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An Iterative Learning-Control Scheme for Impedance Control of Robotic Manipulators

Abstract

In this paper, an iterative learning-control law is proposed for impedance control of robotic manipulators. In most of the learning-controller designs in the literature, a reference trajectory is given and a learning algorithm is designed to force the trajectory tracking error to converge to zero as the action is repeated. In contrast, our approach allows the performance of the learning system to be specified by a target impedance. A design method for analyzing the learning-impedance system is developed, and sufficient conditions for guaranteeing the convergence of the error to zero are derived. The robustness of the learning impedance-control system to the fluctuation of the dynamics, output measurement noise, and error in the initial conditions is also analyzed in details. Experimental results on a system using an industrial robot (SEIKO TT3000) are presented to illustrate the theoretical results.

1. Introduction

Most industrial robots employed in assembly applications perform an assigned task repetitively, with each task requiring a prescribed period of time. However, whatever errors that may exist in following a trajectory will be repeated in the subsequent operations. Since most of the present industrial robots have largely repeatable dynamics due to the superiority of repeatability precision, a learning controller can be designed to improve the performance of the robot as the operation is repeated. Learning-control schemes are easy to implement, and do not require exact knowledge of the dynamic model of the robot.

Many research efforts have been devoted to defining and analyzing learning-control schemes. A recent survey of the works by Arimoto is available (Spong, Lewis, and Abdallah 1993). In most of the proposed learning schemes, learning systems are designed to track a given desired trajectory as an operation is repeated. Examples of learning-motion control and hybrid position-force control designs have been described (Craig 1984; Heinzinger et al. 1992; Sugie and Ono 1991; Uchiyama 1978, Arimoto, Kawamura, and Miyazaki 1984; Bondi, Casalino, and Gambardella 1988; Arimoto 1990, 1991; Aicardi, Cannata, and Gasalino 1992; Cheah, Wang, and Soh 1992, 1994; Jeon and Tomizuka 1993; Kawamura, Miyazaki, and Arimoto 1985; Wang and Cheah 1996; Wang, Soh, and Cheah 1995; Arimoto, Liu, and Naniwa 1993). However, in certain applications such as impedance control (Hogan 1985) of robotic manipulators, the control objective is specified explicitly by a reference model (or target impedance) rather than by a desired trajectory. Impedance control does not attempt to track motion and force trajectories, but rather to regulate a mechanical impedance that is specified by a target model at the robot end effector (Hogan 1985):

$$M_m\ddot{X} + C_m(\dot{X} - \dot{X}_d) + K_m(X - X_d) = F, \qquad (1)$$

where M_m, C_m, and $K_m \in R^{n\times n}$ are matrices that specify the desired dynamic relationship between the position error and the external force $F \in R^n$ exerted on the robot end effector. By controlling the manipulator position and specifying its relationship with the external forces, the contact-force response can be kept in a desired range. Impedance control is one of the major approaches used in the controller design for force-control problems of robotic manipulators (Spong, Lewis, and Abdallah 1993). It provides a unified approach to all aspects of manipulation (Hogan 1985). When using this approach, both free motion and contact tasks can be controlled using a single control algorithm. The nature of the trajectory learning formulation has limited the research of this important prob-

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lem, because in the impedance-control problem, a desired model is specified rather than the trajectory (Hogan 1985). Recently, a learning-control scheme for impedance control of a robotic manipulator was proposed by Cheah and Wang (1994, 1995). However, it was implicitly assumed in those papers that the manipulator must take the same initial position and velocity at every operation trial; also, it was assumed that there were neither fluctuation of dynamics nor measurement noise. From a practical point of view, it is important to establish the robustness of any learning-control scheme with respect to bounded errors, fluctuations of the dynamics, and measurement noise. To address the robustness issue of impedance learning-control design, Wang and Cheah proposed such a robust learning-control scheme (1996).

In this paper, we extend the impedance learning-control scheme described by Wang and Cheah (1996) for the impedance control of a robotic manipulator. Sufficient conditions for the selection of learning gains to guarantee the convergence of the learning system are derived. We study in detail the robustness of the proposed algorithm to dynamics fluctuations, output measurement noise, and error in the initial conditions. Furthermore, we show that the impedance error converges to a bound that tends to zero in the absence of dynamics fluctuations, output measurement noise, and error in the initial conditions. The proposed learning-impedance controller is applied to a SEIKO TT3000 industrial robot, and experimental results are presented to verify the theoretical developments.

2. The Robot Dynamic Model and Problem Formulation

The dynamic motion equations of a rigid-link manipulator with n degrees of freedom can be described in joint space by the following equation (Wang, Soh, and Cheah 1995):

$$M(q_k(t))\ddot{q}_k(t) + V(q_k(t), \dot{q}_k(t)) = u_k(t) + f_k(t), \qquad (2)$$

where $q_k(t) \in R^n$ represents the joint angle at the k^{th} operation; $t \in R$ is the operation time; $M(\cdot) \in R^{n\times n}$ is the inertia matrix; $V(\cdot,\cdot) \in R^n$ is the vector of centrifugal, Coriolis, and gravitational forces; $f_k(t) \in R^n$ is the external force; and $u_k(t) \in R^n$ is the control torque. Now, let $X_k(t) \in R^n$ be the Cartesian space vector defined by Lewis, Abdallah, and Dawson (1993):

$$X_k(t) = h(q_k(t)), \qquad (3)$$

where $h(q_k(t)) \in R^n$ is the manipulator kinematics describing the relationship between the joint and Cartesian space. Then, the derivative of $X_k(t)$ is given as

$$\dot{X}_k(t) = J(q_k(t))\dot{q}_k(t), \qquad (4)$$

where $J(\cdot) = \frac{\partial h(\cdot)}{\partial q_k} \in R^{n\times n}$ is the manipulator Jacobian matrix, which is assumed to be nonsingular. The external force $F_k(t) \in R^n$ in Cartesian space is related to the external force in joint space as

$$f_k(t) = J^T(q_k(t))F_k(t). \qquad (5)$$

It is assumed that the relationship between $F_k(t)$ and $X_k(t)$ at the contact point is described by

$$F_k(t) = -M_E(X_k(t))\ddot{X}_k(t) + k_E(X_k(t), \dot{X}_k(t), t), \qquad (6)$$

where $M_E(\cdot) \in R^{n\times n}$ is the inertia of the environment, and $k_E(\cdot,\cdot,\cdot) \in R^n$ is a function that describes the environmental damping and stiffness. Both $M_E(\cdot)$ and $k_E(\cdot,\cdot,\cdot)$ can be nonlinear in their arguments. We also suppose that a feedback law has been designed to stabilize the closed-loop system (Arimoto 1992). This feedback law can be written as

$$u_k(t) = K_v(\dot{q}_d(t) - \dot{q}_k(t)) + K_p(q_d(t) - q_k(t)) + m_k(t), \qquad (7)$$

where $q_d(t) \in R^n$ is the reference-joint angle, $\dot{q}_d(t) \in R^n$ is the reference-joint velocity, K_v and $K_p \in R^{n\times n}$ are the feedback-gain matrices, and $m_k(t) \in R^n$ is the learning-control input, which is added to learn the desired impedance of the system as the operation is repeated. When the control input (eq. (7)) is applied to the robotic manipulator described by eq. (2), we obtain

$$M(q_k(t))\ddot{q}_k(t) + N(q_k(t), \dot{q}_k(t), t) = m_k(t) + f_k(t), \qquad (8)$$

where $N(q_k(t), \dot{q}_k(t), t) = V(q_k(t), \dot{q}_k(t)) + K_v(\dot{q}_k(t) - \dot{q}_d(t)) + K_p(q_k(t) - q_d(t))$.

We assume that the robot parameters and the environment parameters ($M_E(\cdot)$, $k_E(\cdot,\cdot,\cdot)$) are unknown. However, the learning system satisfies the following properties (Arimoto 1990; Bondi, Casalino, and Gambardella 1988):

(A1) Every operation ends in a finite time interval; i.e., $t \in [0, T]$.

(A2) The desired motion is specified a priori over the time duration $t \in [0, T]$ by the following target impedance at the end effector:

$$M_m\ddot{X}(t) + C_m(\dot{X}(t) - \dot{X}_d(t)) + K_m(X(t) - X_d(t)) = F(t), \qquad (9)$$

where M_m, C_m, and $K_m \in R^{n\times n}$ are matrices that specify the dynamic response of the system, which should be chosen so that the target impedance is asymptotically stable.

(A3) The inertia matrix $M(\cdot)$ is positive definite and bounded such that

$$\kappa_1 I \le M(q) \le \kappa_2 I, \qquad (10)$$

where κ_1 and κ_2 are scalars such that $\kappa_2 > \kappa_1 > 0$ (Craig 1988). Hence, $M^{-1}(q_k)$ exists and is positive definite and bounded. Furthermore, $J(\cdot)$ and its derivative $\dot{J}(\cdot)$ are bounded for all $t \in [0, T]$.

