_L b_A + \Delta h_2)b_B\frac{\mathrm{e}^{\Delta h_1}-1}{h_1}\\ &\quad\times\sum_{j=0}^{k}\|\delta u_i(j)\|\alpha^j(\mathrm{e}^{\Delta h_1}\alpha)^{(k-j)}\end{aligned} \tag{48}$$

In α-norm, the above equation becomes, noting that $\|\delta u_i\|_\alpha \le \|\delta v_i\|_\alpha$,

$$\|\delta v_{i+1}\|_\alpha \le \hat{\rho}\|\delta u_i\|_\alpha \le \hat{\rho}\|\delta v_i\|_\alpha \tag{49}$$

where

$$\hat{\rho} = \rho + (\mathrm{e}^{\Delta h_1}-1)(b_L b_{g_x} + \Delta h_2)b_B\frac{1-(\mathrm{e}^{\Delta h_1}\alpha)^n}{h_1(1-(\mathrm{e}^{\Delta h_1}\alpha))}$$

When Δ tends to zero, $\lim_{\Delta\to 0}(\mathrm{e}^{\Delta h_1}-1) = 0$. Thus the sampling interval Δ can be chosen small enough so that $\hat{\rho} < 1$. In this case, equation (49) is a contraction mapping of $\|\delta v\|_\alpha$. Therefore, when operations increase, $\|\delta v\|_\alpha$ and $\|\delta u\|_\alpha$ converge to zero

$$\lim_{i\to\infty}\sup\|\delta v_i\|_\alpha = \lim_{i\to\infty}\sup\|\delta u_i\|_\alpha = 0$$

Similarly, we can show that the states converge to the desired trajectory. From equation (47), setting $t = (k+1)\Delta$, and multipling both sides by α^{k+1},

$$\begin{aligned}\|\delta x_i(k+1)\|\alpha^{k+1} &\le \frac{\alpha b_B(\mathrm{e}^{\Delta h_1}-1)}{h_1}\\ &\quad\times\sum_{j=0}^{k}\|\delta u_i(j)\|\alpha^j(\mathrm{e}^{\Delta h_1}\alpha)^{(k-j)}\end{aligned}$$

In α-norm,

$$\|\delta x_i\|_\alpha \le \mu b_B\|\delta u_i\|_\alpha$$

where $\mu = \alpha(\mathrm{e}^{\Delta h_1}-1)(1-(\mathrm{e}^{\Delta h_1}\alpha)^n)/(h_1(1-\mathrm{e}^{\Delta h_1}\alpha))$. When operations increase, we have

$$\lim_{i\to\infty} \sup \|\delta x_i\|_\alpha = 0$$

Finally, the convergence of the output tracking error can be established as, using equation (2),

$$\lim_{i\to\infty} \sup \|\delta y_i\|_\alpha = 0$$

This completes the proof. □

Remark 8: In the above development, convergence is established without considering uncertainties, measurement noises and disturbances. However, robustness of this sampled-data learning law can be easily established against the presence of state disturbances, output measurement noise and initial state uncertainties. Proof of the convergence follows similar lines to those for Theorem 3. The results can be stated in the following theorem, with proof omitted.

Theorem 4: *Consider the system* (36) *and* (37) *with the ILC* (41) *and* (42). *Under the assumptions* 1–3 *and* 4′ *if the learning gain* $L(\cdot)$ *is chosen such that the inequality* (43) *holds for all* $(x,k) \in R^p \times \mathcal{N}$ *and* Δ *is small enough, as* $i \to \infty$, *we have, at the sampling time instances*

$$\lim_{i\to\infty} \sup \|\delta u_i\|_\lambda = \hat{\mu}_u(b_{x0}, b_\eta, b_\xi) \tag{50}$$

$$\lim_{i\to\infty} \sup \|\delta x_i\|_\lambda = \hat{\mu}_x(b_{x0}, b_\eta, b_\xi) \tag{51}$$

$$\lim_{i\to\infty} \sup \|\delta y_i\|_\lambda = \hat{\mu}_y(b_{x0}, b_\eta, b_\xi) \tag{52}$$

where the tracking error bounds $\hat{\mu}_u(b_{x0}, b_\eta, b_\xi)$, $\hat{\mu}_x(b_{x0}, b_\eta, b_\xi)$ *and* $\hat{\mu}_y(b_{x0}, b_\eta, b_\xi)$ *are some constants proportional to constants* b_{x0}, b_η, *and* b_ξ, *linearly and independently.*