(A4) $M(\cdot)$, $N(\cdot,\cdot,\cdot)$, $J(\cdot)$, $\dot{J}(\cdot)$, $M_E(\cdot)$, and $k_E(\cdot,\cdot,\cdot)$ are local Lipschitzian functions of their arguments (Arimoto 1990) for $t \in [0, T]$.

(A5) The system dynamics is invertible such that for a given reference model (eq. (9)), there exists a unique control input $u_e(t) \in R^n$ corresponding to the solutions $X_e(t) = h(q_e(t))$, $\dot{X}_e(t) = J(q_e(t))\dot{q}_e(t)$ and $F_e(t) = -M_E(X_e(t))\ddot{X}_e(t) + k_E(X_e(t), \dot{X}_e(t), t)$ of the target impedance.

(A6) Repeatability of initialization is satisfied throughout repeated training. That is,

$$q_k(0) = q_d(0), \quad \dot{q}_k(0) = \dot{q}_d(0), \quad \forall k = 0, 1, \cdots. \tag{11}$$

(A7) Invariance of the system dynamics is assured throughout repeated operations.

The objective of learning-impedance-control design is to develop an iterative learning law such that as $k \to \infty$, for $t \in [0, T]$,

$$w_k(t) \to 0, \tag{12}$$

where

$$\begin{aligned} w_k(t) = {} & -M_m\ddot{X}_k(t) + C_m(\dot{X}_d(t) - \dot{X}_k(t)) \\ & + K_m(X_d(t) - X_k(t)) + F_k(t) \end{aligned} \tag{13}$$

is defined as the impedance error.

REMARK 1. In the conventional formulation, a learning controller is designed to track the desired trajectory as the action is repeated. In general, a feedforward control input is learned so that (Wang, Soh, and Cheah 1995)

$$X_k(t) \to X_d(t), \qquad F_k(t) \to F_d(t), \tag{14}$$

as $k \to \infty$, for all $t \in [0, T]$. In this proposed learning approach, the control objective can be specified by a target impedance so that the impedance error described in eq. (12) converges to zero as the action is repeated. That is, a feedforward control input is learned so that

$$w_k(t) \to 0, \tag{15}$$

as $k \to \infty$, for all $t \in [0, T]$. As seen from eq. (13), the objective of learning in this paper is the relationship of force and position in a dynamic manner. Note that the reaction force is not treated as an independent target variable to be controlled, as in the case of pure force control.

Furthermore, since the desired variables $X_e(t)$ and $F_e(t)$ cannot be derived from the reference model (eq. (9)) because $k_E(\cdot,\cdot,\cdot)$ and $M_E(\cdot)$ are unknown, it is not possible to specify the desired trajectories $X_e(t)$ and $F_e(t)$ from the reference model. Therefore, conventional trajectory learning-control schemes cannot be applied directly for learning the desired model explicitly from the desired trajectories.

REMARK 2. The stability of the robot system can be guaranteed by a predesigned on-line feedback controller (Arimoto 1992) described by eq. (7), and thus the velocity is bounded. Therefore, assumptions (A3) and (A4) (Arimoto 1990; Craig 1988) are valid.

3. Learning-Impedance Control

The iterative learning-control input $m_k(t)$ for learning the target impedance (eq. (9)) is updated using the following equation:

$$m_{k+1}(t) = m_k(t) + L(q_k(t))w_k(t), \tag{16}$$

where $L(\cdot) \in R^{n\times m}$ is the learning gain. The choice of the learning gain $L(\cdot)$ is specified by the following theorem.

THEOREM 1. Consider the learning-impedance control system described by eqs. (8), (5), (6), (16), and (13) with the desired model specified by eq. (9). Let $L(\cdot)$ be any bounded learning gain that satisfies the condition:

$$\begin{aligned} \| I_n - L(q_k(t))[M_m + M_E(X_k(t))]J(q_k(t))[M(q_k(t)) \\ + J^T(q_k(t))M_E(X_k(t))J(q_k(t))]^{-1} \| \le p < 1. \end{aligned} \tag{17}$$

Then, the impedance error $w_k(t)$ generated by the learning-control input $m_k(t)$ converges such that

$$w_k(t) \to 0, \tag{18}$$

uniformly for all $t \in [0, T]$, as $k \to \infty$.

The following definition is needed in the subsequent development.

DEFINITION 1. The convergence of the impedance error is derived using α-norms. The α-norm for a function $b(t)$, with α being a positive scalar, is defined as

$$\| b(t) \|_\alpha \stackrel{\Delta}{=} \sup_{t\in[0,T]} e^{-\alpha t} \| b(t) \|. \tag{19}$$

Note that

$$\|b(t)\|_\alpha \le \|b(t)\|_\infty \le e^{\alpha T}\|b(t)\|_\alpha, \tag{20}$$

where $\|b(t)\|_\infty = \sup_{t\in[0,T]} \|b(t)\|$. Thus, the α-norm of a function is equivalent to its ∞-norm.

Proof. For clarity of the proof, the dependence of the system parameters on time is implied unless otherwise specified. Using eq. (13), define the desired state $[X_e^T, \dot{X}_e^T]^T$ and the desired force F_e corresponding to the desired impedance (eq. (9)) as

$$w_e = -M_m\ddot{X}_e + C_m(\dot{X}_d - \dot{X}_e) + K_m(X_d - X_e) + F_e = 0, \quad (21)$$

where

$$F_e = -M_E(X_e)\ddot{X}_e + k_E(\dot{X}_e, X_e, t). \quad (22)$$

The desired state $[X_e^T, \dot{X}_e^T]^T$ is unknown since F_e is an unknown desired force; F_e is unknown because $M_E(\cdot)$ and $k_E(\cdot,\cdot,\cdot)$ are unknown. The definitions of the desired state and force are used for analysis purposes only, and are not used in the control law. Since $w_e = 0$, from eqs. (13) and (21), we have

$$\begin{aligned} w_k &= w_k - w_e = M_m(\ddot{X}_e - \ddot{X}_k) + C_m(\dot{X}_e \\ &- \dot{X}_k) + K_m(X_e - X_k) - (F_e - F_k), \end{aligned} \quad (23)$$

where, from eqs. (22) and (6), we have

$$\begin{aligned} F_e - F_k = &-M_E(X_k)(\ddot{X}_e - \ddot{X}_k) - (M_E(X_e) \\ &- M_E(X_k))\ddot{X}_e + k_E(X_e, \dot{X}_e, t) - k_E(X_k, \dot{X}_k, t). \end{aligned} \quad (24)$$

Differentiating eq. (4) with respect to time, we have

$$\ddot{X}_k = J(q_k)\ddot{q}_k + \dot{J}(q_k)\dot{q}_k. \quad (25)$$

Then, from eqs. (8), (5), (6) and (25), we obtain

$$\begin{aligned} M(q_k)\ddot{q}_k + N(q_k, \dot{q}_k, t) &= m_k \\ &+ J^T(q_k)(-M_E(X_k)(J(q_k)\ddot{q}_k + \dot{J}(q_k)\dot{q}_k) \\ &+ k_E(X_k, \dot{X}_k, t)). \end{aligned} \quad (26)$$

Hence,

$$\begin{aligned} (M(q_k) + J^T(q_k)M_E(X_k)J(q_k))\ddot{q}_k + \bar{N}(q_k, \dot{q}_k, t) \\ - J^T(q_k)k_E(X_k, \dot{X}_k, t)) = m_k, \end{aligned} \quad (27)$$

where $\bar{N}(q_k, \dot{q}_k, t) = N(q_k, \dot{q}_k, t) + J^T(q_k)M_E(h(q_k))\dot{J}(q_k)\dot{q}_k$. From eqs. (16), (23), (3), (4), (24), (25), and (27), we have