Remark 9: The results developed above are for continuous-time dynamic systems. In discrete-time domain, many ILCs have been developed on the basis of the causal pairs of actions and results (Rogers and Owens 1992, Amann *et al.* 1995, Xu 1997, Longman 1998, Wang 1998b, Cheah and Wang 1998a) for different systems and applications.

5. Experiments

In the experimental study, we test the anticipatory scheme using two experimental setups. Both experiments provide positive results to confirm the proposed theory.

5.1. *Experiment 1*

We use a mechanism of a dc-motor driving a single rigid link through a gear, as shown in figure 1. An optical encoder is mounted on the link side to measure the link angle position. All parameters are unknown, except we know that the dynamics of the system is governed by the following second-order differential equation

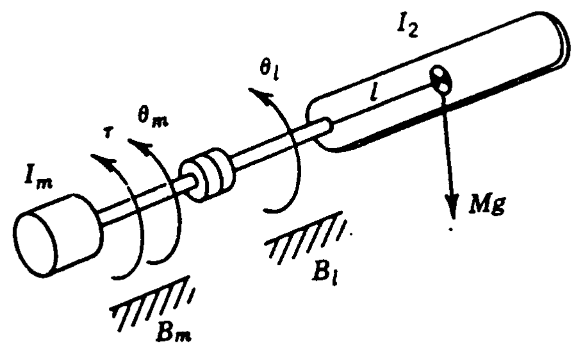

Figure 1. Single-link mechanism.

$$\left(J_\mathrm{m} + \frac{J_\mathrm{l}}{n^2}\right)\ddot{\theta}_\mathrm{m} + \left(B_\mathrm{m} + \frac{B_\mathrm{l}}{n^2}\right)\dot{\theta}_\mathrm{m} + \frac{Mgl}{n}\sin\left(\frac{\theta_\mathrm{m}}{n}\right) = u \tag{53}$$

and the link angle position is related to the motor angle as

$$\theta_\mathrm{l} = \theta_\mathrm{m}/n \tag{54}$$

where $\theta_\mathrm{m}, J_\mathrm{m}, B_\mathrm{m}$ and $\theta_\mathrm{l}, J_\mathrm{l}, B_\mathrm{l}$ are the motor and link angles, inertia and damping coefficients, respectively, n is the gear ratio, u is the motor torque, M is the lumped mass and l is the centre of mass from the axis of motion. The motor is controlled by a PC with a power amplifier.

The operation cycle is set as $\Gamma = 3$ s and the desired trajectory is given as

$$\theta_\mathrm{ld}(t) = \frac{\pi t^2}{6} - \frac{\pi t^3}{27} \text{ rad} \tag{55}$$

The starting position (0°) of the link is vertically upwards and the ending position (90°) is horizontal, pointing out where the gravity effect is the greatest.

The state differential equation (36) can be derived from (53) with $x = (x_1, x_2)^\mathrm{T} = (\theta_\mathrm{m}, \dot{\theta}_\mathrm{m})^\mathrm{T}$, $y = (y_1, y_2)^\mathrm{T} = (\theta_\mathrm{l}, \dot{\theta}_\mathrm{l})^\mathrm{T}$ and the following functions

$$f(x(t), t) = \begin{pmatrix} x_2(t) \\ \dfrac{1}{J_\mathrm{m} + J_\mathrm{l}/n^2}\left[[-(B_\mathrm{m} + B_\mathrm{l}/n^2)x_2(t) - \dfrac{Mgl}{n}\sin(x_1(t)/n)\right] \end{pmatrix} \tag{56}$$

$$B(x(t), t) = \begin{pmatrix} 0 \\ 1 \\ \dfrac{1}{J_\mathrm{m} + J_\mathrm{l}/n^2} \end{pmatrix} \tag{57}$$

The state disturbances $\eta(k)$ include the frictions and amplifier and circuit uncertainties. The output equation takes the form (37) with

$$g(x(t), t) = \begin{pmatrix} x_1(t)/n \\ x_2(t)/n \end{pmatrix} \tag{58}$$

The output noises $\xi(t)$ include the sensor noises and numerical differentiation errors.