$$\begin{aligned} \delta m_{k+1} &= \delta m_k - L(q_k)\{M_m(\ddot{X}_e - \dot{X}_k) + C_m(\dot{X}_e - \dot{X}_k) \\ &\quad + K_m(X_e - X_k) - \delta F_k\} \\ &= \delta m_k - L(q_k)\{(M_m + M_E(X_k))(J(q_k)\delta\ddot{q}_k \\ &\quad + \delta J(q_k)\ddot{q}_e + \dot{J}(q_k)\delta\dot{q}_k + \delta\dot{J}(q_k)\dot{q}_e) \\ &\quad + C_m(J(q_k)\delta\dot{q}_k + \delta J(q_k)\dot{q}_e) + K_m\delta h(q_k) \\ &\quad + \delta M_E(X_k)\ddot{X}_e - \delta k_E(X_k, \dot{X}_k, t)\} \\ &= [I - L(q_k)(M_m + M_E(X_k))J(q_k)(M(q_k) \\ &\quad + J^T(q_k)M_E(X_k)J(q_k))^{-1}]\delta m_k \\ &\quad - L(q_k)(M_m + M_E(X_k))\{\delta J(q_k)\ddot{q}_e \\ &\quad + \dot{J}(q_k)\delta\dot{q}_k + \delta\dot{J}(q_k)\dot{q}_e \\ &\quad - J(q_k)R^{-1}(q_k)(\delta\bar{N}(q_k, \dot{q}_k, t) \\ &\quad - J^T(q_k)\delta k_E(X_k, \dot{X}_k, t) \\ &\quad - \delta J^T(q_k)k_E(X_e, \dot{X}_e, t)) \\ &\quad - J(q_k)\delta R^{-1}(q_k)(\bar{N}(q_e, \dot{q}_e, t) \\ &\quad - J^T(q_e)k_E(X_e, \dot{X}_e, t) + m_e)\} \\ &\quad - L(q_k)(C_m(J(q_k)\delta\dot{q}_k \\ &\quad + \delta J(q_k)\dot{q}_e) + K_m\delta h(q_k) \\ &\quad + \delta M_E(X_k)\ddot{X}_e - \delta k_E(X_k, \dot{X}_k, t)), \end{aligned} \quad (28)$$

where $\delta m_{k+1} = m_e - m_{k+1}$, $\delta m_k = m_e - m_k$, $f_e = J^T(q_e)F_e$, $\delta J(q_k) = J(q_e) - J(q_k)$, $\delta\dot{J}(q_k) = \dot{J}(q_e) - \dot{J}(q_k)$, $\delta q_k = q_e - q_k$, $\delta\dot{q}_k = \dot{q}_e - \dot{q}_k$, $\delta F_k = F_e - F_k$, $\ddot{X}_e = J(q_e)\ddot{q}_e + \dot{J}(q_e)\dot{q}_e$, $R(q_k) = M(q_k) + J^T(q_k)M_E(X_k)J(q_k)$, $\delta\bar{N}(q_k, \dot{q}_k, t) = \delta N(q_k, \dot{q}_k, t) + J^T(q_k)M_E(X_k)(\dot{J}(q_k)\delta\dot{q}_k + \delta\dot{J}(q_k)\dot{q}_e) + J^T(q_k)\delta M_E(X_k)\dot{J}(q_e)\dot{q}_e + \delta J^T(q_k)M_E(X_e)\dot{J}(q_e)\dot{q}_e$, $\delta M_E(X_k) = M_E(X_e) - M_E(X_k)$, $\delta R^{-1}(q_k) = R^{-1}(q_e) - R^{-1}(q_k)$, $\delta N(q_k, \dot{q}_k, t) = N(q_e, \dot{q}_e, t) - N(q_k, \dot{q}_k, t)$, $\delta k_E(X_k, \dot{X}_k, t) = k_E(X_e, \dot{X}_e, t) - k_E(X_k, \dot{X}_k, t)$, $\delta h(q_k) = h(q_e) - h(q_k)$ and

$$m_e = M(q_e)\ddot{q}_e + N(q_e, \dot{q}_e, t) - f_e. \quad (29)$$

Taking norms on eq. (28), and using the bounds and Lipschitz conditions, yields

$$\begin{aligned} \|\delta m_{k+1}\| \le\; &\|I - L(q_k)(M_m + M_E(X_k))J(q_k)(M(q_k) \\ &+ J^T(q_k)M_E(X_k)J(q_k))^{-1}\| \cdot \|\delta m_k\| \\ &+ \|L(q_k)\| \cdot \|M_m \\ &+ M_E(X_k)\| \cdot (\|\delta J(q_k)\| \cdot \|\ddot{q}_e\| \\ &+ \|\dot{J}(q_k)\| \cdot \|\delta\dot{q}_k\| + \|\delta\dot{J}(q_k)\| \cdot \|\dot{q}_e\| \\ &+ \|J(q_k)\| \cdot \|R^{-1}(q_k)\| \cdot (\|\delta\bar{N}(q_k, \dot{q}_k, t)\| \\ &+ \|J^T(q_k)\| \cdot \|\delta k_E(X_k, \dot{X}_k, t)\| \\ &+ \|\delta J^T(q_k)\| \cdot \|k_E(X_e, \dot{X}_e, t)\|) \\ &+ \|J(q_k)\| \cdot \|\delta R^{-1}(q_k)\| \cdot \|\bar{N}(q_e, \dot{q}_e, t) \\ &- J(q_e)k_E(X_e, \dot{X}_e, t) + m_e\| \\ &+ \|L(q_k)\| \cdot (\|C_m\| \cdot (\|J(q_k)\| \cdot \|\delta\dot{q}_k\| \\ &+ \|\delta J(q_k)\| \cdot \|\dot{q}_e\|) \\ &+ \|K_m\| \cdot \|\delta h(q_k)\| + \|\delta M_E(X_k)\| \cdot \|\ddot{X}_e\| \\ &+ \|\delta k_E(X_k, \dot{X}_k, t)\|) \\ \le\; &p\|\delta m_k\| + b_L c_1 \left\| \begin{bmatrix} \delta q_k \\ \delta\dot{q}_k \end{bmatrix} \right\|, \end{aligned} \quad (30)$$

where $c_1 = (b_{Mm} + b_{ME})(b_{q2}c_J + b_{J1} + c_{J1}b_{q1} + b_J b_R c_{\bar{N}} + b_J b_R b_J c_{kE} c_x + b_J b_R c_J b_{kE} + b_J c_R b_{\bar{N}} + b_J c_R b_J b_{KE} + b_J c_R b_1) + b_{Cm}b_J + b_{Cm}c_J b_{q1} + b_{Km}c_h + c_{kE}c_x + c_{ME}c_h b_{x2}$, $c_{\bar{N}} = c_N + b_J b_{ME}(b_{J1} + c_{J1}b_{q1}) + b_J c_{ME} b_{J1} b_{q1} + c_J b_{ME} b_{J1} b_{q1}$, $c_x = c_h + b_J + c_J b_{q1}$, $b_L, b_J, b_{J1}, b_R, b_{Mm}, b_{Cm}, b_{Km}, b_{\bar{N}}, b_{ME}, b_{kE}$ are the norm bound for $L(\cdot)$, $J(\cdot)$, $\dot{J}(\cdot)$, $R^{-1}(\cdot)$, M_m, C_m, K_m, $\bar{N}(\cdot,\cdot,\cdot)$, $M_E(\cdot)$, and $k_E(\cdot,\cdot,\cdot)$, respectively; $b_{q1} = \sup_{t\in[0,T]} \|\dot{q}_e(t)\|$, $b_{q2} = \sup_{t\in[0,T]} \|\ddot{q}_e(t)\|$, $b_{X2} = \sup_{t\in[0,T]} \|\ddot{X}_e(t)\|$, $b_1 = \sup_{t\in[0,T]} \|m_e(t)\|$, and c_J, c_{J1}, $c_{\bar{N}}$, c_R, c_h, c_{ME}, c_{kE} are the Lipschitz constants for $J(\cdot)$, $\dot{J}(\cdot)$, $\bar{N}(\cdot,\cdot,\cdot)$, $R^{-1}(\cdot)$, $h(\cdot)$, $M_E(\cdot)$, and $k_E(\cdot,\cdot,\cdot)$, respectively. From eqs. (26), (3), and (4), we have

$$\begin{bmatrix} \delta\dot{q}_k \\ \delta\ddot{q}_k \end{bmatrix} = \delta f(q_k,\dot{q}_k,t) + \delta g(q_k)m_e + g(q_k)\delta m_k, \quad (31)$$

where $f(q_k,\dot{q}_k,t) = \begin{bmatrix} \delta\dot{q}_k \\ -R^{-1}(q_k)(\bar{N}(q_k,\dot{q}_k,t) \\ -J^T(q_k)k_E(h(q_k),J(q_k)\dot{q}_k,t)) \end{bmatrix}$, $\delta f(q_k,\dot{q}_k,t) = f(q_e,\dot{q}_e,t) - f(q_k,\dot{q}_k,t)$, $g(q_k) = \begin{bmatrix} 0 \\ -R^{-1}(q_k) \end{bmatrix}$, and $\delta g(q_k) = g(q_e) - g(q_k)$. Integrating eq. (31) with respect to time and taking norms, we have

$$\begin{aligned} \left\|\begin{bmatrix} \delta q_k \\ \delta\dot{q}_k \end{bmatrix}\right\| &\le \int_0^t \{\|\delta f(q_k,\dot{q}_k,\tau)\| + \|\delta g(x_k)\|\cdot\|m_e\| \\ &+ \|g(q_k)\|\cdot\|\delta m_k\|\}d\tau \\ &\le \int_0^t \{c_2\left\|\begin{bmatrix} \delta q_k \\ \delta\dot{q}_k \end{bmatrix}\right\| + b_g\|\delta m_k\|\}d\tau, \end{aligned} \quad (32)$$

where $c_2 = c_f + c_g b_{me}$, c_f, and c_g are the Lipschitz constants for $f(\cdot,\cdot,\cdot)$ and $g(\cdot)$, respectively, $b_{me} = \sup_{t\in[0,T]} \|m_e(t)\|$, and b_g is the norm bound for $g(\cdot)$. Using the Bellman-Gronwall lemma, we obtain

$$\left\|\begin{bmatrix} \delta q_k \\ \delta\dot{q}_k \end{bmatrix}\right\| \le b_g \int_0^t e^{c_2(t-\tau)} \| \delta m_k \| \, d\tau. \quad (33)$$