In the experiment, the sampled-data ILC (42) is used with the sampling time interval set to $\Delta = 50$ ms. The set of sampling instances is $\mathcal{N} = \{0, 1, \ldots, 60\}$. The first equation of (42) is given as

$$v_{i+1}(k) = u_i(k) + l_1(\dot{\theta}_{\mathrm{ld}}(k+1) - \dot{\theta}_{li}(k+1)) + l_2(\theta_{\mathrm{ld}}(k+1) - \theta_{li}(k+1)) \tag{59}$$

The learning control gain matrix has two gains as $L = [l_1, l_2]$. The convergence condition (43) becomes

$$\left| 1 - \frac{l_2 n \Delta}{n J_{\mathrm{m}} + J_1} \right| < \rho < 1 \tag{60}$$

It is easy to choose a learning gain to satisfy the inequality. In the experiment, $l_1 = l_2 = 2$ are chosen. No on-line feedback control is used and the initial input is set to zero, i.e. $u_0(k) = 0$, for $k \in \mathcal{N}$.

The experimental results are given in figures 2 and 3. They clearly show that, through iterations, the position trajectory converges to the desired trajectory defined in (55).

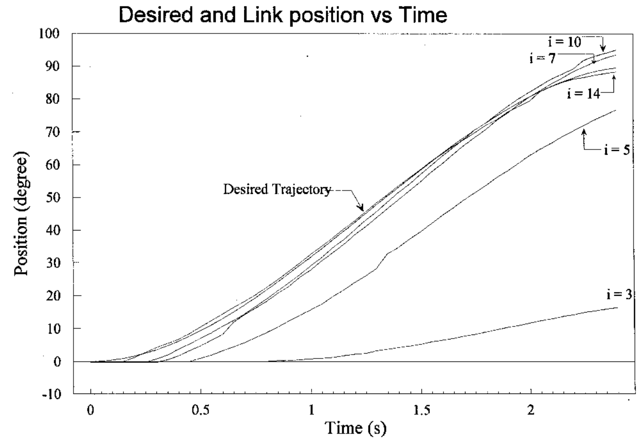

Figure 2. Link angle trajectories.

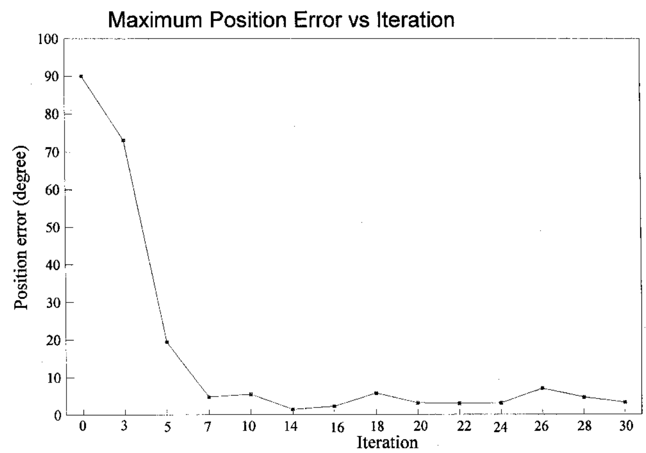

Figure 3. RMS of position tracking error.

5.2. *Experiment 2*

This experiment is performed using an industrial robot, SEIKO TT3000, which is a SCARA type robotic manipulator, as shown in figure 4. Joints 2 and 3 control the two links moving in a horizontal plane. The dynamics of these two links possess the non-linear and coupling characteristics and thus joints 2 and 3 are used to test our proposed controllers. The dynamics of these two links are given as