Substituting eq. (33) into eq. (30), multiplying both sides by $e^{-\alpha t}$, defining $c \triangleq \max\{c_2, b_L b_g c_1\}$, and letting $\alpha > c_2$, we get

$$\begin{aligned} e^{-\alpha t}\|\delta m_{k+1}\| &\le p e^{-\alpha t}\|\delta m_k\| \quad (34) \\ &+ c\int_0^t e^{-\alpha\tau}\|\delta m_k\| e^{-(\alpha-c)(t-\tau)}d\tau, \end{aligned}$$

for all $t \in [0,T]$. Hence, we have

$$\| \delta m_{k+1} \|_\alpha \le \hat{p} \| \delta m_k \|_\alpha, \quad (35)$$

where

$$\hat{p} = [p + \frac{c}{\alpha - c}(1 - e^{-(\alpha-c)T})]. \quad (36)$$

If we choose p to be less than 1, $\alpha > c$ and large enough so that $0 \le \hat{p} < 1$, then eq. (35) converges such that

$$m_k \to m_e, \quad (37)$$

uniformly for all $t \in [0,T]$, as $k \to \infty$. Applying the same argument to eqs. (33) and (31), we have

$$\left\|\begin{bmatrix} \delta q_k \\ \delta\dot{q}_k \end{bmatrix}\right\|_\alpha \le \frac{b_g}{\alpha - c_2}(1 - e^{-(\alpha-c_2)T}) \|\delta m_k\|_\alpha, \quad (38)$$

$$\left\|\begin{bmatrix} \delta\dot{q}_k \\ \delta\ddot{q}_k \end{bmatrix}\right\|_\alpha \le (b_g + \frac{c_2 b_g}{\alpha - c_2}(1 - e^{-(\alpha-c_2)T})) \|\delta m_k\|_\alpha. \quad (39)$$

Therefore, from eqs. (23) and (24), we have

$$\| w_k \|_\alpha \le \bar{c} \, \|\delta m_k\|_\alpha, \quad (40)$$

where $\bar{c} = (b_{Mm} + b_{ME})b_J(b_g + \frac{c_2 b_g}{\alpha - c_2}(1 - e^{-(\alpha-c_2)T})) + ((b_{Mm} + b_{ME})(c_J b_{q2} + b_{J1} + c_{J1}b_{q1}) + (b_{Cm} + c_{KE})(b_J + c_J b_{q1}) + (b_{Km} + c_{ME}b_{x2} + c_{KE})c_h)(\frac{b_g(1-e^{-(\alpha-c_2)T})}{\alpha-c_2})$. Hence, we have

$$w_k(t) \to 0, \quad (41)$$

uniformly for all $t \in [0,T]$, as $k \to \infty$. □

REMARK 3. In practical implementations, the learning-control law described by eqs. (16) and (13) involves the measurement of position and force and off-line computation of the velocity and acceleration.

REMARK 4. From eq. (23), the reference trajectory error $X_e(t) - X_k(t)$ can be written as

$$\begin{aligned} X_e(t) - X_k(t) &= [p^2 M_m + pC_m \\ &+ K_m]^{-1}(F_e(t) - F_k(t) + w_k(t)), \end{aligned} \quad (42)$$

where $p = d/dt$. Therefore, in the special case of a free-motion or noncontact task where the contact force is zero, in addition to the convergence of the reference-model error $w_k(t)$, the reference-trajectory error also converges to zero such that

$$X_e(t) - X_k(t) = [p^2 M_m + pC_m + K_m]^{-1} w_k(t) \to 0. \quad (43)$$

Hence, using the model-reference approach, a unified learning controller can be developed for both contact and noncontact tasks without the need to switch the learning controllers from noncontact to contact tasks. This is important, since the current learning-control designs provide methods to control robots during contact and free motion separately. That is, during noncontact motion (Arimoto 1990), a learning controller suitable for the noncontact phase of motion is applied, and during contact, another learning controller suitable for contact motion is applied. From a practical point of view, most tasks involve a transition from free motion to contact motion, and every contact task ends with a transition from contact to free motion. Therefore, when these different control schemes are applied to the robots, the learning algorithms are needed to switch from one control to another, and the overall control is discontinuous in nature.

REMARK 5. If the reference model is specified for certain applications as discussed by Yao, Chan, and Wang (1992),

$$M_m(\ddot{X}_d(t) - \ddot{X}(t)) + C_m(\dot{X}_d(t) - \dot{X}(t)) + K_m(X_d(t) - X(t)) = F_d(t) - F(t), \quad (44)$$

the results still hold, as the feedforward terms are cancelled in eq. (23).

4. Robustness of Learning-Impedance Control

In the development of the previous section, it was assumed that the learning-control system must take the same initial state at every operation trial and that there are neither fluctuations of dynamics nor measurement noise. To establish the technical soundness of the learning-control system, it is important to analyze the robustness of the system with respect to the small but persistent error of initialization, fluctuations of the dynamics, and measurement noise during operation. In other words, it is important to relax the conditions posed in postulates (A6) and (A7). We assume that the update law in eq. (16) is contaminated by noise in the following manner (Arimoto 1990; Heinzinger et al. 1992):

(A'6) The initial-state error of the system is bounded such that $\left\| \begin{bmatrix} q_d(0) - q_k(0) \\ \dot{q}_d(0) - \dot{q}_k(0) \end{bmatrix} \right\| \le b_{q0}$ for any operation k.

(A'7) When the operation is not repeated under the same condition, bounded fluctuations may be present in the robot dynamics. That is, from eq. (8), we have

$$M(q_k(t))\ddot{q}_k(t) + N(q_k(t), \dot{q}_k(t), t) + d_k(t) = m_k(t) + f_k(t), \quad (45)$$

where $d_k(t) \in R^n$ is a term due to the fluctuations of the dynamics; and d_k is bounded by a constant b_d on $t \in [0, T]$.

(A'8) The impedance error is contaminated by measurement noise in the following way:

$$\hat{w}_k(t) = w_k(t) + n_k(t),$$

where $n_k(t) \in R^m$ is bounded by a constant b_n for $t \in [0, T]$.

Taking postulate (A'8) into consideration, the iterative learning-control input $m_k(t)$ for learning the target impedance (eq. (9)) is described by the following equation:

$$m_{k+1}(t) = m_k(t) + L(q_k(t))(w_k(t) + n_k(t)), \quad (46)$$

where $L(\cdot) \in R^{n \times m}$ is the learning gain. The following theorem states the robustness result for the learning-impedance control system.

THEOREM 2. Consider the learning-impedance control system described by eqs. (45), (5), (6), (46), and (13) with bounded measurement noise, dynamics fluctuations, and error in initial conditions. Let $L(\cdot)$ be any bounded learning gain that satisfies condition (17) in Theorem 1. Then, the impedance error due to the control $m_k(t)$ is bounded such that for all $t \in [0, T]$

$$\lim_{k\to\infty} \| w_k \|_\alpha \le \hat{c}_1 b_{q0} + \hat{c}_2 b_n + \hat{c}_3 b_d, \quad (47)$$

where $\hat{c}_1 \cdots \hat{c}_3$ are constants that will be defined later.