$$\begin{pmatrix} (m_2+m_3)a_2^2 + m_3 a_3^2 + 2 m_3 a_2 a_3 \cos\theta_2 & m_3 a_3^2 + m_3 a_2 a_3 \cos\theta_3 \\ m_3 a_3^2 + m_3 a_2 a_3 \cos\theta_3 & m_3 a_3^2 \end{pmatrix} \begin{pmatrix} \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{pmatrix} + \begin{pmatrix} -m_3 a_2 a_3 (2\dot{\theta}_2\dot{\theta}_3 + \theta_3^2) \sin\theta_3 \\ m_3 a_2 a_3 \dot{\theta}_2^2 \sin\theta_3 \end{pmatrix} + \begin{pmatrix} f_{v2}\dot{\theta}_2 \\ f_{v3}\dot{\theta}_3 \end{pmatrix} + \begin{pmatrix} F_{c2} \\ F_{c3} \end{pmatrix} = \begin{pmatrix} \tau_2 \\ \tau_3 \end{pmatrix} \tag{61}$$

where θ_j, f, F_{cj} and $\tau_j, j = 2, 3$ are the joint angles, viscous frictions, Coulomb frictions and control torques of joints 2 and 3, respectively. m_j and a_j, $j = 2, 3$, are the masses and centres of mass of links 2 and 3, respectively. The values of these parameters are unknown for our ILC design.

The second-order differential equation (61) can be rewritten in the form (36) in terms of state variables

$$x(t) = [\theta_2, \theta_3, \dot{\theta}_2, \dot{\theta}_3]^{\mathrm{T}}$$

The output equation (2) is given as

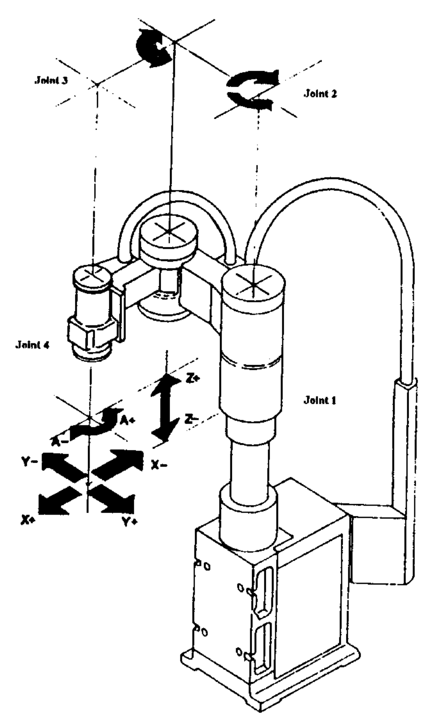

Figure 4. Experimental robot arm.

Figure 5. Robot control system.

$$y(t) = \left[\frac{\theta_2}{n_2}, \frac{\dot{\theta}_2}{n_2}, \frac{\theta_3}{n_3}, \frac{\dot{\theta}_3}{n_3}\right]^{\mathrm{T}} \tag{62}$$

where n_2 and n_3 are the unknown gear ratios of joints 2 and 3.

The robot control system consists of the industrial robot SEIKO TT3000 and an open architecture controller, as shown in figure 5. The robot controller has two levels of computer systems. The lower level is the real-time platform using a VME bus-based system with multiprocessor motion controllers. It includes the host computer MVME 147 consisting of an MC68030-based system, 4MB DRAM and 25 MHz system clock, and the target computer MVME104 consisting of an MC68010-based system, 10 MHz CPU clock frequency and 512 k byte random access memory. The MVME104 is also responsible for I/O operations, including four channels encoder input ports and four channels A/D converters. The second level is the PC platform, which is mainly responsible for task management and data processing.

The joint angle values are measured in real-time at MVME104 and then passed to the motion control algorithm which is in MVME147 for control input calculations. The control signal is converted to analogue signals for PWM power amplifiers with working switching frequency as high as 40 KHz. These outputs are channelled to control the joint motors. The sensing and control inputs are run at the frequency of $f = 244$ Hz.

The desired velocity trajectory of joint 2 is given as follows

$$\dot{\theta}_{2\mathrm{d}}(t) = \begin{cases} 0.05t^2, & \text{for } 0 \le t < 300/f \\ -0.05(t - 600/f)^2, & \\ & \text{for } 300/f \le t < 600/f \\ 0.15(0.3\cos(t - 600/f) + 0.7), & \\ & \text{for } t \ge 600/f \end{cases}$$

and the desired velocity trajectory of joint 3 is given as

$$\dot{\theta}_{3\mathrm{d}} = -\dot{\theta}_{2\mathrm{d}}(t)$$

The desired position trajectories for joints 2 and 3 are given as

$$\theta_{j\mathrm{d}}(t) = \int_0^t \dot{\theta}_{j\mathrm{d}}(\tau)\,\mathrm{d}\tau + \theta_{j\mathrm{d}}(0)$$

with $j = 2, 3$ and integration constants (desired initial joint angle positions) $\theta_{2\mathrm{d}}(0) = 0.5$ rad and $\theta_{3\mathrm{d}}(0) = 1.786$ rad.

The controllers for both joints are given in the form of, for $j = 2, 3$,

$$u_j(t) = u_{j\mathrm{fb}}(t) + u_{j\mathrm{ff}}(t) \tag{63}$$

where $u_{j\mathrm{fb}}(t)$ are the feedback PD controllers to ensure stability. The position and velocity gains are chosen as $k_{\mathrm{p2}} = k_{\mathrm{p3}} = 50$ and $k_{\mathrm{v2}} = k_{\mathrm{v3}} = 50$. $u_{j\mathrm{ff}}(t)$ is the feed-forward input which is updated using the anticipatory ILC law of the form (42). The sampling interval is chosen as $\Delta = 0.02$ s. The learning gain matrix is chosen as

$$L(\cdot) = \mathrm{diag}[l_{\mathrm{p2}}, l_{\mathrm{v2}}, l_{\mathrm{p3}}, l_{\mathrm{v3}}] = \mathrm{diag}[60, 6, 37.5, 1.25]$$

The experimental results, in figures 6–9, clearly show the convergence of the tracking errors in both joints 2 and 3, respectively. This experiment demonstrates the effectiveness of the proposed anticipatory ILC scheme and its robustness against uncertainties in robot repeatability, frictions and measurement noises.

6. Conclusion

In this paper, continuous-time non-linear dynamic systems with relative degree one are considered and the convergence of iterative learning control is studied. The proposed anticipatory ILC scheme uses a causal pair of the input action taken and its produced result

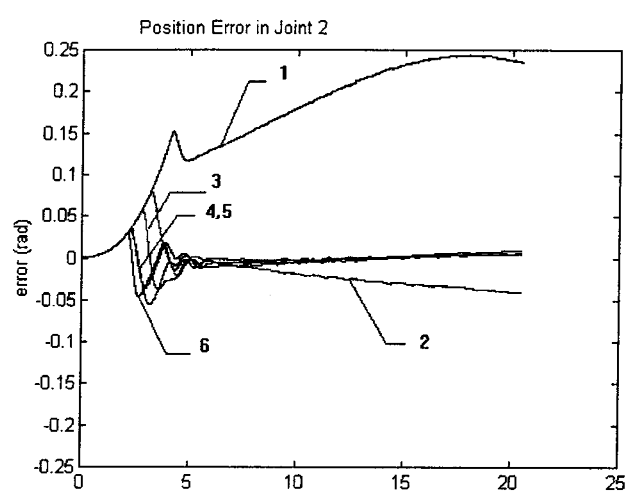

Figure 6. Tracking errors for joint 2.

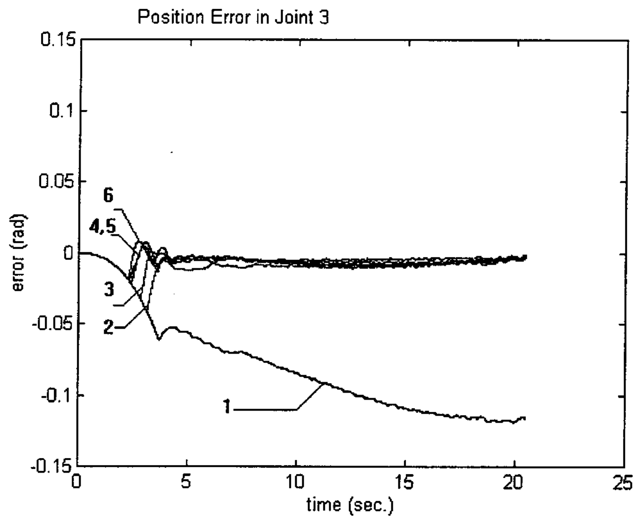

Figure 7. Tracking errors for joint 3.