Proof. From eqs. (46), (23), (3), (4), (5), (6), (25), and (45), we have

$$\begin{aligned} \delta m_{k+1} = {} & [I - L(q_k)(M_m + M_E(X_k))J(q_k)(M(q_k) \\ & + J^T(q_k)M_E(X_k)J(q_k))^{-1}]\delta m_k \\ & - L(q_k)(M_m + M_E(X_k))\{\delta J(q_k)\ddot{q}_e \\ & + \dot{J}(q_k)\delta\dot{q}_k + \delta\dot{J}(q_k)\dot{q}_e \\ & - J(q_k)R^{-1}(q_k)(\delta\bar{N}(q_k, \dot{q}_k, t) \\ & - J^T(q_k)\delta k_E(X_k, \dot{X}_k, t) \\ & - \delta J^T(q_k)k_E(X_e, \dot{X}_e, t)) \\ & - J(q_k)\delta R^{-1}(q_k)(\bar{N}(q_e, \dot{q}_e, t) \\ & - J^T(q_e)k_E(X_e, \dot{X}_e, t) + m_e) \\ & - R^{-1}(q_k)d_k\} + L(q_k)(C_m(J(q_k)\delta\dot{q}_k \\ & + \delta J(q_k)\dot{q}_e) + K_m\delta h(q_k) \\ & + \delta M_E(X_k)\ddot{X}_e - \delta k_E(X_k, \dot{X}_k, t) + n_k). \end{aligned} \quad (48)$$

Using a similar argument to the one used in eq. (30), we can show from eq. (48) that

$$\| \delta m_{k+1} \| \le p \| \delta m_k \| + b_L c_1 \left\| \begin{bmatrix} \delta q_k \\ \delta\dot{q}_k \end{bmatrix} \right\| + b_L b_R b_d + b_L b_n. \quad (49)$$

Similar to the derivation of eq. (32), from eq. (45), we obtain

$$\left\| \begin{bmatrix} \delta q_k \\ \delta\dot{q}_k \end{bmatrix} \right\| \le \left\| \begin{bmatrix} \delta q_k(0) \\ \delta\dot{q}_k(0) \end{bmatrix} \right\| + \int_0^t \{c_2 \left\| \begin{bmatrix} \delta q_k \\ \delta\dot{q}_k \end{bmatrix} \right\| + b_g\|\delta m_k\| + b_g b_d\} d\tau. \quad (50)$$

Using the Bellman-Gronwall lemma (Flett 1980), we obtain

$$\left\| \begin{bmatrix} \delta q_k \\ \delta\dot{q}_k \end{bmatrix} \right\| \le \left\| \begin{bmatrix} \delta q_k(0) \\ \delta\dot{q}_k(0) \end{bmatrix} \right\| e^{c_2 t} + b_g \int_0^t e^{c_2(t-\tau)}(\| \delta m_k \| + b_d) d\tau. \quad (51)$$

Substituting eq. (51) into eq. (49) and multiplying both sides by $e^{-\alpha t}$, then defining $c \triangleq \max\{c_2, b_L b_g c_1\}$ while letting $\alpha > c_2$, we get

$$\begin{aligned} e^{-\alpha t}\|\delta m_{k+1}\| \quad \le \quad & p e^{-\alpha t}\|\delta m_k\| \\ & + b_L c_1 e^{(c_2-\alpha)t} \left\| \begin{bmatrix} \delta q_k(0) \\ \delta \dot{q}_k(0) \end{bmatrix} \right\| \\ & + c \int_0^t e^{-\alpha\tau}\|\delta m_k\| e^{(c-\alpha)(t-\tau)} d\tau \\ & + b_L c_1 b_g b_d \int_0^t e^{-\alpha\tau} e^{(c_2-\alpha)(t-\tau)} d\tau \\ & + b_L b_R b_d e^{-\alpha t} + b_L b_n e^{-\alpha t}, \end{aligned} \tag{52}$$

for all $t \in [0, T]$. Hence, we have

$$\| \delta m_{k+1} \|_\alpha \quad \le \quad \hat{p} \, \| \delta m_k \|_\alpha + \varepsilon, \tag{53}$$

where $\hat{p} = [p + \frac{c}{\alpha - c}(1 - e^{-(\alpha - c)T})]$, $\varepsilon = b_L c_1 b_{q0} + c_3 b_d + b_L b_n$, and $c_3 = \frac{b_L b_g c_1}{\alpha - c_2}(1 - e^{-(\alpha - c_2)T}) + b_L b_R$. If we choose p to be less than 1, and $\alpha > c$ and large enough so that $0 \le \hat{p} < 1$, then eq. (53) converges such that the α-norm of the error between $m_k(t)$ and $m_e(t)$ converges to a neighborhood of radius $(\frac{1}{1-\hat{p}})\varepsilon$ for $t \in [0, T]$; i.e.,

$$\lim_{k\to\infty} \| \delta m_k \|_\alpha \le (\frac{1}{1-\hat{p}})\varepsilon. \tag{54}$$

Similar to eqs. (39) and (40), we have

$$\lim_{k\to\infty} \left\| \begin{bmatrix} \delta q_k \\ \delta \dot{q}_k \end{bmatrix} \right\|_\alpha \quad \le \quad c_4 b_{q0} + c_5 b_n + c_6 b_d, \tag{55}$$

$$\lim_{k\to\infty} \left\| \begin{bmatrix} \delta \dot{q}_k \\ \delta \ddot{q}_k \end{bmatrix} \right\|_\alpha \quad \le \quad c_8 b_{q0} + c_9 b_n + c_{10} b_d, \tag{56}$$

where $c_4 = 1 + \frac{b_g b_L c_1 c_7}{1-\hat{p}}$, $c_5 = \frac{b_g b_L c_7}{1-\hat{p}}$, $c_6 = c_7 + \frac{b_g c_3 c_7}{1-\hat{p}}$, $c_7 = \frac{1}{\alpha - c_2}(1 - e^{(c_2-\alpha)T})$, $c_8 = c_2 c_4 + \frac{b_g b_L c_1}{1-\hat{p}}$, $c_9 = c_2 c_5 + \frac{b_g b_L}{1-\hat{p}}$, and $c_{10} = c_2 c_6 + b_g + \frac{b_g c_3}{1-\hat{p}}$. Therefore, we can write

$$\lim_{k\to\infty} \sup \| w_k \|_\alpha \quad \le \quad \hat{c}_1 b_{q0} + \hat{c}_2 b_n + \hat{c}_3 b_d, \tag{57}$$

where $\hat{c}_1 = c_{12}c_4 + c_{11}c_8$, $\hat{c}_2 = c_{12}c_5 + c_{11}c_9$, $\hat{c}_3 = c_{12}c_6 + c_{11}c_{10}$, $c_{11} = (b_{Mm} + b_{ME})b_J$, and $c_{12} = (b_{Mm} + b_{ME})(c_J b_{q2} + b_{J1} + c_{J1} b_{q1}) + (b_{Cm} + b_{KE})(b_J + c_J b_{q1}) + (b_{Km} + c_{ME} b_{x2} + c_{kE})c_h$. □

REMARK 6. Equation (57) shows the dependence of the bounds of $w_k(t)$ on the bounds of the errors in the initial conditions, disturbances, and measurement noises. Note that if b_{q0}, b_d, and b_n tend to zero, then the impedance error tends to zero.

5. Experimental Studies

In a practical robot system, many disturbances are present. Although the robustness analysis of the learning-control system to certain practical issues is developed, implementing the proposed learning scheme in real-time experiments allows for the investigation of the feasibility of the actual implementations and the robustness of the proposed scheme. In this section, the proposed learning-impedance controller is applied to an industrial robot, and experimental results are presented and discussed.

5.1. The Experimental Setup

The robot used in this experiment is the industrial robot SEIKO TT3000, as shown in Figure 1. This robot is of the Selective Compliance Assembly Robot Arm (SCARA)-type manipulator with three degrees of freedom, as illustrated in the schematic diagram of Figure 2. The first joint is a prismatic joint, and the second and third joints are revolute joints.

The dynamics model of the robotic arm (Lewis, Abdallah, and Dawson 1993) can be described by eq. (2). The parameters of this SCARA robot are as follows:

$$M(q) = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}, \quad V(q, \dot{q}) = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix},$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}, \quad f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix},$$

where

$$\begin{aligned} m_{11} &= m_1 + m_2 + m_3, \\ m_{12} &= 0, \\ m_{13} &= 0, \\ m_{22} &= (m_2 + m_3)a_2^2 + m_3 a_3^2 + 2m_3 a_2 a_3 \cos(\theta_3), \end{aligned}$$

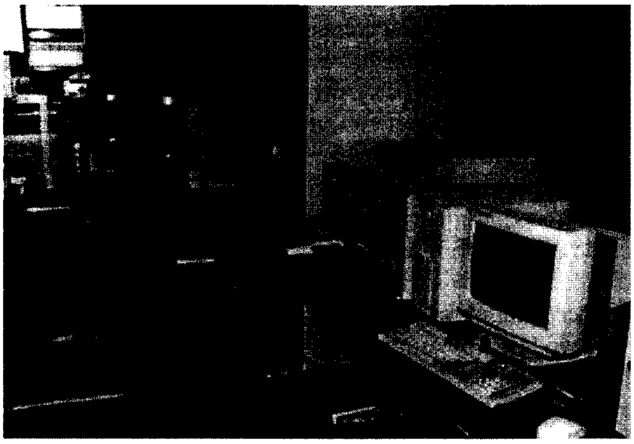

Fig. 1. The experimental setup.

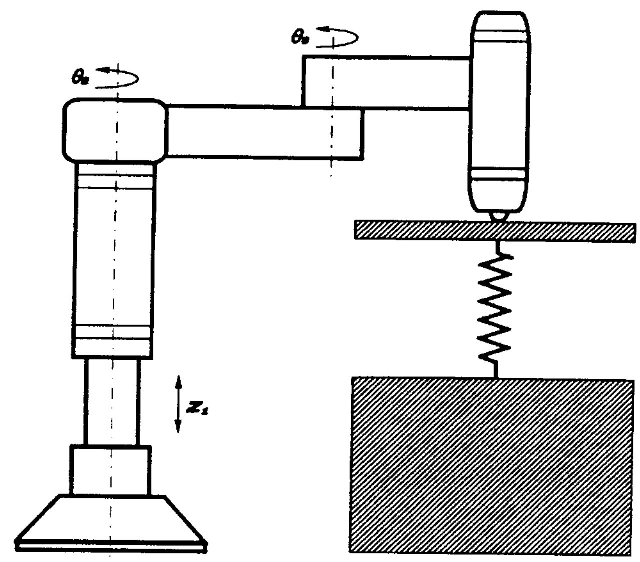

Fig. 2. A SCARA robot.

$$
\begin{aligned}
m_{23} &= m_3 a_3^2 + m_3 a_2 a_3 \cos(\theta_3),\\
m_{33} &= m_3 a_3^2,\\
v_1 &= m_{11} g,\\
v_2 &= -m_3 a_2 a_3 \sin(\theta_3)(\dot{\theta}_3^2 + 2\dot{\theta}_2\dot{\theta}_3),\\
v_3 &= m_3 a_2 a_3 \sin(\theta_3)\dot{\theta}_2^2,
\end{aligned}
\tag{58}
$$

and m_1, m_2, and m_3 are the masses of links 1, 2, and 3, re-pectively (in kilograms), a_2 and a_3 are the lengths of links 2 nd 3, respectively (in meters), and g is the constant acceler-tion due to gravity (in meters per second per second). The ierarchical structure of the robot-control system is shown in igure 3. At the top of the system hierarchy is the robot su-ervisory computer using a 486 PC,and at the lower level are nultiprocessors using a VME bus-based system. The super-isory computer is mainly used for task planning and high-evel programming. The lower level is used for real-time data ollection and control. This VME bus-based system consists f the host MVME 147 computer and the target MVME104 omputer. The MVME 147 computer is an MC68030-based ystem with 4 MB of DRAM and a 25-MHz system clock; ne MVME104 computer is an MC68010-based system with 12 kB of RAM and a 10-MHz system clock. The MVME104 omputer is also responsible for input and output operations sing encoder-input ports and digital-to-analog converters. hree encoders are employed for position measurements of ach joint, and a differentiator is used to estimate the velocity nd acceleration from the position measurements. The pulse er revolution measurements for encoders 2 and 3 are 600 nd 800, respectively. For the prismatic joint, one pulse cor-esponds to a displacement of 0.01044 mm. To measure the ontact force, a force sensor made by the Lord Corporation is nounted on the end effector of the robot.

5.2. Experimental Results

Experimental investigation was conducted on the SEIKO TT3000 industrial robot. To effectively verify the proposed control laws, the end effector was set to follow a path that involved free-motion tracking, transition from free motion to contact motion, contact motion on the constraint plane with compliance, transition from contact motion to free motion, and finally, free-motion tracking again. The path of the end effector is illustrated in Figure 4. The joint space is chosen as the task space, since the contact task in this experiment can be conveniently described by joint axis 1 (or the z-axis), as shown in Figures 2 and 4. Therefore,

$$
X(t) = \begin{bmatrix} z_1(t) \\ \theta_2(t) \\ \theta_3(t) \end{bmatrix}. \tag{59}
$$

Mathematically, the task can be specified by the target impedance (eq. (9)) as follows:

$$
M_m = \begin{bmatrix} 50 & & \\ & 40 & \\ & & 40 \end{bmatrix} \; C_m = \begin{bmatrix} 200 & & \\ & 200 & \\ & & 200 \end{bmatrix}
$$
$$
K_m = \begin{bmatrix} 800 & & \\ & 1000 & \\ & & 1000 \end{bmatrix}, \tag{60}
$$

where the reference trajectories $X_d(t) = [z_{1d}^T(t), \theta_{2d}^T(t)$, and $\theta_{3d}^T(t)]^T$ are described by

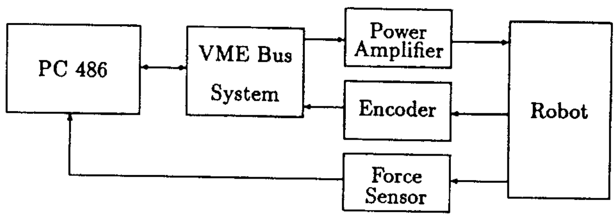

Fig. 3. A block diagram of the experimental system.

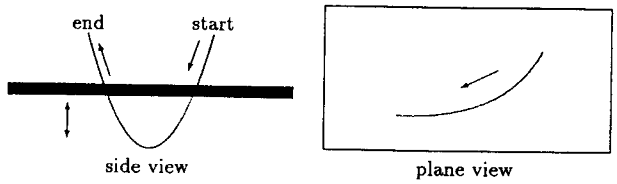

Fig. 4. The end-effector path.

$$z_{1d}(t) = \begin{cases} -0.045(\frac{6f_s^5}{1500^5}t^5 - \frac{15f_s^4}{1500^4}t^4 + \frac{10f_s^3}{1500^3}t^3) \\ \text{for } 0 \le t < \frac{1500}{f_s} \\ -0.045 + 0.045(\frac{6f_s^5}{1500^5}t^5 - \frac{15f_s^4}{1500^4}t^4 \\ +\frac{10f_s^3}{1500^3}t^3) \\ \text{for } \frac{1500}{f_s} \le t < \frac{3000}{f_s} \\ 0 \\ \text{for } \frac{3000}{f_s} \le t \le \frac{3600}{f_s} \end{cases}$$

$$\theta_{2d}(t) = \begin{cases} 2.017 + 0.6(\frac{6f_s^5}{3000^5}t^5 - \frac{15f_s^4}{3000^4}t^4 \\ +\frac{10f_s^3}{3000^3}t^3) \\ \text{for } 0 \le t < \frac{3000}{f_s} \\ 2.617 \\ \text{for } \frac{3000}{f_s} \le t \le \frac{3600}{f_s} \end{cases}$$

$$\theta_{3d}(t) = \begin{cases} 1.885 - 0.5(\frac{6f_s^5}{3000^5}t^5 - \frac{15f_s^4}{3000^4}t^4 \\ +\frac{10f_s^3}{3000^3}t^3) \\ \text{for } 0 \le t < \frac{3000}{f_s} \\ 1.385 \\ \text{for } \frac{3000}{f_s} \le t \le \frac{3600}{f_s}. \end{cases} \tag{61}$$

Here, M_m is specified in N s^2/m for joint 1 and N ms^2/rad for joints 2 and 3; C_m is specified in N s/m for joint 1 and N ms/rad for joints 2 and 3; K_m is specified in Newtons per meter for joint 1 and NM/rad for joints 2 and 3; $z_{1d}(t)$ is specified in meters, and $\theta_{2d}(t)$ and $\theta_{3d}(t)$ are specified in radians. The sampling frequency f_s was chosen to be 244 Hz, and the period T of the whole operation was 3,600/f_s sec. In this experiment, a steel ball was attached to the force sensor, and hence the frictional force along the constraint plane was negligible. In other words,

$$F = \begin{bmatrix} f_1(t) \\ 0 \\ 0 \end{bmatrix}. \tag{62}$$

The impedance learning-control law described by eqs. (7), (16), and (13) is applied to the robotic system with the controller gains chosen as follows:

$$K_p = \begin{bmatrix} 10 & & \\ & 10 & \\ & & 250 \end{bmatrix}, \quad K_v = \begin{bmatrix} 12 & & \\ & 12 & \\ & & 100 \end{bmatrix},$$

$$L = \begin{bmatrix} 2 & & \\ & 2 & \\ & & 60 \end{bmatrix}. \tag{63}$$

The impedance error was calculated as

$$\begin{aligned} w_k(t) = &-M_m\ddot{X}_k(t) + C_m(\dot{X}_d(t) - \dot{X}_k(t)) \\ &+ K_m(X_d(t) - X_k(t)) + F_k(t), \end{aligned} \tag{64}$$

and the experimental results of the impedance errors, the reference trajectory errors $(X_d - X_k)$,and the contact force $f_1(t)$ are shown in Figures 5–11. In the first trial, i.e., $k = 0$, $m_0(t)$ was set to zero for all $t \in [0, T]$; hence, the controller was a PD-feedback law with no learning control. As the operation was repeated, the impedance errors decreased, as shown in Figures 5, 6, and 7. Note that the impedance error for joint 1 decreased even though the contact points from free motion to and from contact motion were changing at every iteration, as seen from the contact force in Figure 11. From Figures 8, 9, and 10, the results also show that the reference-trajectory errors decreased when the impedance errors decreased. It should be noted that in Figure 8, the reference-trajectory error for joint 1 converges to a steady-state value described by eq. (42) in the presence of contact force. The experimental results illustrate the validity of the theoretical developments, and show that the learning controller reduces the impedance error tremendously. These results also illustrate the superiority of the proposed learning-control scheme as compared to no learning control (on the first trial).