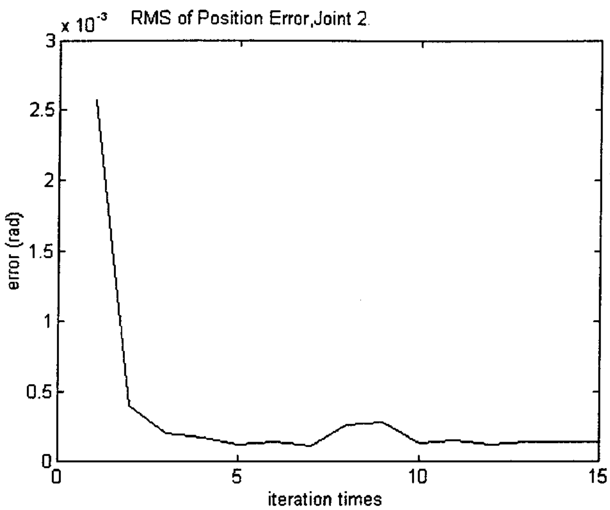

Figure 8. RMS of tracking errors for joint 2.

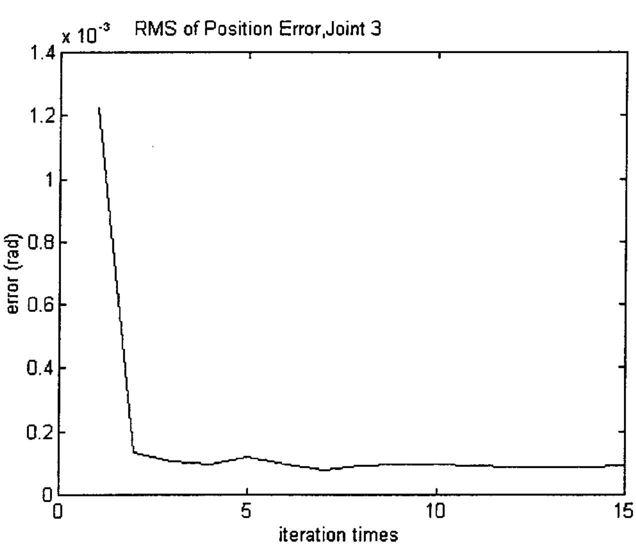

Figure 9. RMS of tracking errors for joint 3.

in state variable measurement to compute the required input action for the next operation cycle. The time shift ahead in the output errors installs anticipatory characteristics in the updating laws (4), (10) and (42). When the sampled data form is used, this ILC scheme saves computation time and achieves convergent results. Experimental results show its effectiveness and robustness against uncertainties in the robot repeatability, frictions and measurement noises.

References

AHN, H. S., CHOI, C. H., and KIM, K. B., 1993, Iterative learning control for a class of nonlinear systems. *Automatica*, **29**, 1575–1578.

AMANN, N., OWENS, D. H., and ROGERS, E., 1995, Iterative learning control for discrete time systems using optimal feedback and feedforward actions. *Proceedings of the 34th Conference on Decision and Control*, New Orleans, LA, pp. 1696–1701.

ARIMOTO, S., 1990, Learning control theory for robotic motion. *International Journal of Adaptive Control and Signal Processing*, **4**, 543–564.

ARIMOTO, S., NANIWA, T., and SUZUKI, H., 1991, Selective learning with a forgetting factor for robotic motion control. *Proceedings of the 1991 IEEE International Conference on Robotics and Automation,* Sacramento, CA, pp. 728–733.

BIEN, Z., and HUH, K. M., 1989, High-order iterative learning control algorithm. *IEE Proceedings, Part D*, **136**, 105–112.

CHEAH, C. C., and WANG, D., 1998a, Discrete-time model reference learning control: theory and experiment. *Dynamics and Control*, **8**, 145–162.

CHEAH, C. C., and WANG, D., 1998b, Impedance learning control for robotic manipulators. *IEEE Transactions on Robotics and Automation*, **14**, 452–464.