In the experiments, the measurements, joint positions, and force are processed for on-line feedback as well as for off-line learning. The on-line feedback controller (eq. (7)) is a PD controller that is used for stabilization. This on-line controller only requires velocity calculation from the measured joint positions. The off-line learning controller (eq. (16)) involves the computation of impedance error defined by eq. (13), which requires the measured force as well as the computations of acceleration. Although second numerical differentiations are required, off-line computation makes it possible to filter out the severe noises. In our experiments, the numerical calculations of velocity and acceleration serve well for the off-line learning-control updates by using an off-line FFT filter (Zhang 1996). Furthermore, because of the presence of noise, including that from the numerical differentiations, the impedance errors do not converge to zeros, as shown in Figures 5, 6, and 7. This is expected from the results of Theorem 2, where the noise disturbances appear as constants on the right-hand side of eq. (47).

6. Conclusion

A learning-control algorithm was developed for impedance control of robotic manipulators. Given a target impedance, the learning controller was able to learn and eventually drive

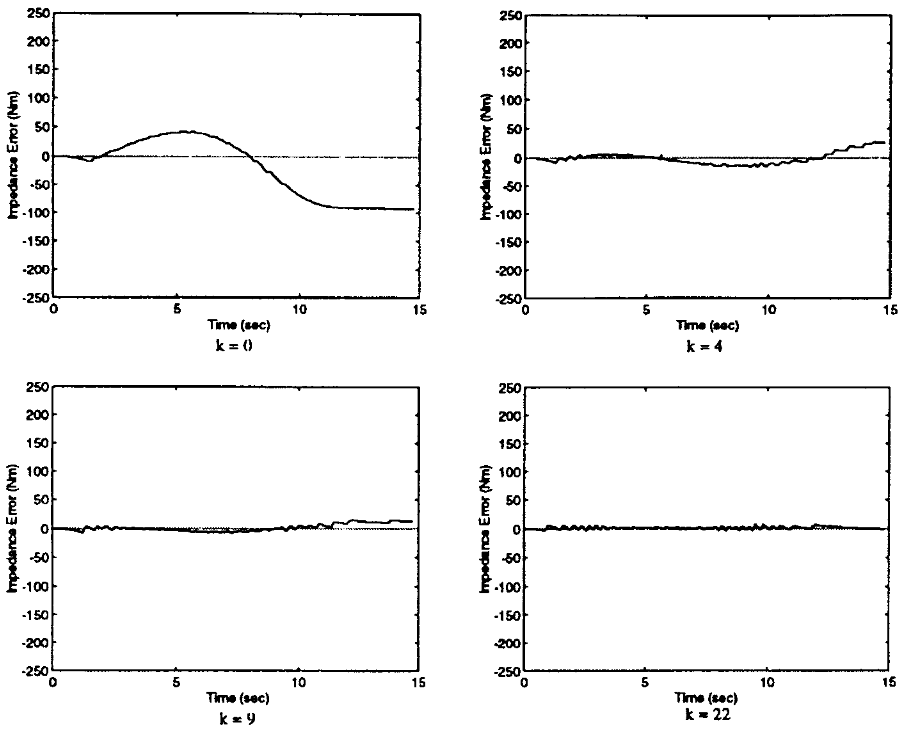

Fig. 7. The impedance error of joint 3.

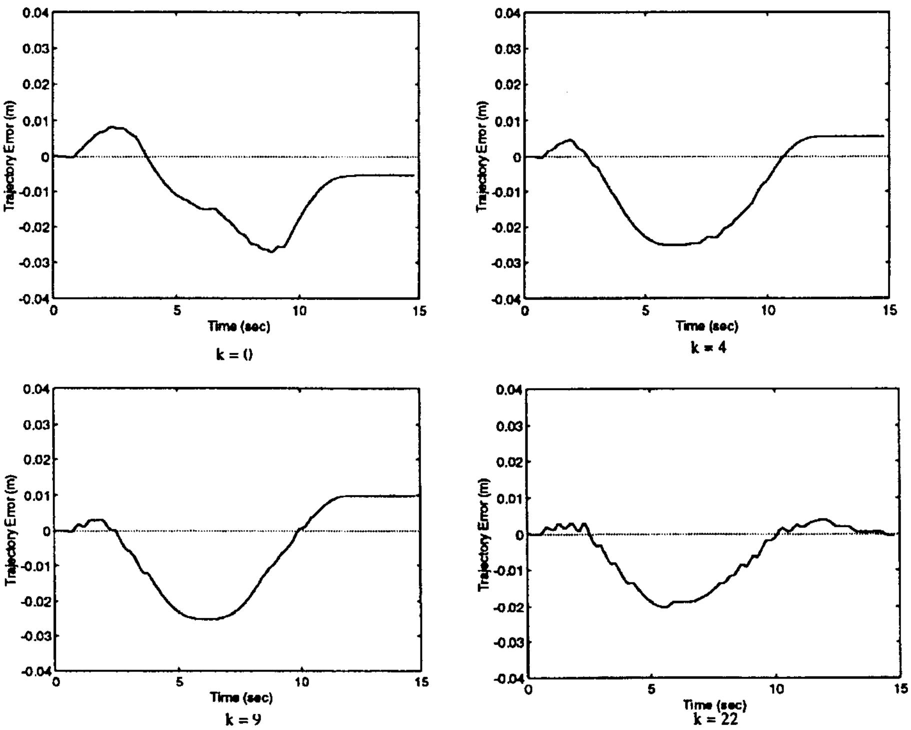

Fig. 8. The reference trajectory error of joint 1.

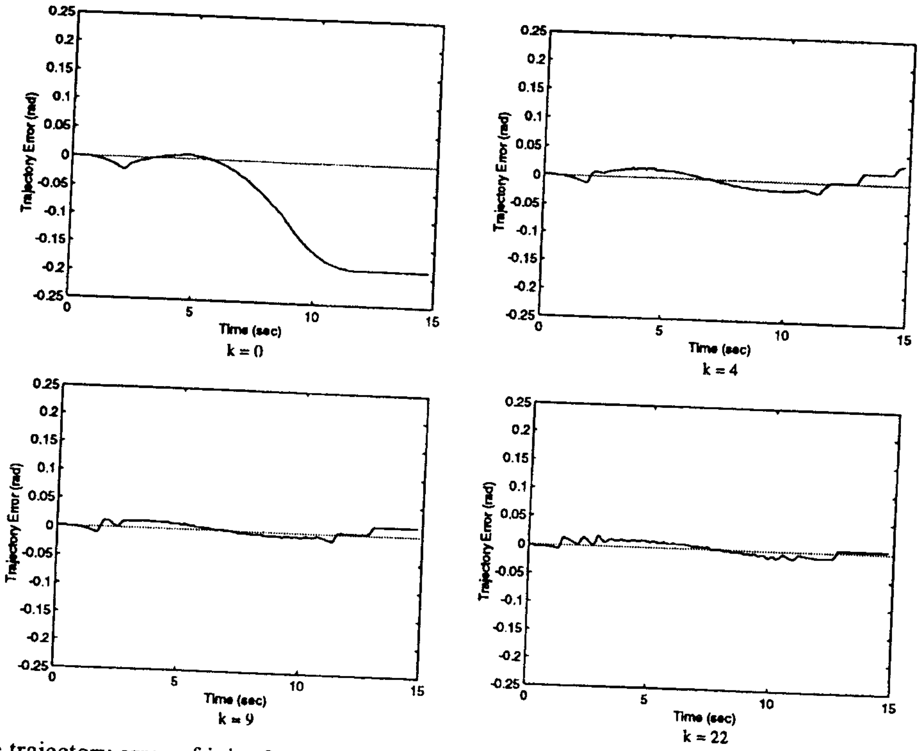

Fig. 9. The reference trajectory error of joint 2.

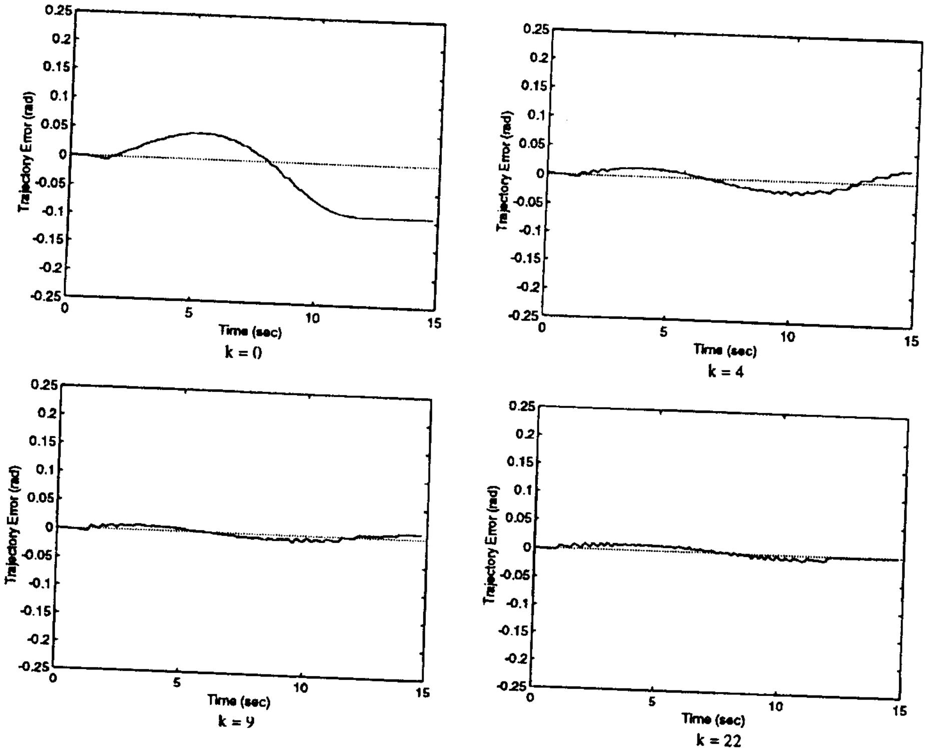

Fig. 10. The reference trajectory error of joint 3.

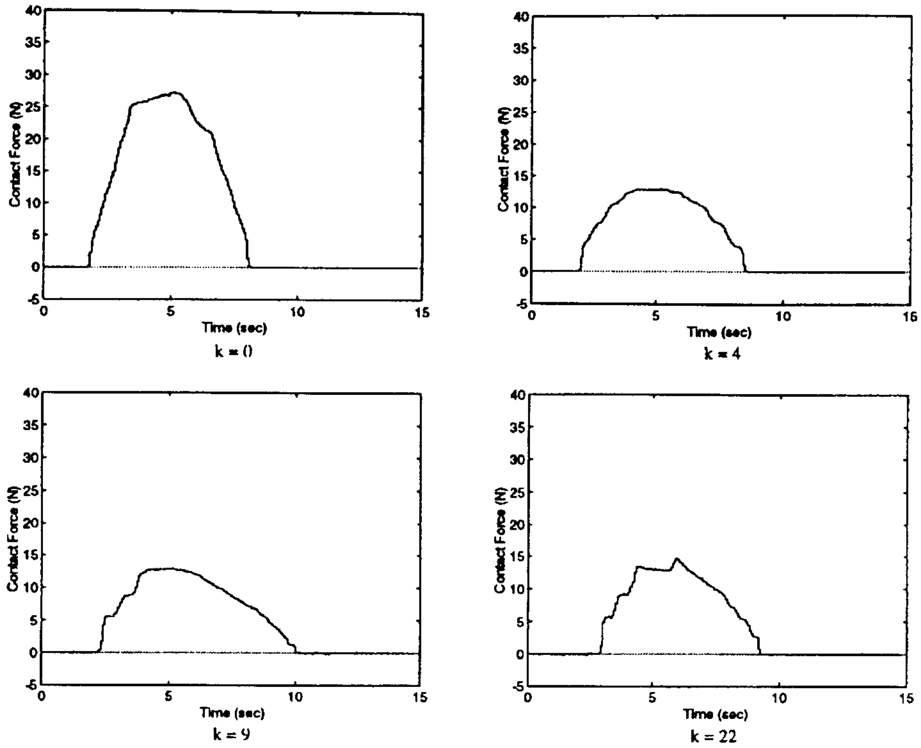

Fig. 11. The contact force.

the closed-loop response to reach the target impedance as the operations were repeated. A sufficient condition for guaranteeing the convergence of the proposed learning-impedance control scheme was derived. The robustness of the proposed algorithm was analyzed in the presence of error in the initial conditions, dynamics fluctuations, and measurement noise. The learning impedance-control scheme was successfully applied to an industrial SEIKO TT3000 robot, and the experimental results verified the theoretical developments.

References

Aicardi, M., Cannata, G., and Gasalino, G. 1992. Hybrid learning control for constrained manipulators. *Adv. Robot.* 6(1):69–94.

Arimoto, S. 1990. Learning control theory for robot motion. *Int. J. Adaptive Control Signal Processing* 4:543–564.

Arimoto, S. 1991. Learning for skill refinement in robotic systems. *IEICE Trans. Fundamentals* E74(2):235–243.

Arimoto, S. 1992. Passivity and learnability for mechanical systems—a learning control theory for skill refinement. *IEICE Trans. Fundamentals* E75-A(5):552–560.

Arimoto, S., Kawamura, S., and Miyazaki, F. 1984. Bettering operation of robots by learning. *J. Robot. Sys.* 1(2):440–447.

Arimoto, S., Liu, Y. H., and Naniwa, T. 1993 (Sydney, Australia). Principle of orthogonalization for hybrid control of robot arms. *Proc. of the IFAC World Congress.* Sydney, Australia: IFAC, vol. 1, pp. 507–512.

Bondi, P., Casalino, G., and Gambardella, L. 1988. On the iterative learning control theory for robotic manipulators. *IEEE J. Robot. Automat.* 4:14–22.

Cheah, C. C., and Wang, D. 1994 (Baltimore, MD). Model reaching learning control for a class of nonlinear systems. *Proc. of the American Control Conf.* Baltimore, MD: American Automatic Control Council, pp. 882–886.

Cheah, C. C., and Wang, D. 1995 (Nagoya, Japan). Learning impedance control for robotic manipulators. *Proc. of the IEEE Int. Conf. on Robot. and Automat.* Washington, DC: IEEE, pp. 2150–2155.

Cheah, C. C., Wang, D., and Soh, Y. C. 1992. Learning control of constrained robots. *Proc. IEEE Int. Symp. on Intell. Control.* Los Alamitos, CA: IEEE, pp. 91–96.

Cheah, C. C., Wang, D., and Soh, Y. C. 1994. Convergence and robustness of a discrete-time learning control scheme for constrained manipulators. *J. Robot. Sys.* 11(3):223–238.

Craig, J. J. 1984 (San Diego, CA). Adaptive control of manipulators through repeated trials. *Proc. of the American Control Conf.* Baltimore, MD: American Automatic Control Council, pp. 1566–1573.

Craig, J. J. 1988. *Adaptive Control of Mechanical Manipulators.* Reading, MA: Addison-Wesley.

Flett, T. M. 1980. *Differential Analysis.* Cambridge: Cambridge University Press.

Heinzinger, G., Fenwick, D., Paden, B., and Miyazaki, F. 1992. Stability of learning control with disturbances and uncertain initial conditions. *IEEE Trans. Automat. Control* 137(1):110–114.

Hogan, N. 1985. Impedance control: An approach to manipulation, part i, part ii, part iii. *ASME J. Dyn. Sys. Meas. Control* 107:1–24.

Jeon, D., and Tomizuka, M. 1993. Learning hybrid force and position control of robot manipulators. *IEEE Trans. Robot. Automat.* 9(4):423–431.

Kawamura, S., Miyazaki, F., and Arimoto, S. 1985 (Tokyo). Hybrid position/force control of manipulators based on learning method. *Proc. Int. Conf. on Adv. Robot.*, pp. 235–242.

Lewis, F. L., Abdallah, C. T., and Dawson, D. M. 1993. *Control of Robot Manipulators.* New York: MacMillan.

Spong, M. W., Lewis, F. L., and Abdallah, C. T. 1993. *Robot Control: Dynamics, Motion Planning, and Analysis.* Los Alamitos, CA: IEEE.

Sugie, T., and Ono, T. 1991. An iterative learning control law for dynamical systems. *Automatica* 27(4):729–732.

Uchiyama, M. 1978. Formation of high-speed motion pattern of a mechanical arm by trial. *Trans. SICE* 21(6):706–712.

Wang, D., and Cheah, C. C. 1996 (Montpelier, France). A robust learning control scheme for manipulators with target impedance at end effectors. *Proc. of the World Automat. Congress.* Montpellier, France: ASME, vol. 6, pp. 851–856.

Yao, B., Chan, S. P., and Wang, D. 1992 (Chicago, IL). A unified approach to variable structure control of robot manipulators. *Proc. of the American Control Conf.* Chicago, IL: American Automatic Control Council, pp. 1282–1286.

Zhang, N. 1996. Experimental studies of robot control. Master's thesis, Nanyang Technological University, Singapore.