CHEAH, C. C., WANG, D., and SOH, Y. C., 1994, Convergence and robustness of a discreet-time learning control scheme for constrained manipulators. *Journal of Robotic Systems*, **11**, 223–238.

CHIEN, C. J., 1998, On the iterative learning control of sampled-data systems. In Z. Bien and J. Xu (Eds), *Iterative Learning Control: Analysis, Design, Integration and Applications* (Boston, MA; Kluwer), pp. 71–82.

CHIEN, C. J., and LIU, J. S., 1996, A p-type iterative learning controller for robust output tracking of non-linear time-varying systems. *International Journal of Control*, **64**, 319–334.

FLETT, T. M., 1980, *Differential Analysis* (Cambridge: Cambridge University Press).

HAC, A., 1990, Learning control in the presence of measurement noise. *Proceedings of American Control Conference*, pp. 2846–2851.

HAUSER, J. E., 1987, Learning control for a class of nonlinear systems. *Proceedings of the 26th IEEE Conference on Decision and Control*, Los Angeles, CA, pp. 859–860.

HEINZINGER, G., FENWICK, D., PADEN, B., and MIYAZAKI, F., 1992, Stability of learning control with disturbances and uncertain initial conditions. *IEEE Transactions on Automatic Control*, **37**, 110–114.

KUC, T.-Y., LEE, J. S., and NAM, K., 1992, An iterative learning control theory for a class of nonlinear dynamic systems. *Automatica*, **28**, 1215–1221.

LONGMAN, R. W., 1998, Designing iterative learning and repetitive controllers. In Z. Bien and J. Xu (Eds), *Iterative Learning Control: Analysis, Design, Integration and Applications* (Boston, MA: Kluwer), pp. 107–146.

MOORE, K. L., DAHLEH, M., and BHATTACHARYYA, S. P., 1992, Iterative learning control: a survey and new results. *Journal of Robotic Systems*, **9**, 563–594.

OH, S.-R., LIM, M.-S., AHN, H. S., and KIM, K. B., 1994, Experimental study on iterative learning control algorithms for direct drive robot arm. *Proceedings of the Asian Control Conference*, Tokyo, pp. 553–556.

ROGERS, E., and OWENS, D. H., 1992, *Stability Analysis for Linear Repetitive Processes* (Berlin: Springer-Verlag).

SAAB, S. S., 1994, On the P-type learning control. *IEEE Transactions on Automatic Control*, **39**, 2298–2302.

TSO, S. K., and MA, L. Y. X., 1993, Discrete learning control for robots: strategy, convergence and robustness. *International Journal of Control*, **57**, 273–291.

WANG, D., 1995, An effect-driven discrete time ILC scheme for continuous time nonlinear systems. Technical report, EEE, Nanyang Technological University, Singapore.

WANG, D., 1998a, An anticipatory iterative learning control scheme: theory and experiments. *Proceedings of Iterative Learning Control Workshop and Roundtable*, Tampa, FL, pp. 79–80.

WANG, D., 1998b, Convergence and robustness of discrete-time nonlinear systems with iterative learning control. *Automatica*, **32**, 1445–1448.

WANG, D., 1999, On anticipatory iterative learning control designs for continuous time nonlinear dynamic systems. *Proceedings of the 38th IEEE Conference on Decision and Control*, Phoenix, AZ, 1605–1610.

WANG, D., and CHEAH, C. C., 1998, An iterative learning control scheme for impedance control of robotic manipulators. *International Journal of Robotics Research*, **19**, 1091–1104.

XU, J. X., 1997, Analysis of iterative learning control for a class of nonlinear discrete-time systems. *Automatica*, **33**, 1905–1907.

XU, J, X., 1998, Direct learning of control efforts for trajectories with different time scales. *IEEE Transactions on Automatic Control*, **43**, 1027–1030.

XU, J. X., and ZHU, T., 1999, Dual-scale direct learning control of trajectory tracking for a class of nonlinear uncertain systems. *IEEE Transactions on Automatic Control*, **44**, 1884–1888.

ZHANG, N., 1996, Experimental research on robot control. Master's thesis, EEE, Nanyang Technological University, Singapore.